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CHRISTO ET ECCLESIÆ

E. P. Clark.

Boston September 6th 1823.

The Gift of
Joseph Horace Clark,
of Cambridge,
(H. C. 1857,)
7 Feb., 1860.

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E. P. Clark.

Boston September 6th 1823.

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A

NEW AND COMPLETE

SYSTEM OF ARITHMETICK.

COMPOSED

FOR THE USE OF THE CITIZENS

OF THE

UNITED STATES.

BY NICOLAS PIKE, A. M. A. A. S.

QUID MUNUS REIPUBLICÆ MAJUS MELIUSVE AFFERRE POSSUMUS, QUAM SI JUVENTUTEM DOCEMUS, ET BENE ERUDIMUS?
—— E VARIIS SUMENDUM EST OPTIMUM......CICERO.

FOURTH EDITION;
REVISED, CORRECTED, AND IMPROVED,
BY CHESTER DEWEY, A. A. S.
PROFESSOR OF MATHEMATICKS, AND NATURAL PHILOSOPHY IN WILLIAMS COLLEGE.

TROY, N. Y.

PRINTED AND PUBLISHED BY WM. S. PARKER,
SOLD AT THE TROY BOOKSTORE, AND BY THE PRINCIPAL BOOKSELLERS IN THE UNITED STATES.
....

1822.

PREFACE

TO THE FIRST EDITION.

It may, perhaps, by some, be thought needless, when Authors are so multiplied, to attempt publishing any thing further on Arithmetick, as it may be imagined there can be nothing more than the repetition of a subject already exhausted. It is however the opinion of not a few, who are conspicuous for their knowledge in the Mathematicks, that the books, now in use among us, are generally deficient in the illustration and application of the rules; of the truth of which, the general complaint among Schoolmasters is a strong confirmation. And not only so, but as the United States are now an independent nation, it was judged that a System might be calculated more suitable to our meridian, than those heretofore published.

Although I had sufficient reason to distrust my abilities for so arduous a task, yet not knowing any one who would take upon himself the trouble, and apprehending I could not render the publick more essential service, than by an attempt to remove the difficulties complained of, with diffidence I devoted myself to the work.

I have availed myself of the best authors which could be obtained but have followed none particularly, except Bonnycastle's Method of Demonstration.

Although I have arranged the work in such order as appeared to me the most regular and natural, the student is not obliged to pay a strict adherence to it; but may pass from one Rule to another, as his inclination or opportunity for study, may require.

The Federal Coin, being purely decimal, most naturally falls in after Decimal Fractions.

I have given several methods of extracting the Cube Root, and am indebted to a learned friend, who declines having his name made publick, for the investigation of two very concise Algebraick Theorems for the extraction of all Roots, and of a particular Theorem for the Sursolid.

Among the Miscellaneous Questions, I have given some of a philosophical nature, as well with a view to inspire the pupil with a relish for philosophical studies, as to the usefulness of them in the common business of life.

Being sensible the following Treatise will stand or fall, according to its real merit or demerit, I submit it to the judgment of the candid.

With pleasure I embrace this opportunity, to express my gratitude to those learned Gentlemen, who have honoured this Treatise with their approbation, as well as to such Gentlemen, as have encouraged it by their subscriptions; and to request the reader to excuse any errours he may meet with; for although great pains have been taken in correcting, yet it is difficult to prevent errours from creeping into the press, and some may have escaped my own observation; in either case, a hint from the candid will much oblige their

Most obedient,

And humble Servant,

THE AUTHOR.

PREFACE

TO THE FOURTH EDITION.

PIKE'S ARITHMETICK is universally acknowledged to be the most complete system ever published in the United States. It early obtained a very high reputation, and has continued to receive the approbation of the publick, wherever it has been used. It is designed for the instruction of our youth in academies and higher schools, as well as for the use of the man of business and the gentleman. All those rules, which are so frequently employed in the various departments of business, are introduced into it. It is the source too, from which the later Arithmeticks have chiefly been compiled. By them, however, it has not been superseded, so much more full and extensive are its rules and their application. In the demonstration and illustration of the rules, it stands preeminent.

The continued demand for the work has induced the publisher and proprietor of the copy right, to present to the publick a new and improved edition. In the revision of the work much labour has been bestowed, and in the language of a Mathematician well acquainted with the work, "to excellent purpose. It is still Pike's Arithmetick, but altogether more perfect than it was before. As a complete system, it may be pronounced superior to any ever published." The imperfections of the previous editions, which have been noticed by the most distinguished teachers of Arithmetick, are to a great degree remedied in the present edition.

The alterations and improvements consist in the following particulars. Several rules have been added, as well as a variety of Tables, of much practical importance. Some Tables have been corrected and others have been enlarged. Several simple and obvious rules were redundant and have been omitted. The Rule of Three and Interest have been much improved. Demonstrations of a large proportion of the rules were not given by Mr. Pike: where the subject would readily admit, they have been supplied. The illustrations of the Rules are more copious, and in many cases simplified. Most of the Algebraick demonstrations, which are useless to the mere student in Arithmetick, have been exchanged for arithmetical illustrations. Logarithms, Trigonometry, Algebra, and Conic Sections, are omitted. These subjects were so briefly treated by Mr. Pike, as to possess little value. As they require a large volume of themselves, and are very fully treated of in Day's Course of Mathematicks, and in the system of Mathematicks now publishing at the University in Massachusetts, the publisher has been uniformly advised to omit them entirely.

A concise System of Book Keeping by single and double Entry has been added to the work, which we hesitate not to say will greatly enhance its value.

It is confidently believed that this edition will merit the approbation of the publick, and receive that patronage which has been so liberally bestowed on the previous editions.

THE PUBLISHER.

TROY, OCTOBER 31, 1822.

RECOMMENDATIONS.

DARTMOUTH UNIVERSITY, 1786.

AT the request of Nicolas Pike, Esq. we have inspected his System of Arithmetick, which we cheerfully recommend to the publick, as easy, accurate, and complete. And we apprehend there is no treatise of the kind extant, from which so great utility may arise to Schools.

B. WOODWARD, Math. and Phil. Prof.
JOHN SMITH, Prof. of the Learned Languages.

I do most sincerely concur in the preceding recommendation.

J. WHEELOCK, President of the University.

PROVIDENCE, RHODE ISLAND, 1785.

WHOEVER may have the perusal of this treatise on Arithmetick may naturally conclude I might have spared myself the trouble of giving it this recommendation, as the work will speak more for itself than the most elaborate recommendation from my pen can speak for it: But as I have always been much delighted with the contemplation of mathematical subjects, and at the same time fully sensible of the utility of a work of this nature, was willing to render every assistance in my power to bring it to the publick view: And should the student read it with the same pleasure with which I perused the sheets before they went to the press, am persuaded he will not fail of reaping that benefit from it which he may expect, or wish for, to satisfy his curiosity in a subject of this nature. The author, in treating on numbers, has done it with so much perspicuity and singular address, that I am convinced the study thereof will become more a pleasure than a task.

The arrangement of the work, and the method by which he leads the *tyro* into the first principles of numbers, are novelties I have not met with in any book I have seen. Wingate, Hatton, Ward, Hill, and many other authors whose names might be adduced, if necessary, have claimed a considerable share of merit; but when brought into a comparative point of view with this treatise, they are inadequate and defective. This volume contains, besides what is useful and necessary in the common affairs of life, a great fund for amusement and entertainment. The Mechanick will find in it much more than he may have occasion for; the Lawyer, Merchant and Mathematician, will find an ample field for the exercise of their genius; and I am well assured it may be read to great advantage by students of every class, from the lowest school to the University. More than this need not be said by me, and to have said less, would be keeping back a tribute justly due to the merit of this work.

BENJAMIN WEST.

UNIVERSITY IN CAMBRIDGE, 1786.

HAVING, by the desire of Nicolas Pike, Esq. inspected the following volume in manuscript, we beg leave to acquaint the publick, that in our opinion it is a work well executed, and contains a complete system of Arithmetick. The rules are plain, and the demonstrations perspicuous and satisfactory; and we esteem it the best calculated, of any single piece we have met with, to lead youth, by natural and easy gradations, into a methodical and thorough acquaintance with the science of figures. Persons of all descriptions may find in it every thing, respecting numbers, necessary to their business; and not only so, but if they have a speculative turn, and mathematical taste, may meet with much for their entertainment at a leisure hour.

We are happy to see so useful an American production, which, if it should meet with the encouragement it deserves, among the inhabitants of the United States, will save much money in the country, which would otherwise be sent to Europe, for publications of this kind.

We heartily recommend it to schools, and to the community at large, and wish that the industry and skill of the Author may be rewarded, for so beneficial a work, by meeting with the general approbation and encouragement of the publick.

JOSEPH WILLARD, D. D. President of the University.
E. WIGGLESWORTH, S. T. P. Hollis.
S. WILLIAMS, L. L. D. Math. et Phil. Nat. Prof. Hollis.

YALE COLLEGE, 1786.

UPON examining Mr. Pike's System of Arithmetick and Geometry, in manuscript, I find it to be a work of such mathematical ingenuity, that I esteem myself honoured in joining with the Rev. President Willard, and other learned gentlemen, in recommending it to the publick as a production of genius, interspersed with originality in this part of learning, and as a book, suitable to be taught in schools: of utility to the merchant, and well adapted even for the University instruction. I consider it of such merit, as that it will probably gain a very general reception and use throughout the republick of letters.

EZRA STILES, President.

BOSTON, 1786.

FROM the known character of the Gentlemen who have recommended Mr. Pike's System of Arithmetick, there can be no room to doubt, that it is a valuable performance; and will be, if published, a very useful one. I therefore wish him success in its publication.

JAMES BOWDOIN.

UNION COLLEGE, OCT. 10, 1822.

PIKE'S ARITHMETICK is too well known and too highly appreciated to require any recommendation; and by furnishing an edition of that work, in which common language is substituted for algebraic signs, Professor Dewey has conferred a favour on those who may wish to acquire or teach Arithmetick withou Algebra; by whom it is presumed this edition will be patronised.

E. NOTT, President.

SCHENECTADY, OCT. 16, 1822.

MR. WM. S. PARKER,

I HAVE for many years been fully acquainted with *Pike's System of Arithmetick*, and am persuaded of its excellence; I do not know of any treatise of more *practical* utility; the arrangements of its parts is natural, its rules are plain and easily understood and applied, and it contains all that is of any importance to the Mercantile or Scientific Arithmetician. To those who have not the elementary knowledge of Algebra, the translation of the Algebraic expression into plain Arithmetical language must be very acceptable and profitable. This improvement, together with the notes and emendations of Professor Dewey, cannot fail to ensure the public confidence and patronage. A hand so able as his, cannot touch without improving an elementary treatise, and wherever he is known, his name must be a sufficient credential.

Wishing you all success, and abundant remuneration for your labours, I am, Sir, your friend and servant.

T. M'AULEY, S. T. D.
Late Professor of Mathematicks, Union College.

AMHERST, MASS. FEB. 9, 1822.

I HAVE long been acquainted with Pike's Arithmetic, and think it the best of any extant, for those who wish to acquire a thorough knowledge of Arithmetic as a science and an art. The plan of improvement adopted and pursued by Professor DEWEY, in the present edition, is, in my opinion, such as to render the work more perfect and more useful. By supplying defects, omitting redundancies, and illustrating what was obscure, he has given to the present edition a superior value. I cheerfully recommend the work to the patronage of the public, and especially to the patronage of the Instructors of youth in Academies and Schools, as combining more excellences than any other Arithmetic now in use.

ZEPH. SWIFT MOORE,
President of the Collegiate Institution, at Amherst, Mass.

LENOX, MS. APRIL 20, 1822.

I HAVE seen Pike's Arithmetic revised by Mr. Professor DEWEY of Williams College. I entirely approve of all the alterations, additions, and illustrations. I cannot but believe, that the work thus presented to the public, will be superior to any thing of the kind extant. While it initiates the scholar into the theory of this science, it is distinguished for a happy conciseness, lucid method, and graceful simplicity, which cannot fail to make it a valuable companion for the Merchant, Mechanic, or Farmer.

LEVI GLEZEN,
Preceptor of Lenox Academy.

Extract of a Letter from Mr. Benedict, Tutor of Williams College, to the Publisher, dated

WILLIAMS COLLEGE, JANUARY 2, 1822.

Mr. PARKER,

"From the experience which I have had in instructing youth, I have had occasion to acquaint myself with many, if not most of the Systems of Arithmetick in use in this country. I can therefore speak with some more confidence than I otherwise should, from having proved their excellences and defects by actual trial of them. It is most certain that as a complete System on this important part of education, the work under consideration stands preeminent. It is impossible that Arithmetick should be so treated of, as not to leave much to be done by the instructor. Still, as I think, Pike's System will enable the teacher to benefit his scholars, to give them sound theoretical and practical knowledge in this branch, to induce them to think and reason closely, and increase their power of arithmetical invention, far more than any one within the compass of my knowledge. Excellent as it was when it came from its author, it had its defects. By the revision of it by Lord, little else was done than to change the sterling to federal notation. Much remained to be done. In some parts, Mr. Pike had been needlessly minute, and loaded the work with a multiplicity of rules on one subject, which the accountant could not but make for himself, as occasion demanded, with perfect ease. Though his illustrations and demonstrations are usually very good, in some cases they were obscure; and in some parts, as for instance that of interest, there was a great deficiency. I have examined the work with Mr. Dewey's corrections, with considerable care. He has bestowed great labour upon it, and I think to excellent purpose. It is still Pike's Arithmetick; but altogether more perfect than it was before. I do believe that as a complete System, it may be pronounced superior to any one ever published. I most earnestly wish you success in its publication; and I feel a confidence that good judges will not hesitate on perusing it, to give it an unqualified recommendation."

Yours respectfully,
GEORGE W. BENEDICT.

CONTENTS.

EXPLANATION

OF THE CHARACTERS MADE USE OF IN THIS TREATISE.

$=$ The sign of equality: as 12 pence $=$ 1 shilling, signifies that 12 pence are equal to one shilling; and, in general, that whatever precedes it is equal to what follows.

$+$ The sign of Addition: as $5+5=10$, that is, 5 added to 5 is equal to 10. Read 5 plus 5, or 5 more 5 equal to 10.

$-$ The sign of Subtraction: as, $12-4=8$, that is, 12 lessened by 4 is equal to 8, or 4 from 12 and 8 remains. Read 12 minus 4, or 12 less 4 equal to 8.

$\times$ The sign of Multiplication: as $6\times5=30$, that is, 6 multiplied by 5 is equal to 30. Read 6 into 5 equal to 30.

$\div$ or 5)30(The sign of Division: as $30\div5=6$, that is, 30 divided by 5 is equal to 6. Read 30 by 5 equal to 6.

$\frac{875}{25}$ Numbers placed fractionwise, do likewise denote division, the numerator or upper number being the dividend, and the denominator or lower number, the divisor; thus, $\frac{875}{25}$ is the same as $875\div25=35$.

: :: : The sign of proportion, thus, $2 : 4 :: 8 : 16$, that is, as 2 is to 4 so is 8 to 16.

$\div\!\div$ Signifies Geometrical Progression.

$\overline{9-2}+6=13$ Shews that the difference between 2 and 9 added to 6 is equal to 13. Read 9 minus 2 plus 6 equal to 13. And that the line above (called a *Vinculum*) connects all the numbers over which it is drawn.

$12-\overline{3+4}=5$ Signifies that the sum of 3 and 4 taken from 12 leaves or is equal to 5.

$\overline{}|^2$ Signifies the second power, or Square.

$\overline{}|^3$ Signifies the third power, or Cube.

$\overline{}|^m$ Signifies any power in general, as $\overline{6}|^2=$square of 6; and $\overline{50}|^3=$cube of 50, &c. thus m signifies either the square or cube, or any other power.

$\sqrt{}$, or $\overline{}|^{\frac{1}{2}}$ Prefixed to any number or quantity, signifies that the square root of that number is required. It likewise (as also the character for any other root) stands for the expression of the root of that number or quantity to which it is prefixed. As $\sqrt{36}=6$, and $\sqrt{108+36}=12$, and $\overline{36}|^{\frac{1}{2}}=6$, &c.

$\sqrt[3]{}$, or $\overline{}|^{\frac{1}{3}}$ Prefixed to any number, signifies that the cube root of that number is required, or expressed. As $\sqrt[3]{216}=6$, and $\sqrt[3]{513+216}=9$, &c. or $\overline{216}|^{\frac{1}{3}}=$ 6, &c.

$\sqrt[m]{}$, or $\overline{}|^{\frac{n}{m}}$ Signifies any root in general. As $\overline{36}|^{\frac{1}{2}}=$square root, $\overline{216}|^{\frac{1}{3}}=$cube root, &c. Thus, $\frac{n}{m}$ signifies either the square root, cube root, or any oth root whatever.

$abcd$ When several letters are set together, they are supposed to be multiplied into each other; as those in the margin are the same as $a\times b\times c\times d$, and represent the continual product of quantities or numbers.

$\frac{1}{a}$ Is the reciprocal of a, and $\frac{a}{b}$ is the reciprocal of $\frac{b}{a}$.

If a be the root, then $a\times a=aa$ or a^2 is the square of a, and $a\times a\times a=aaa$ or a^3 is the cube of a, &c.

Note. The figure above is called the index of the power.

It is usual to write shillings at the left hand of a stroke, and pence at the right; thus, 13/4 is thirteen shillings and four pence.

Note. The use of these characters must be perfectly understood by the pupil, as he may have occasion for them.

A

NEW AND COMPLETE

SYSTEM OF ARITHMETICK.

ARITHMETICK is the Art or Science of computing by numbers, and consists both in Theory and Practice. The Theory considers the nature and quality of numbers, and demonstrates the reason of practical operations. The Practice is that, which shews the method of working by numbers, so as to be most useful and expeditious for business, and is comprised under five principal or fundamental Rules, viz. NOTATION or NUMERATION, ADDITION, SUBTRACTION, MULTIPLICATION, and DIVISION; the knowledge of which is so necessary, that scarcely any thing in life, and nothing in trade can be done without it.

NUMERATION

1. TEACHES the different value of figures by their different places, and to read or write any sum or number by these ten characters, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.—0 is called a cypher, and all the rest are called figures or digits.* The names and significations of these characters, and the origin or generation of the numbers they stand for, are as follow; 0 nothing; 1 one, or a single thing called an unit; $1+1=2$, two; $2+1=3$, three; $3+1=4$, four; $4+1=5$, five; $5+1=6$, six; $6+1=7$, seven; $7+1=8$, eight; $8+1=9$, nine; $9+1=10$, ten; which has no single character; and thus, by the continual addition of one, all numbers are generated.

2. The value of figures when alone, is called their *simple* value, and is invariable. Besides the *simple* value, they have a *local* value, that is, a value which varies according to the place they stand

* These *figures* or *digits* were obtained from the Arabians, and were introduced into Europe in the ninth century. The Arabs probably derived the decimal notation from India. The sexagesimal division had previously been in general use in Europe. This mode of division is yet retained in a few cases, as in the division of time, where *sixty* minutes make an hour, *sixty* seconds a minute, &c. The figures are doubtless called *digits* from *digitus*, a finger, because counting used to be performed on the fingers.

in when connected together. In a combination of figures, reckoning from the right to the left, the figure in the first place represents its *simple* value; that in the second place, ten times its simple value, and so on; each succeeding figure being ten times the value of it in the place immediately preceding. There is no reason in the nature of numbers that their local value should vary according to this law. They might have been made to increase in 3, 4, 5, &c. fold, or in any other ratio. The tenfold increase is assumed because it is most convenient.

3. The values of the places are estimated according to their order: The first is denominated the place of units; the second, tens; the third, hundreds; and so on, as in the table. Thus in the number—5293467; 7, in the first place signifies only seven; 6, in the second place, signifies 6 tens, or sixty; 4, in the third place, four hundred; 3, in the fourth place, three thousand; 9, in the fifth place, ninety thousand; 2, in the sixth place, two hundred thousand; 5, in the seventh place, is five millions; and the whole, taken together, is read thus; five millions, two hundred and ninety three thousand, four hundred and sixty seven.

The process of Numeration may be more clearly seen by the following

TABLE.

Hundreds of thousands of billions, &c.	Tens of thousands of billions.	Thousands of billions.	Hundreds of billions.	Tens of billions.	Billions.	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	Millions.	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
2	4	5,	9	3	8.	6	7	5,	2	6	7.	8	9	1,	3	4	5.

Six places of figures, beginning on the right, are called a *period*, and each successive six places another period. Each period is considered as divided into two *half periods* of three figures each. These are distinguished by the comma, and the point for a period. There is an obvious reason for this division into periods, for at the beginning of each period, there is a new denomination of units, of which the tens, hundreds, thousands, &c. are numerated as in the first period.

4. A cypher, though it is of no signification itself, yet, it possesses a place, and, when set on the right hand of figures, in whole

numbers, increases their value in the same tenfold proportion; thus, 9 signifies only nine; but if a cypher is placed on its right hand, thus, 90, it then becomes ninety; and, if two cyphers be placed on its right, thus, 900, it is nine hundred, &c.

5. To enumerate any parcel of figures, observe the following Rule.

First, commit the words at the head of the table, viz. units, tens, hundreds, &c. to memory, then, to the simple value of each figure, join the name of its place, beginning at the left hand, and reading towards the right.—*More particularly*—1. Place a dot under the right hand figure of the 2d, 4th, 6th, 8th, &c. half periods, and the figure over such dot will, universally, have the name of thousands. —2. Place the figures, 1, 2, 3, 4, &c. as indices over the 2d, 3d, 4th, &c. period. These indices will then shew the number of times the millions are increased.—The figure under 1, bearing the name of millions, that under 2, the name of billions (or millions of millions) that under 3, trillions.

EXAMPLE.

Sextillions.	Quintilli.	Quatrill.	Trillions.	Billions.	Millions.	Units.
th. un.	th. un.	th. un.	th. un.	th. un.	th. un.	c.x.t.c.x.u.
6	5	4	3	2	1	
913,208	000,341	620,057	219,356	809,379	120,406	129,763
Thousands.	Thousands.	Thousands.	Thousands.	Thousands.	Thousands.	Thousands.

NOTE 1. Billions is substituted for millions of millions: Trillions, for millions of millions of millions; Quatrillions, for millions of millions of millions of millions, and so on.

These names of periods of figures, derived from the Latin numerals, may be continued without end. They are as follows, for twenty periods, viz. Units, Millions, Billions, Trillions, Quatrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novemdecillions, Vigintillions.

THE APPLICATION.

Write down, in proper figures, the following numbers.

Fifteen. - - - - - - - - - - - - -	15
Two hundred and feventy nine. - - - - - - - -	279
Three thoufand four hundred and three. - - - - - - -	
Thirty feven thoufand, five hundred and fixty feven. - - -	37567
Four hundred, one thoufand and twenty eight. - - -	
Nine millions, feventy two thoufand and two hundred. - -	9072200
Fifty five millions, three hundred, nine thoufand and nine. - -	
Eight hundred millions, forty four thoufand, and fifty five. -	
Two thoufand, five hundred and forty three millions, four hundred and thirty one thoufand, feven hundred and two.	2543431702

Write down in words at length the following numbers.

8	437	709040	3476194	7584397647
17	3010	879066	84094007	49163189186
129	76506	4091875	690748591	500098400700

Notation by Roman Letters.

I. One.	XV. Fifteen.	CC. Two hundred.
II. Two.	XVI. Sixteen.	CCC. Three hundred.
III. Three.	XVII. Seventeen.	CCCC. Four hundred.
IV. Four.	XVIII. Eighteen.	D. or IↃ. Five hundred.
V. Five.	XIX. Nineteen.	DC. Six hundred.
VI. Six.	XX. Twenty.	DCC. Seven hundred.
VII. Seven.	XXX. Thirty.	DCCC. Eight hundred.
VIII. Eight.	XL. Forty.	DCCCC. Nine hundred.
IX. Nine.	L. Fifty.	M. or CIↃ. One Thousand.
X. Ten.	LX. Sixty.	IↃↃ. Five Thousand.
XI. Eleven.	LXX. Seventy.	IↃↃↃ. Fifty thousand.
XII. Twelve.	LXXX. Eighty.	IↃↃↃIↃↃↃ. Five hund. thou.
XIII. Thirteen.	XC. Ninety.	MDCCCVIII. One Thousand,
XIV. Fourteen.	C. Hundred.	eight hundred and eight.

A less literal number placed after a greater always augments the value of the greater; if put before, it diminishes it. Thus, VI. is 6; IV. is 4; XI. is 11; IX. is 9, &c.

The practice of counting on the fingers doubtless originated the method of Notation by Roman Letters. The letter I was taken for one finger, or one; and hence II, for two; III, for three; IIII, for four; and V, as representing the opening between the thumb and fore-finger, and being also an easier combination of the marks for the fingers, was taken for five. As IV is a simpler expression for *four* than the above, it was doubtless adopted for this reason, and on the general principle too that a less literal number placed before a greater should diminish the greater so much, and, placed after a greater should augment it so much. Hence as IV, is *four*; VI is *six*; VIII is *eight*, and so on. *Ten* was expressed by X, because it is two Vs united, and twice five is ten. *Fifty* was expressed by L, because it is half of C or Ⅽ, as it was anciently written, and C is the initial of the Latin *centum*, one hundred.

Five hundred is expressed by D, because it is half of the Gothic CIↃ or M., the initial of *mille*, one thousand.

ADDITION

IS the putting together of two or more numbers, or sums, to make them one total, or whole sum.

SIMPLE ADDITION

Is the adding of several integers or whole numbers together, which are all of one kind, or sort; as 7 pounds, 12 pounds, and 20 pounds being added together, their aggregate, or sum total, is 39 pounds.

Rule.

Having placed units under units, tens under tens, &c. draw a line underneath, and begin with the units; after adding up every figure in that column, consider how many tens are contained in their sum, and placing the excess under the units, carry so many as you have tens, to the next column, of tens: Proceed in the same manner through every column, or row, and set down the whole amount of the last row.*

* This Rule as well as the method of proof, is founded on the known axiom, "the whole is equal to the sum of all its parts." The method of placing the numbers, and carrying for the tens, is evident from the nature of notation; for, any other disposition of the numbers would alter their value; and carrying one, for every ten, from an inferiour to a superiour column, is, evidently, right; because one unit in the latter case is equal to the value of ten units in the former.

Besides the method of proof here given, there is another, by casting out the nines; thus:

1. Add the figures in the upper row together, and find how many nines are contained in their sum.

2. Reject the nines, and set down the remainder, directly even with the figures in the row.

3 Do the same with each of the given numbers, and set all the excesses of nines in a column, and find their sum; then, if the excess of nines in this sum, found, as before, is equal to the excess of nines in the sum total; the question is supposed to be right.

Example.

	Exc. of nines
5738	5
9156	3
8471	2
5324	5
28689	6

This method depends upon a property of the number 9, which, except 3, belongs to no other digit whatever; viz, that any number divided by 9, will leave the same remainder, as the sum of its figures, or digits, divided by 9: which may be thus demonstrated.

Demonstration. Let there be any number, as 5432; this, separated into its several parts, becomes $5000+400+30+2$; but $5000=5\times1000=5\times999+1=5\times999+5$. In like manner $400=4\times99+4$, and $30=3\times9+3$. Therefore, $5432=5\times999+5,+4\times99+4,+3\times9+3+2=5\times999+4\times99+3\times9+5+4+3+2$.

And $\frac{5432}{9}=\frac{5\times999+4\times99+3\times9+5+4+3+2}{9}$; but $5\times999+4\times99+3\times9$ is divisible by 9; therefore, 5432, divided by 9, will leave the same remainder, as $5+4+3+2$, divided by 9; and the same will hold good of any other number whatever.

The same property belongs to the number 3: However, this inconveniency attends this method, that, although the work will always prove right, when it is so; it will not, always, be right, when it proves so; I have, therefore, given this demonstration more for the sake of the curious, than for any real advantage.

In casting out the nines, proceed thus. Begin with the uppermost row of the example at the left hand; 5 and 7 are 12, from which take out nine, and 3 remains: 3 added to 3 make 6, which must be added to the 8, because 6 is less than 9, and the sum is 14: cast out nine and 5 remains, which is to be placed at the right against the row, as in the example. In the next row, 9 the first figure, may be omitted because it is 9; then 1 and 5 make 6, which added to the 6, make 12, from which take out 9, and 3 remains to be placed on the right of the row as before. Proceed thus with all the rows and with the sum at bottom. Then add the remainders against the several rows, casting out 9 as often as it

Proof. Begin at the top of the sum and reckon the figures downwards, in the same manner as they were added upwards, and, if it be right, this aggregate will be equal to the first. Or, cut off the upper line of figures, and find the amount of the rest; then, if the amount and upper line, when added, be equal to the sum total, the work is supposed to be right.

Addition and Subtraction Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	5	6	7	8	9	10	11	12	13	14
3	5	6	7	8	9	10	11	12	13	14	15
4	6	7	8	9	10	11	12	13	14	15	16
5	7	8	9	10	11	12	13	14	15	16	17
6	8	9	10	11	12	13	14	15	16	17	18
7	9	10	11	12	13	14	15	16	17	18	19
8	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	16	17	18	19	20	21
10	12	13	14	15	16	17	18	19	20	21	22

When you would add two numbers, look one of them in the left hand column and the other at top, and in the common angle of meeting, or, at the right hand of the first, and under the second, you will find the sum—as, 5 and 8 is 13.

When you would subtract: Find the number to be subtracted in the left hand column, run your eye along to the right hand till you find the number from which it is taken, and right over it at top you will find the difference—as 8, taken from 13, leaves 5.

occurs, and, if the remainder be the same as that against the sum, as it is in this example, the work is presumed to be right.

An easier method of casting out the nines, is to begin as before, and when the sum *exceeds nine*, to add the figures themselves of this sum as before, and so proceed, and this new sum will always be equal to the remainder after nine is taken from the first sum. Thus, as before, 5 and 7 are 12,—now add the numbers of this sum, which, being 1 and 2, make 3, equal to the remainder after 9 is taken from 12; then 3 and 3 added to 8 make 14,—add the 1 and 4, and the sum is 5, the same as the remainder above. In the next row,—omitting the 9, the sum is 12, the numbers of which, 1 and 2, make 3, the remainder as above. The same will hold true in any case.

Note. It should be noticed that the method of proof for this rule, and various others, depends upon the *accuracy* of both operations. It does not follow beause the result is the same by both operations, that there *can be* no error. For both operations may be incorrectly performed, and the results, though alike, erroneous. The *best* proof that any result is right, is the *correct performance of all the operations*.

EXAMPLES.

1.	2.	3.	4.	5.	6.
£.	lb.	Cwt.	Miles.	Yards.	£.
1	12	123	1234	12345	987654321
2	34	456	5678	67890	123456789
3	56	785	9098	98765	234567891
4	78	12	7654	43210	345678910
5	90	345	3210	12345	456789123
6	1	678	62	67890	567879287
7	23	901	4713	74100	678900028
8	45	234	131	64786	789400690
9	67	567	9128	19876	548769138
Sum. 45			40908		

In the first Example, the student adds together the several numbers, and finds the sum to be 45; and, as there is but one column, he must set down 45 for the answer.

In the 4th Ex. the student will add the numbers of the column on the right hand, which he will find to be 38; he will set the 8 under the column, and carry 3 to the next column. The next column with the 3 to be carried, he will find to be 40; he must set down the 0, and carry 4 to the next column. This will be found to be 29; the 9 is to be set under the column, and the 2 carried to the next column, which makes 40; the cypher is to be put under the column, and the 4 will take the next higher place, for it is evident the whole must be set down. The same course must be pursued in each example.

7.	8.	9.	10.
1234567	1234567	67	1234567
2345678	723456	123	9876543
3456789	34565	4567	2102865
4567890	4566	89093	4321234
5678209	333	654321	5682098
6789098	90	1234567	6543218
	1997577		

11. What is the sum of 3406, 7980, 345 and 27? Ans. 11758.

12. A man borrowed of his neighbour, thirty dollars at one time, one hundred and 66 at another, and seventy five at another: how much did he borrow in the whole? Ans. 271 dolls.

13. Four boys collected chesnuts; A. had 4096, B. 16784, C. 11590, and D. five hundred and 57; how many were there in the whole? Ans.

14. Four boys, on counting their apples, found that A. had 67, B. 11 more than A, C. had 101, and D. had sixteen more than C; how many had they all?

15. The Deluge happened 2348 years before the birth of our Saviour, and America was discovered 1492 years after it; how many years intervened?

SUBTRACTION.

TEACHES to take a less number from a greater, to find a third, shewing the inequality or difference between the given numbers. The *greater* number is called the *Minuend*. The *less* number is called the *Subtrahend*. The difference, or, what is left after the subtraction is made, is called the *Remainder*.

SIMPLE SUBTRACTION

Teaches to find the difference between any two numbers, which are of the same kind.

Rule.

Place the larger number uppermost, and the less underneath, so that units may stand under units, tens under tens, &c. then, drawing a line underneath, begin with the units, and subtract the lower from the upper figure, and set down the remainder; but if the lower figure be greater than the upper, add ten, and subtract the lower figure therefrom: To this difference, add the upper figure, which being set down, you must add one to the ten's place of the lower line, for that which you added before; and thus proceed through the whole.*

Proof.

In either simple, or compound Subtraction, add the remainder and the less line together, whose sum, if the work be right, will be equal to the greater line: Or subtract the remainder from the greater line, and the difference will be equal to the less.

Examples.

	1.	2.	3.	4.	5.	6.
	£.	£.	Miles.	Yards.	Feet.	Cwt.
From	25	305	4670	58934	879647	9187641
Take	12	103	4020	6182	164348	91843
Rem.	13			52752		
Proof.	25			58934		

* *Dem.* When all the figures of the less number are less than their correspondent figures in the greater, the difference of the figures, in the several like places, must, all taken together, make the true difference sought; because, as the sum of the parts is equal to the whole; so must the sum of the differences, of all the similar parts, be equal to the difference of the whole.

2. When any figure in the greater number is less than its correspondent figure in the less, the ten which is added by the Rule, is the value of an unit in the next higher place, by the nature of notation; and the one which is added to the next place of the less number, is to diminish the correspondent place of the greater, accordingly; which is only taking from one place and adding as much to another, whereby the total is never changed: And, by this means, the greater is resolved into such parts, as are, each, greater than, or equal to, the similar part of the less; and the difference of the correspondent figures, taken together, will, evidently, make up the difference of the whole.

The truth of the method of proof is evident; for the difference of two numbers added to the less, is manifestly, equal to the greater.

The operation on the first three Examples is sufficiently plain. In the 4th Ex. I begin on the right hand, and take 2 from 4, and set down the difference 2, under the column. As 8 is greater than 3, I add 10 to 3, which makes 13, and from it take the 8, and 5 is the difference to be set down. As I add 10 to the 3, I now add 1 to the 1 in the next higher place, because 10 in one place is equal only to 1 in the next higher place, and take the 2 from the 9, and the difference is 7. The rest of the work is obvious. The same proofs must be followed in every similar case.

7.	8.	9.
100200300400500600700800900	10000	1000000
98076054032011023045067089	9999	1

10. What is the difference of 40875 and 38968? Ans. 1907.

11. What number must be added to 6892, so that the sum shall be 8265? Ans. 1373.

12. America was discovered in 1492; how many years have elapsed since?

13. If you lend your friend 3646 dollars, and afterwards are paid 2998 dollars; how much is still due? Ans. 648 dollars.

14. If a man was seventy five years old in the year 1821, in what year was he born? Ans. 1746.

15. The Independence of the United States was declared July 4th, 1776; how many years have passed since? Ans.

16. Sir Isaac Newton died in the year 1727, aged eighty five; in what year was he born? Ans. 1642.

MULTIPLICATION

TEACHES to find the amount of one number increased as many times as there are units in another, and thus performs the work of many additions in the most compendious manner; brings numbers of great denominations into small, as pounds into shillings, pence or farthings, &c. and, by knowing the value of one thing, we find the value of many.

The number given to be multiplied, is called the *Multiplicand.*

The number given to multiply by, is called the *Multiplier.*

The multiplicand and multiplier are often called *factors.*

The result of the operation, or the number found by multiplying, is called the *Product.*

Multiplication is distinguished into Simple and Compound.

SIMPLE MULTIPLICATION

Is the multiplying of any two numbers together, without having regard to their signification; as 7 times 8 is 56, &c.

MULTIPLICATION AND DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

To learn this Table for Multiplication: Find your multiplier in the left hand column, and your multiplicand at top, and in the common angle of meeting, or against your multiplier, along at the right hand, and under your multiplicand, you will find the product, or answer.

To learn it for Division: Find the divisor in the left hand column, and run your eye along the row to the right hand until you find the dividend; then, directly over the dividend, at top, you will find the quotient, shewing how often the divisor is contained in the dividend.

RULE.

Having placed the multiplier under the multiplicand so that units stand under units, tens under tens, &c. and drawn a line under them, then,

1. *When the multiplier does not exceed* 12; begin at the right hand of the multiplicand and multiply each figure by the multiplier, setting down the unit figure under units, and so on, and carrying for the tens to the next place, as in addition, and the work is done.*

2. *When the multiplier exceeds* 12; multiply each figure of the multiplicand by every figure in the multiplier as before, placing the first figure of each product exactly under its multiplier: then

* *Dem.* When the multiplier is a single digit, it is plain that we find the product; for, by multiplying every figure, that is, every part of the multiplicand, we multiply the whole: and, the writing down of the products, which are less than ten, or the excess of tens, in the places of the figures multiplied, and carrying the number of tens to the product of the next place, is only gathering together the similar parts of the respective products, and is, therefore, the same, in effect, as though we wrote down the multiplicand as often as the multiplier expresses, and added them together; for the sum of every column is the product of the figures in the place of that column; and the products, collected together, are evidently equal to the whole required product.

add together these several products as they stand, and their sum will be the total product.*

* If the multiplier be a number, made up of more than one figure; after we have found the product of the multiplicand by the first figure of the multiplier, as above, we suppose the multiplier divided into parts, and, after the same manner, find the product of the multiplicand by the second figure of the multiplier; but as the figure, by which we are multiplying, stands in the place of tens, the product must be ten times its simple value; and, therefore, the first figure in this product must be noted in the place of tens, or, which is the same, directly under the figure we are multiplying by. And, proceeding in the same manner with all the figures of the multiplier, separately, it is evident we shall multiply all the parts of the multiplicand by all the parts of the multiplier; therefore, these several products, being added together, will be equal to the whole required product.

The reason of the method of proof, depends upon this proposition, that if two numbers are to be multiplied together, either of them may be made the multiplier or multiplicand, and the product will be the same.

A small attention to the nature of numbers will make this truth evident; for $5\times9=45=9\times5$; and, in general, $2\times3\times4\times5\times6$, &c. $=3\times2\times6\times5\times4$, &c. without any regard to the order of the terms; and this is true of any number of factors whatever.

The following examples are subjoined, to make the reason of the rule appear as clearly as possible.

$$\begin{array}{rcr}
64753 & & \\
5 & & \\
\hline
15 = & & 3\times5 \\
25 = & & 50\times5 \\
35 = & & 700\times5 \\
20 = & & 4000\times5 \\
30 = & & 60000\times5 \\
\hline
323765 = & & 64753\times5 \\
\hline
\end{array}
\qquad
\begin{array}{rl}
237956 & \\
3728 & \\
\hline
1903648 = 8 & \text{times the multiplicand.} \\
475912 = 20 & \text{times ditto.} \\
1665692 = 709 & \text{times ditto.} \\
713868 = 3000 & \text{times ditto.} \\
\hline
887099968 = 3728 & \text{times ditto.} \\
\hline
\end{array}$$

Another method of proving the rule is as follows. Let the factors be 64753 and 5. Now $64753=60000+4000+700+50+3$. The sum of the products of these quantities severally multiplied by 5, is the true product. Then $60000+4000+700+50+3$ is one factor. 5 the multiplier the other factor.
$300000+20000+3500+250+15=323765=64753\times5$.

Or let the factors be 45 and 24. Then $45=40+5$, and $24=20+4$, and

$$\begin{array}{l}
40+5 \text{ multiplicand.} \\
20+4 \text{ multiplier.} \\
\hline
800+100=45\times20 \\
\quad 160+20=45\times4 \\
\hline
800+260+20 = 1030=15\times21.
\end{array}
\qquad
\begin{array}{l}
\text{Let the factors be 24 and 24. Then,} \\
20+4 \\
20+4 \\
\hline
400+80 \quad = 24\times20 \\
\quad 80+16 = 24\times4. \\
\hline
400+160+16 = 576=24\times24.
\end{array}$$

Multiplication may also be proved, by casting out the nines; but is liable to the inconvenience before mentioned.

It may likewise be, very naturally, proved by division; for the product, being divided by either of the factors, will, evidently, give the other; and it might not be amiss for the pupil to be taught division, at the same time with multiplication; as it not only serves for proof; but also gives him a readier knowledge of them both. But it would have been contrary to good method to have given this rule in the text, because the pupil is supposed, as yet, to be unacquainted with division.

PROOF.

Multiply the multiplier by the multiplicand.

Multiply 3851 by 3.	By addition.
3851 Multiplicand.	3851
3 Multiplier.	3851
	3851
11553 Product.	
	11553 Sum.

Having placed the numbers according to the rule,—then say, 3 times 1 is 3, and place 3 directly under units; then 3 times 5 is 15, set down 5 and carry the *one* to the next product. Then, 3 times 8 is 24, to which the 1 is to be added, making 25; set down 5 and carry 2. Then 3 times 3 is 9, and the 2 to be carried, make 11, which set down, and the work is done. The result is the same as is obtained by addition.

Multiply 6053 by 11.
11

Prod. 66583

Proceeding as before, multiply 3 by 11, and of the product, 33, set down 3 under units, and carry 3; then 5 by 11, and to the product, 55, add the 3 to be carried, set down 8, and carry 5; then 0 by 11, and as the product is 0, set down the 5, which was to be carried; then 6 by 11, and, as there is none to carry, set down the product, 66, and the operation is finished.

Multiply 67013 by 29.
67013 Multiplicand.
29 Multiplier.

603117 Product by 9, the units of the multiplier.
134026 Product by 2, the tens of the multiplier.

1943377 Product or answer.

In this example, the multiplicand is first multiplied by 9, the units of the multiplier, and the product set down, as in the preceding examples. The multiplicand is then multiplied by 2, the tens of the multiplier, as before, the first figure of the product is placed under the 2, in the place of tens. The two products are then added, and their sum is the whole product or answer.

EXAMPLES.

	1.	2.	3.	4.
	37934	769308	4980076	763896
	2	3	4	5
Prod.	75868			

	5.	6.	7.	8.
	67589	503764	3918295	9164785
	6	7	8	9
Prod.	405534			

	9.	10.	11.
	4879567	5864794	8583478646
	10	11	12
Prod.		64512734	

	12.	13.	14.
	6357534	8324629	46293845
	47	59	106
	44502738		277763070
	25430136		46293845
Prod.	298804098		4907147570

	15.	16.	17.
	647906	760483	91867584
	4873	9152	6875
	3157245938		

18. Multiply 103 by sixty seven. Ans. 6901.

19. Said Jack to Harry, you have only 77 chesnuts, but I have seven times as many; how many have I? Ans. 539.

20. If four bushels of wheat make a barrel of flour, and the price of wheat be one dollar a bushel, what will 225 barrels of flour cost? Ans. 900 dolls.

21. Eighty nine men shared equally in a prize, and received 17 dolls. each; how much was the prize? Ans. 1513 dolls.

22. Multiply 308879 by twenty thousand five hundred and three. Ans. 6332946137.

In some cases the operations of multiplication are shortened by particular rules. Several Cases follow.

NOTE. A *composite number* is the product of two or more numbers, as 27, which 3×9, and, as 315, which = 5×7×9.

CASE I.

When the multiplier is a composite number, multiply the multiplicand by one of those figures, first, and that product by the other, &c. and the last product will be the total required.*

* The reason of this method is obvious: For any number, multiplied by the component parts of another number, must give the same product, as though it were multiplied by that number at once: Thus, in example first, 5 times the product of 7, multiplied into the given number, makes 35 times that given number, as plainly as 5 times 7 makes 35.

EXAMPLES.

1.	2.	3.
Mult. 59375 by 35.	39187 by 48.	91632 by 56.

```
           59375
               7
7×5 = 35 -------
          415625
               5
         -------
         2078125
         -------
```

4.	5.
3065 by 63.	6061 by 121.

6.
14567 by 144.

CASE II.

When there are cyphers on the right hand of either the multiplicand, or multiplier, or both: Neglect those cyphers; then place the significant figures under one another, and multiply by them only; add them together, as before directed, and place to the right hand as many cyphers as there are in both the factors.

EXAMPLES.

	1.	2.	3.
	67910	956700	930137000
	5600	320	9500
Prod.	380296000	306144000	8836301500000

CASE III.

To multiply by 10, 100, 1000, *&c.:* Set down the multiplicand underneath, and join the cyphers in your multiplier to the right hand of them.*

EXAMPLES.

	1.	2.	3.	4.
	57935	84935	613975	8473965
	10	100	1000	10000
Prod.	579350			

CASE IV.

To multiply by 99, 999, *&c. in one line:* Place as many dots at the right hand of the multiplicand, as there are figures of 9 in your multiplier, which dots suppose to be cyphers; then, beginning with the right hand dot, subtract the *given* multiplicand from the *new* one, and the remainder will be the product.†

* This is evident from the nature of numbers, since every cypher annexed to the right of a number increases the value of that number in a tenfold proportion.

† Here it may easily be seen that, if you multiply any sum by 9, the product will be but 9 tenths of the product of the same sum, multiplied by 10: and as the annexing of a dot or cypher, to the right hand of the multiplicand, supposes it to be increased tenfold: therefore, subtracting the given multiplicand from the tenfold multiplicand, it is evident that the remainder will be ninefold the said given multiplicand, equal to the product of the same by 9; and the same will hold true of any number of nines.

EXAMPLES.

1.	2.	3.
6473..	857389...	5384976....
99	999	9999
640827		53844375024

That these examples may appear as clear as possible, I will illustrate them by giving another.

Mult. 371967... by 999	According to the rule, it will stand thus.	371967... Minuend. 371967 Subtrah.
371595033		371595033 Rem. or total Pro.

CASE V.

To multiply by 13, 14, 15, *&c. to* 19; *also from* 101 *to* 109, *from* 1001 *to* 1009, *&c.:* Multiply with the unit figure only, of the multiplier, removing the product one place to the right hand of the multiplicand, and so many places further as there may be cyphers between the significant figures; then add all together, and their sum will be the product.*

EXAMPLES.

	1.	2.	3.
	75964×13	7598×104	6735×1005
	227892	30392	33675
Prod.	987532		

CASE VI.

To multiply by 21, 31, 41, *&c. to* 91, *also by the same figures with any number of cyphers between them:* Multiply by the left hand figure, only, of the multiplier, and set the unit figure of the product one place to the left, and as many places further as there are cyphers between the significant figures; and add the numbers together for the product.

EXAMPLES.

	1.	2.	3.
	73918 ×21	56934 ×301	45936 ×4001
	147836	170802	4.
Prod.	1552278	17137134	3167×500001.

* The reason of this Rule, and of the following one also, will be evident on inspecting an example under each rule.

CASE. VII.

To multiply any number, by any number, giving only the Product: Put down the product figure of the first figure of the multiplicand by the *first* of the multiplier. To the product of the *second* of the multiplicand by the *first* of the multiplier, add the number to be carried, and the product of the *first* of the multiplicand by the *second* of the multiplier; then, carrying for the tens in the sum, put down the rest. To the product of the *third* of the multiplicand by the *first* of the multiplier, add the number to be carried, and the product of the *second* of the multiplicand by the *second* of the multiplier, also the product of the *first* of the multiplicand by the *third* of the multiplier, carry the tens, and put down the rest, and so proceed till you have multiplied the *highest* of the multiplicand by the *lowest* of the multiplier. Then multiply the *highest* of the multiplicand by the *second* of the multiplier: Add the number to be carried, and the product of the *last but one* of the multiplicand by the *third* of the multiplier, and the product of the *last but two* of the multiplicand by the *fourth* of the multiplier, &c. Then to the product of the *last but one* of the multiplicand by the *fourth* of the multiplier; and so proceed till you have multiplied the *last* of the multiplicand by the *last* of the multiplier, which finishes the work.

Example.

Mult. 5321415
By 2354

Prod. 12526610910

Explanation.

$5\times4=20$
$1\times4+2+5\times5=31$
$4\times4+3+1\times5+5\times3=39$
$1\times4+3+4\times5+1\times3+5\times2=40$
$2\times4+4+1\times5+4\times3+1\times2=31$
$3\times4+3+2\times5+1\times3+4\times2=36$
$5\times4+3+3\times5+2\times3+1\times2=46$
$5\times5+4+3\times3+2\times2=42$
$5\times3+4+3\times2=25$
$5\times2+2=12$

DIVISION

TEACHES to separate any number or quantity given, into any number of parts assigned; or to find how often one number is contained in another; and is a concise way of performing several Subtractions.

There are four parts to be noticed in Division, viz.

The *Dividend*, is the number given to be divided.

The *Divisor*, is the number given to divide by.

The *Quotient*, or answer to the question, shows how many times the divisor is contained in the dividend.

If there be any thing left after the operation is performed, it is called the *Remainder*; sometimes there is a remainder and sometimes there is not. The remainder, when there is any, is of the same denomination as the dividend.

Division is both Simple and Compound.

PROOF.

Multiply the divisor and quotient together, and add the remainder, if there be any, to the product; if the work be right, that sum will be equal to the dividend.

SIMPLE DIVISION

Is the dividing of one number by another, without regard to their values: As 56, divided by 8, produces 7 in the quotient: That is, 8 is contained 7 times in 56.*

RULE.

Having drawn a curve line on each side of the dividend and placed the divisor on the left hand of it,

Seek how many times the divisor is contained in the least number of the figures of the dividend on the left hand that do contain

* According to the rule, we resolve the dividend into parts, and find, by trial, the number of times the divisor is contained in each of those parts; and the only thing which remains to be proved, is, that the several figures of the quotient, taken as one number, according to the order, in which they are placed, are the true quotient of the whole dividend by the divisor; which may be thus demonstrated.

Dem. The complete value of the first part of the dividend, is, by the nature of notation, 10, 100, 1000, &c. times the simple value of what is taken in the operation; accordingly, as there are 1, 2, or 3, &c. figures standing before it; and, consequently, the true value of the quotient figure, belonging to that part of the dividend, is also 10, 100, 1000, &c. times its simple value; but the true value of the quotient figure, belonging to that part of the dividend, found by the rule, is also 10, 100, 1000, &c. times its simple value; for there are as many figures set before it, as the number of remaining figures in the dividend: therefore the first quotient figure, taken in its complete value from the place it stands in, is the true quotient of the divisor in the complete value of the first part of the dividend. For the same reason, all the rest of the figures of the quotient, taken according to their places, are, each, the true quotient of the divisor, in the complete value of the several parts of the dividend belonging to each; because, as the first figure, on the right hand of each succeeding part of the dividend, has a less number of figures standing before it, so ought their quotients to have; and so they are actually ordered; consequently, taking all the quotient figures in order, as they are placed by the rule, they make one number, which is equal to the sum of the true quotients of all the several parts of the dividend; and is, therefore, the true quotient of the whole dividend by the divisor.

That no obscurity may remain, in the demonstration, it is illustrated by the following example.

it, and place the answer on the right of the dividend for the quotient; multiply the divisor by this quotient figure, place the product under those left hand figures of the dividend; then subtract it therefrom, and bring down the next figure of the dividend to the right hand of the remainder: If, when you have brought down a figure to the remainder, it is still less than the divisor, a cypher must be placed in the quotient, and another figure be brought down; after which, you must seek, multiply, and subtract, till you have brought down every figure of the dividend.

Divifor 25)74503 Dividend		
1ft part of the dividend is =	74000	
25×2000 =	50000	—— 2000 the 1ft quotient.
1ft remainder =	24000	
add	500	
2d part of the dividend =	24500	
25×900 =	22500	—— 900 the 2d quotient.
2d remainder =	2000	
add	00	
3d part of the dividend =	2000	
25×80 =	2000	—— 80 the 3d quotient.
	00	
add	3	
4th part of the dividend =	3	
25×0	0	—— 0 the 4th quotient.
Laft remainder = 3 —		2980 = Sum of all the quotients, or, the Anfwer.

Explan. It is evident the dividend is refolved into thefe parts, 74000+500+00+3; for the firft part of the dividend is confidered only as 74; but yet it is, truly, 74000; and therefore its quotient, inftead of 2, is 2000, and the remainder 24000; and fo of the reft; as may be feen in the operation.

When there is no remainder to a divifion, the quotient is the abfolute and perfect anfwer to the queftion; but where there is a remainder, it may be obferved, that it goes fo much towards another time as it approaches the divifor; thus, if the remainder be half the divifor, it will go half of a time more, and fo on; in order, therefore, to complete the quotient, put the laft remainder to the end of it, above a line, and the divifor below it. Hence the origin of vulgar fractions, which are treated of hereafter.

It is fometimes difficult to find how often the divifor may be had in the numbers of the feveral fteps of the operation: The beft way will be to find how often the firft figure of the divifor may be had in the firft, or two firft figures of the dividend, and the anfwer, made lefs by one or two, is, generally, the figure wanted: but, if, after fubtracting the product of the divifor and quotient from the dividend, the remainder be equal to, or exceed the divifor, the quotient figure muft be increafed accordingly; or, if the product of the divifor and quotient figure exceed the dividend, then the quotient figure muft be proportionably leffened.

The reafon of the method of proof is plain; for, fince the quotient is the number of times the dividend contains the divifor, the product of the quotient and divifor, muft, evidently, be equal to the dividend.

Examples.

1.

Divisor. Dividend. Quotient.
3)175817(58605
15
——
25
24
——
18
18
——
17
15
—
2 Rem.

Proof.
58605 Quotient.
×3 Divisor + 2
——
175817

In this example, I find that 3, the divisor, cannot be contained in the first figure of the dividend; therefore, I take two figures, viz. 17, and inquire how often 3 is contained therein, which finding to be 5 times, I place the 5 in the quotient, and multiply the divisor by it, setting the first figure of the multiplication under the 7 in the dividend, &c. I then subtract 15 from 17, and find a remainder of 2, to the right hand of which I bring down the next figure of the dividend, viz. 5; then I inquire how often the divisor 3, is contained in 25, and, finding it to be 8 times, I multiply by 8, and proceed as before, till I bring down the 1, when, finding I cannot have the divisor in 1, I place 0 in the quotient, and bring down 7 to the 1, and proceed as at the first.

Observe, that, in multiplying by 3, I add in the 2.

2.

29)153598(5296
145
——
85
58
——
279
261
——
188
174
——
14

3.

6493)91876375(14150
6493
——
26946
25972
——
9743
6493
——
32507
32465
——
425

There are several other methods made use of to prove division; as follow, viz.

Rule I.

Subtract the remainder from the dividend; divide this number by the quotient, and the quotient, found by this division, will be equal to the former divisor, when the work is right.

Rule II.

Add the remainder and all the products of the several quotient figures multiplied by the divisor together, according to the order in which they stand in the work, and the sum, when the work is right, will be equal to the dividend.

Here the numbers to be added are the products of the divisor by every figure of the quotient, separately; and each by its place, possesses its complete value; therefore, the sum of the parts together with the remainder, must be equal to the whole. I will illustrate the whole by an example proved according to the several different methods.

4.	5.	6.
23)503775(	35)197184(	85)994466(
7.	8.	9.
236)3798567(	3479)483956795(	5679)19647394(

10.	11.
38473)119184693(	641976)9187642959(

12.
5823789(791822376496(

13.
123456789)121932631112635269(

Some operations are more readily performed by the following particular rules.

CASE I.

When there is one cypher, or more, at the right hand of the divisor: It or they must be cut off; also cut off the same number of figures from the dividend, and then proceed as in Case first: But the figures which were cut off from the dividend must be placed at the right hand of the remainder.†

```
79)9 8 7 6 5 4 3 2 1(12501953
   7 9*                       79+34 remainder.
   ----                 ---------
   1 9 7                 112517577
   1 5 8*                87513671
   ------                     +34
   . 3 9 6              ---------
   . 3 9 5*              987654321 Proof by Multiplication.
   . ------
   . . . 1 5 4
   . . . . 7 9*                     987654321
   . . . .------                          -34
   . . . . 7 5 3                    ---------
   . . . . 7 1 1*    12501953)987654287(79 Proof by Division.
   . . . . ------                   87513671
   . . . . . 4 2 2                  --------
   . . . . . 3 9 5*                 112517577
   . . . . . ------                 112517577
   . . . . . . 2 7 1                ---------
   . . . . . . 2 3 7*
   . . . . . . ------
   . . . . . . . 3 4*
   ------------------
   9 8 7 6 5 4 3 2 1   Proof by Addition.
```

We need only to refer to the example, except for the proof by addition, where it may be remarked, that the Asterisms shew the numbers to be added, and the dotted lines their order.

† The reason of this contraction it is easy to conceive; for cutting off the same figures from each, is the same as dividing each of them by 10, 100, 1000, &c. and it is evident, that as often as the whole divisor is contained in the whole dividend, so often must any part of the divisor be contained in the like part of the dividend; this method is only to avoid a needless repetition of cyphers, which would happen in the common way, as may be seen by working one of the examples of this case in the common way without cutting off the cyphers.

EXAMPLES.

1.

```
65|00)3794326|75(58374
      325
      ---
      544
      520
      ---
       243
       195
       ---
        482
        455
        ---
         276
         260
         ---
        1675 Rem.
```

2.

5193|000)8937643|893(

3.

917|0)47658|3(

4.

875|000)91764789430|000(

5.

Quot. Rem.
1|0)9584|6

6.

Quot. Rem.
1|00)76495|80

7.

Quot. Rem.
1|000)93751839|462

Note. In dividing by 10, 100, 1000, &c. when you cut off as many figures from the dividend, as there are cyphers in the divisor, your work is done; those figures, cut off at the right hand, are the remainder, and those on the left, the quotient, as above.

CASE II.

Short Division may be used when the divisor does not exceed 12. It is performed by the following

RULE.

First, seek how often the divisor can be had in the first figure, or figures of the dividend; which, when found, place in the quotient; then, *mentally*, multiply your divisor by the figure placed in the quotient, and subtract the product from the like number of the left hand figures of your dividend, and the remaining units, if any, must be accounted so many tens, which you must suppose to stand at the left hand of the next figure in the dividend, and to be reckoned with it; then, seek how often you can have your divisor in those two figures; but, if nothing remain, you must then seek how often your divisor is contained in the next figure, or figures, and thus proceed till you have done.

EXAMPLES.

Divisor. Dividend.	2.	3.	4.	5.
2)71935	3)51903	5)633795	6)8471937	7)193847
Quot. 35967—1 rem.				

6.	7.	8.	9.
8)5437846	9)45963784	11)91843756	12)1196437847536

CASE III.

When the divisor is such a number that any two or more figures in the Table, being multiplied together, will produce it: Divide the given dividend by one of those figures; the quotient, thence arising, by the other, and so on; and the last quotient will be the answer.*

* This follows from the contraction in caſe 3d, of Simple Multiplication, of which it is only the reverſe; for the fourth part of the half of any thing is evidently the ſame as the eighth part of the whole; and ſo of any other number.

As the learner at preſent is ſuppoſed to be unacquainted with the nature of fractions, and as the quotient is incomplete without the remainder; I ſhall here give a rule for finding the true remainder, without having recourſe to fractions.

RULE I.

Multiply the quotient by the diviſor: Subtract the product from the dividend and the reſult will be the true remainder.

The Rule which is moſt commonly made uſe of when the diviſor is a compoſite number, is

RULE II.

Multiply the laſt remainder by the preceding diviſor, or laſt but one, and to the product add the preceding remainder; multiply this ſum by the next preceding diviſor, and to the product add the next preceding remainder; and ſo on, till you have gone through all the diviſors and remainders, to the firſt.

EXAMPLE.

6)85397 divided by 150		1	the laſt remainder.
	multiply by	*5	the laſt diviſor but one.
5)14232—5			
		5	
5)2846—2	add	2	the ſecond remainder.
569—1		7	
	multiply by	6	the firſt diviſor.
Ans. $569\frac{47}{150}$		42	
	add	5	the firſt remainder.
		47	the true remainder.

To explain this rule from the example, we may obſerve, that every unit in the firſt quotient may be looked upon as containing 6 of the units in the given dividend; conſequently, every unit which remains, will contain the ſame; therefore, this remainder muſt be multiplied by 6, to find the units it contains of the given dividend. Again, each unit in the next quotient will contain 5 of the preceding ones, or 30 of the firſt, that is, 6 times 5; therefore, what remains muſt be multiplied by 30, or, which is the ſame thing, by 6 and 5 continually: Now, this is the ſame as the Rule; for inſtead of finding the remainders, ſeparately, they are reduced from the bottom, upwards, ſtep by ſtep, to one another, and the remaining units, of the ſame claſs, taken as they occur.

EXAMPLES.

1st. method.	2d. method.	3d. method.
9)196473	8)196473	72)196473(2728 Quot.
		144
8)21830	9)24559	524
		504
Quot. 2728—57	Quot. 2728—57	207
		144
		633
		576
		57 Remainder.

I have wrought the above question three ways, that the learner may understand the method of finding the true remainder, according to this case. In the *first*, in dividing by 9, 3 remains, and by 8, 6 remains; which being the last remainder, I multiply it by the first divisor 9, and add in the first remainder 3, and they make 57, the true remainder. In the *second* method, dividing by 8, 1 remains, and by 9, 7 remains; I therefore, multiply 7, the last remainder, by 8, adding in the 1, and they make 57 as before. The *third* method is self evident, and shews that the other remainders are true.

2.	3.	4.	5.
36)79638	25)197835	87)93975	54)93738764

6.	7.	8.
121)75323939	132)38473692	144)891376429732

CASE. IV.

When the divisor is a whole number with some part of unity, as $3\frac{1}{3}$, $5\frac{1}{2}$, $6\frac{2}{3}$, &c. proceed by either of the following methods.

I. Multiply the whole number in the divisor by the number of parts into which unity is divided in the fraction, and to the product add the number of parts of unity taken in the fraction, and the divisor will be reduced to the parts indicated by the fraction, for a new divisor. Multiply the dividend by the parts into which unity is divided in the fraction, for a new dividend. Divide the new dividend by the new divisor, and the quotient will be the answer.

Ex. 1. Divide 1820 by $2\frac{1}{3}$.

Here, I multiply 2 by 3, and add 1 to the product, and have 7 for the new divisor. Then I multiply 1820 also by 3, and have 5460 for the new dividend. Then $5460 \div 7 = 780$, the answer, or $2\frac{1}{3}$ is contained in 1820, exactly 780 times.

Note. It is obvious that the dividend and divisor are proportionally increased by the multiplication, so that the quotient will be the same as if they had not been thus increased.

Ex. 2. Divide 6375 by $5\frac{1}{2}$.

Proceeding as before, the new divisor is 11, and the new dividend is 12750. Then $12750 \div 11 = 1159$ and 1 remains, or $1159\frac{1}{11}$, the number of times $5\frac{1}{2}$ is contained in 6375.

3. Divide 10142 by $3\frac{2}{3}$. Ans. 2766.

4. Divide 178 by $2\frac{3}{8}$.

5. Divide 158765 by $15\frac{7}{8}$.

II. Proceed according to the general rule for division, being careful to add the value of the fractions to the several remainders, at every step of the process.

Ex. 1. Divide 1820 by $2\frac{1}{3}$.

$2\frac{1}{3})1820(780$
$16\frac{1}{3}$
———
$1\frac{2}{3}$
———
$18\frac{2}{3}$
$18\frac{2}{3}$
———
0

Here $7 \times 2\frac{1}{3} = 16\frac{1}{3}$, which being subtracted from 18, leaves $1\frac{2}{3}$. Now $\frac{2}{3}$ belongs to the place of hundredths, and is $\frac{2}{3}$ of 10 for the place of tenths. But $\frac{2}{3}$ of $10 = 6\frac{2}{3}$, to which add the 2, and we have $8\frac{2}{3}$ to be annexed to the 1 remainder, and we have $18\frac{2}{3}$ for the true remainder. Now $2\frac{1}{3}$ is contained in $18\frac{2}{3}$ exactly 8 times. The rest of the process is evident.

Ex. 2. Divide 6375 by $5\frac{1}{2}$.

$5\frac{1}{2})6375(1159\frac{1}{11}$
$5\frac{1}{2}$
———
$\frac{1}{2}$
———
8
$5\frac{1}{2}$
———
$2\frac{1}{2}$
———
32
$27\frac{1}{2}$
———
$4\frac{1}{2}$
———
50
$49\frac{1}{2}$
———
$\frac{1}{2}$

Here once $5\frac{1}{2}$ taken from 6, leaves $\frac{1}{2}$. But $\frac{1}{2}$ in thousandths place is 5 in the place of hundredths, and 5 added to the 3 hundredths is 8. Taking from 8, once $5\frac{1}{2}$, $2\frac{1}{2}$ remain. As before, the $\frac{1}{2}$ becomes 5 in the place of tenths, which added to 7 tenths, make 12, which added to the remainder 2 in the preceding place, give 32. From this 5 times $5\frac{1}{2}$ are to be taken, and $4\frac{1}{2}$ remain. But $4\frac{1}{2}$ in the place of tenths, is 45, which added to 5 make 50. From 50 take 9 times $5\frac{1}{2}$, and $\frac{1}{2}$ remains. Now in $5\frac{1}{2}$ are 11 halves, so that $\frac{1}{2}$ is $\frac{1}{11}$, which annexed to the quotient, gives the whole quotient.

Ex. 3. Divide 347 by $2\frac{2}{5}$ Ans $144\frac{7}{12}$.

4. Divide 13567 by $13\frac{3}{4}$.

Supplement to Contractions in Multiplication.

1. The shortest method of multiplication, when the multiplier is any even part of 100, 1000, &c. is by division: For if the multiplicand be increased by a number of cyphers equal to the number of places in the multiplier, and a part of that product taken for the same proportion, which the multiplier bears to 1, and the same number of cyphers annexed to it, the quotient will be the true product.

1. Multiply 39756 into 125.
$125 = \frac{1}{8}$ of 1000, wherefore,

8)39756000

4969500 Product.

2. Multiply 57638 by $33\frac{1}{3}$
$33\frac{1}{3} = \frac{1}{3}$ of 100, therefore,

3)5763800

$1921266\frac{2}{3}$ Product.

3. Multiply 91378 by $333\frac{1}{3}$. Ans. $30459333\frac{1}{3}$.

The reason of the preceding rule is obvious.

II. If any digit, with cyphers annexed, be divided by 9, the quotient will consist, wholly, ot such digits, and so many 9ths of an unit over; hence the following method of multiplying by repetends of any of the digits.

The reason of the method may be seen by the first example. Thus $80000 \div 9$, or $\frac{80000}{9} = 8888\frac{8}{9}$, and by multiplying by 80000 and dividing by nine, you get eight 9ths, of 645 too much, which must of course be subtracted to obtain the true product. Or, thus, $\frac{80000}{9} - \frac{8}{9} = 8888$; then $645 \times 8888 = 645 \times \frac{80000}{9} - 645 \times \frac{8}{9} = 5733333\frac{3}{9} - 573\frac{3}{9} = 5732760$. But as three 9ths, belong to both numbers, and, as an equal number of 9ths, will belong to both numbers in any case, it is not necessary to write them, as they balance each other in the subtraction.

1.
645 by 8888.
80000

9)51600000

5733333
Subtract 573

Product. 5732760

2.
5394 by 66666.
600000

9)3236400000

359600000
Subt. 3596

Prod. 359596404

3.
3798 by 444

Prod. 1686312

III. When the multiplicand has a fraction belonging to it, such as one fourth, one half, &c. add such a part of the multiplier as the fraction signifies, to the product obtained by multiplying, and the sum is the whole product. When such a fraction belongs to the multiplier, add to the product such a part of the multiplicand as the fraction denotes.

1. Multiply $153\frac{1}{2}$ by 6.
6

918 Product.
3 half of 6.

921 Answer.

It is evident that 6 halves or 3, must be added to 153×6 to obtain 6 times $153\frac{1}{2}$. And the same must be true in every similar case.

2. Multiply 638 by $6\frac{1}{3}$.

$$\begin{array}{rl} & 6\frac{1}{3} \\ \hline 3828 & \text{Product by 6.} \\ 212\frac{2}{3} & \frac{1}{3} \text{ of the multiplicand.} \\ \hline 4040\frac{2}{3} & \text{Ans.} \end{array}$$

As the multiplier is $6\frac{1}{3}$, it is evident, that to 6 times 638, there must be added one third of 638, the multiplicand, for the whole product. The third part of 638 is evidently 212, and 2 remainder, or $\frac{2}{3}$, because it is 2 parts of the 3 in the divisor.

Note. If the sum of the products, or quotients, of two or more numbers multiplied or divided by the same quantity, be required; then multiply or divide the sum of the numbers by the common multiplier or divisor, and the product or quotient will be the answer. If the difference of the products or quotients be required, then multiply or divide the difference of the numbers, as before, for the answer.

1. Required the sum and difference of the products of 27 and 22, multiplied each by 3.

Now $27\times3+22\times3$ = the sum $= 147=3\times49=3\times\overline{27+22}$.

And $27\times3-22\times3$ = the diff. $= 15=3\times5=3\times\overline{27-22}$.

2. Required the sum and difference of the quotients, by dividing 68 and 44, each by 4.

Now $68\div4+44\div4$ = the sum $= 28=112\div4=\overline{68+44}\div4$.

And $68\div4-44\div4$ = the diff. $= 6=24\div4=\overline{68-44}\div4$.

The same course may obviously be pursued in any similar case.

EXAMPLES FOR PRACTICE.

1. Divide 1292 dolls. equally among 17 men. Ans. 76 dolls. each.
2. Divide 2625 dolls. among 35 soldiers. Ans. 75 dolls. each.
3. If 360 cents are to be divided equally among eighteen poor persons, how much will each have? Ans. 20 cents.
4. If a field of 19 acres produce 513 bushels of wheat, how much is that for one third of an acre? Ans. 9 bushels.
5. A man receives 1095 pounds a year, what is it for a day? Ans. 3 pounds.
6. There are $5\frac{1}{2}$ yds. in a rod; how many yards are there in 40 rods? Ans. 220 yds.
7. What number must you multiply by 47, to produce 298804098? Ans. 6357534.
8. What number must be multiplied by 379 to produce 53789678? Ans.
9. In a rod are $16\frac{1}{2}$ feet; how many feet in 320 rods?
10. If 3650 pounds of bread are to be divided equally among 365 soldiers for 10 days, how much will each receive a day? Ans. 1 pound.
11. Multiply 34678 by 250. Ans. 866950.
12. Multiply 125 by 77777. Ans.

TABLES IN COMPOUND ADDITION.

1. Federal Money.*

		marked.		mills.
10 Mills	make one	Cent	m.c.	10= 1 cent.
10 Cents		Dime	d.	100= 10= 1 dime.
10 Dimes		Dollar	$.	1000= 100= 10= 1 dollar.
10 Dollars		Eagle	E.	10000=1000=100=10=1 eagle.

2. English Money.

			marked
4 Farthings	make one	Penny	*qrs. d.*
12 Pence		Shilling	*s.*
20 Shillings		Pound.	£.

* The following account is abſtracted from the "Act eſtabliſhing a Mint, and regulating the Coins of the United States," paſſed April 2nd, 1792.

The money of account of the United States ſhall be expreſſed in dollars or units, diſmes or tenths of a dollar, cents or hundredths of a dollar, and mills or thouſandths of a dollar.

The coins of gold, silver, and copper of the U. S. ſhall be of the following denominations, viz.

Gold.	1. Eagle, of the value of *ten*, dollars.
	2. Half Eagle,..........*five* dolls.
	3. Quarter Eagle,......*two and a half* dolls.
Silver.	4. Dollar, of the value of the *Spaniſh milled* dollar.
	5. Half Dollar,..................*half* the dollar.
	6. Quarter Dollar,..............*one fourth* the dollar.
	7. Dimes,..............................*one tenth* of the doll.
	8. Half Dime,.........................*one twentieth* of the doll.
Copper.	9. Cent,......................................*one hundredth* of the doll.
	10. Half Cent,..............................one half the cent.

The *ſtandard* for all *gold* coins of the U. S ſhall be *eleven* parts of pure gold and *one* part of alloy in *twelve* parts of the coin. The alloy is to be ſilver and copper, but the ſilver is not to exceed one half in the alloy.

The Eagle ſhall contain two hundred and forty ſeven and a half grains of pure gold, or two hundred and ſeventy grains of ſtandard gold; and the other gold coins in the ſame proportion.

The *ſtandard* for all ſilver coins of the U. S. ſhall be one thouſand four hundred and eighty five parts of pure ſilver and one hundred and ſeventy nine parts alloy; and the alloy ſhall be pure copper.

The Dollar ſhall contain three hundred and ſeventy one and one fourth grains of pure ſilver, or of four hundred and ſixteen grains of ſtandard ſilver; and the other ſilver coins in the ſame proportion.

The Copper coins are to be pure copper. The Cent is to contain eleven penny weights of copper; and the Half Cent in proportion.

The proportional value of gold to ſilver in all coins current by law in the U. S. ſhall be fifteen to one, or fifteen pounds weight of pure ſilver ſhall be equal to one pound weight of pure gold.

All coins of gold and ſilver, iſſued from the Mint of the U. S. ſhall be a lawful tender in all payments at the preceding values when of full weight, and if not of full weight, of proportional values.

Farthings.
4 = 1 Penny.
48 = 12 = 1 Shilling.
960 = 240 = 20 = 1 Pound.
A *groat* is 4*d*.

PENCE TABLES.

d.	s.	d.	d.	s.	d.	s.	d.	s.	d.
20 =	1	8	120 =	10	0	1 =	12	11 =	132
30 =	2	6	130 =	10	10	2 =	24	12 =	144
40 =	3	4	140 =	11	8	3 =	36	13 =	156
50 =	4	2	150 =	12	6	4 =	48	14 =	168
60 =	5	0	160 =	13	4	5 =	60	15 =	180
70 =	5	10	170 =	14	2	6 =	72	16 =	192
80 =	6	8	180 =	15	0	7 =	84	17 =	204
90 =	7	6	190 =	15	10	8 =	96	18 =	216
100 =	8	4	200 =	16	8	9 =	108	19 =	228
110 =	9	2	240 =	20	0	10 =	120	20 =	240

3. TROY WEIGHT.*

24 Grains	make one	Pennyweight,	marked	grs. pwt.
20 Pennyweights	- -	Ounce,	- -	oz.
12 Ounces	- - - -	Pound,	- - -	lb

Grains.
24 = 1 Pennyweight.
480 = 20 = 1 Ounce.
5760 = 240 = 12 = 1 Pound.

4. AVOIRDUPOIS WEIGHT.†

16 Drams	- make 1	Ounce,	marked	dr. oz.
16 Ounces	- -	Pound,	- - - - - - -	lb
28 Pounds	- - -	Quarter of a hundred wt.	-	qr.
4 Quarters	- -	Hundred wt. or 112 pounds,	-	Cwt.
20 Hundred wt.	- -	Ton,	- - - - - - -	T.

* By this weight are weighed Gold, Silver, Jewels, Electuaries, and all liquors.
An ounce of gold is divided into 24 parts, called carats, and an ounce of silver, into 20 parts, called pennyweights; therefore, to distinguish fineness of metals, such gold as will abide the fire without loss, is accounted 24 carats fine: If it lose 2 carats in trial, it is called 22 carats fine, &c.

A pound of silver which, loses nothing in trial, is 12 ounces fine; but, if it lose 3 pennyweights, it is 11 oz. 17 pwts. fine, &c.

Alloy is some base metal with which gold or silver is mixed to abate its fineness; 22 carats of gold, and 2 carats of copper, are esteemed the true standard for gold coin in England, the alloy being one eleventh part of the fine gold; and 11 oz. 2 pwts. of fine silver, melted with 18 pwts. of copper, make the true standard for silver coin.

NOTE. 175 Troy ounces, are precisely equal to 192 Avoirdupois ounces, and 175 Troy pounds are equal to 144 Avoirdupois. 1 lb. Troy = 5760 grains, and 1 lb. Avoirdupois = 7000 grains.

† By Avoirdupois are weighed all coarse and drossy goods, grocery and chandlery wares; bread, and all metals, except gold and silver.

A barrel of pork weighs 200 lb. A barrel of beef, 200 lb. A quintal of fish,

Drams.					
16 =	1 Ounce.				
256 =	16 =	1 Pound.			
7168 =	448 =	28 =	1 Quarter.		
28672 =	1792 =	112 =	4 =	1 Hund. wt.	
573440 =	35840 =	2240 =	80 =	20 =	1 Ton.

5. Apothecaries' Weight.*

20 Grains	make one	Scruple,	marked gr. ℈
3 Scruples	- -	Dram,	- - - - ʒ
8 Drams	- - -	Ounce,	- - - - ℥
12 Ounces	- -	Pound,	- - - - lb.

Grains.				
20 =	1 Scruple.			
60 =	3 =	1 Dram.		
480 =	24 =	8 =	1 Ounce.	
5760 =	288 =	96 =	12 =	1 Pound.

6. Cloth Measure.†

2 Inches, and one fourth	- make 1	Nail, marked in.	na.
4 Nails, or 9 Inches	- - -	Quarter of a yard,	qr.
4 Quarters of a yard, or 36 Inches	-	Yard, - - -	yd.
3 Quarters of a yard, or 27 Inches	-	Ell Flemish,	E. Fl.
5 Quarters of a yard, or 45 Inches	-	Ell English, -	E. E.
6 Quarters of a yard, or 54 Inches	-	Ell French, -	E. Fr.
4 Quarters, 1 Inch & one 5th, or 37 Inches and one fifth -	-	Ell Scotch, -	E. Sc.
3 Quarters and two thirds	- - -	Spanish Var.	

1 Cwt. Avoirdupois. 12 particular things make one dozen; 12 dozen 1 gross, and 144 dozen 1 great gross. 20 particular things make 1 score,

		lb.		lb.
A Firkin of Foreign	Butter	56	A Stone of Iron, shot, or horseman's weight,	14
——— ———	Soap	94	——Butcher's Meat,	8
A Barrel of ———	Anchovies	30	A gallon of Train Oil	7½
——— ———	Soap	256	A Tod is - - - -	28
——— ———	Raisins	112	A Weigh - - - -	182
A Punch. of———	Prunes	1120	A Sack - - - -	364
A Fother of———	Lead	19½ Cwt.	A last - - - -	4368

* All the weights now used by Apothecaries, above grains, are Avoirdupois. The Apothecaries' pound and ounce, and the pound and ounce Troy are the same, only differently divided and subdivided.

† All Scotch and Irish linens are bought by the English or American yard, which is the same, and all Dutch linens by the Ell Flemish; but are all sold in America by the American yard; though the Dutch linens are sold in England by the Ell English, and the Scotch and Irish linens, as in America.

The Scotch allow one English yard in every score yards.

Nails, 4 =	1 Quarter.		
16 =	4 =	1 Yard.	
12 =	3 =	1 Flemish Ell.	
20 =	5 =	1 English Ell.	
24 =	6 =	1 French Ell.	

7. Long Measure.*

3 Barley corns	make 1	Inch,	marked bar. in.
12 Inches	-	Foot,	ft.
3 Feet,	-	Yard,	yd.
$5\frac{1}{2}$ Yards, or $16\frac{1}{2}$ feet	-	Rod, Perch, or Pole,	pol.
40 Poles	-	Furlong,	fur.
8 Furlongs	-	Mile,	mile.
$69\frac{1}{2}$ Statute miles, *nearly*		Degree of a great Circle,	deg.
360 Degrees	-	A great Circle of the Earth.	

Or, in Measuring Distances.

$7\frac{92}{100}$ Inches	make 1	Link.
25 Links	-	Pole.
100 Links	-	Chain.
10 Chains	-	Furlong.
8 Furlongs	-	Mile.

Bar. corns, 3 =	1 Inch.					
36 =	12 =	1 Foot.				
108 =	36 =	3 =	1 Yard.			
594 =	198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1 Pole.		
23760 =	7920 =	660 =	220 =	40 =	1 Furlong.	
190080 =	63360 =	5280 =	1760 =	320 =	8 =	1 M.

Inches, $7\frac{92}{100}$ =	1 Link.				
198 =	25 =	1 Pole or Perch.			
792 =	100 =	4 =	1 Chain.		
7920 =	1000 =	40 =	10 =	1 Furlong.	
63360 =	8000 =	320 =	80 =	8 =	1 Mile.

8. Time.†

60 Seconds	make 1	Minute,	marked s. m.
60 Minutes	-	Hour,	h.
24 Hours	-	Day,	d.
7 Days	-	Week,	w.
4 Weeks	-	Month,	mo.
13 Months, 1 day and 6 hours	-	Julian year,	yr.

* The use of Long Measure is to measure the distance of places, or any other thing, where length is considered without regard to breadth.

Note. 60 geometrical miles make a degree. 4 inches a hand. 5 feet a geometrical pace. 6 points make 1 line, 12 lines an inch, 12 inches a foot, and 6 feet one French toise, or Fathom, equal to 6 feet 4 inches, 8.812,875 lines, English measure. 1 English foot equal to 11 inches 3.1154 lines French. 66 feet, or 4 poles, make a Gunter's chain. 3 miles make a league.

† By the Calendar, the year is divided in the following manner:

Thirty days hath September, April, June, and November;
February twenty eight alone, and all the rest have thirty one.

When you can divide the year of our Lord by 4, without any remainder, it is then Bissextile, or Leap Year, in which February has 29 days.

Seconds, 60 =	1 Minute.				
3600 =	60 =	1 Hour.			
86400 =	1440 =	24 =	1 Day.		
604800 =	10080 =	168 =	7 =	1 Week.	
2419200 =	40320 =	672 =	28 =	4 =	1 Month.
Sec.	*Min.*	*h*	*d. h.*	*w. d. h.*	
31557600 =	525960 =	8766 =	365 6 =	52 1 6 =	1 Julian year.*
				m. sec.	
31558154 =	525969 =	8766 =	365 6	9 14 =	1 Period. year.†
31556937 =	525948 =	8765 =	365 5	48 57 =	Tropical year.‡

9. Motion.

60 Seconds	make 1 Prime minute, marked	″ ′
60 Minutes	Degree,	°
30 Degrees	Sign,	s.
12 Signs, or 360 degrees	The whole great circle of the Zodiack.§	

Seconds, 60 =	1 Minute.		
3600 =	60 =	1 Degree.	
108000 =	1800 =	30 =	1 Sign.
1296000 =	21600 =	360 =	12 = Zodiack.

10. Land or Square Measure.

144 Inches	make 1 Square foot.
9 Feet	——— Yard.
30¼ Yards, or 272¼ Feet	——— Pole.
40 Poles	——— Rood.
4 Roods, or 160 Rods, or 4840 yards	——— Acre.
640 Acres	——— Mile.

Inches, 144=	1 Foot.					
1296=	9=	1 Yard.				
39204=	272¼=	30¼=	1 Pole.			
1568160=	10890=	1210=	40=	1 Rood.		
6272640=	43560=	4840=	160=	4=	1 Acre.	
4014489600=	27878400=	3097600=	102400=	2560=	640=	1 Mile.

* The civil ſolar year of 365 days, being ſhort of the true by 5h 48m. 48ſec. occaſioned the beginning of the year to run forward through the ſeaſons nearly 1 day in 4 years. On this account, Julius Cæſar ordained that one day ſhould be added to February, every fourth year, by cauſing the 24th day to be reckoned twice; and becauſe this 24th day was the ſixth, (ſextilis) before the kalends of March, there were in this year, two of theſe ſextiles, which gave the name of Biſſextile to this year, which being thus corrected, was from thence called the Julian year.

† A juſt and equal meaſure of the year is called the periodical year, as being the time of the earth's period about the ſun; in departing from any fixed point in the heavens, and returning to the ſame again.

‡ The ſeveral points of the ecliptick having a retrograde, or backward motion, the equinox will, as it were, meet the ſun; by which mean the ſun will arrive at the Equinox, or firſt point of Aries, before his revolution is completed, and this ſpace of time is called the tropical year.

§ The Zodiack is a great circle of the ſphere, containing the 12 ſigns, through which the ſun paſſes.

11. Solid Measure.*

1728 Inches	make 1	Foot.
27 Feet		Yard.
40 Feet of round Timber, or 50 feet of hewn Timber,		Ton or Load.
128 Solid Feet, i. e. 8 in length, 4 in breadth and 4 in height,		Cord of Wood.

12. Wine Measure.†

2 Pints	make 1	Quart,	marked pts. qts.
4 Quarts		Gallon,	gal.
10 Gallons		Anchor of Brandy,	anc.
18 Gallons		Runlet,	run.
$31\frac{1}{2}$ Gallons		Half an Hogshead,	$\frac{1}{2}$hhd.
42 Gallons.		Tierce,	tier.
63 Gallons		Hogshead,	hhd.
2 Hogsheads		Pipe or butt,	P. or B.
2 Pipes		Tun,	Tun.

Cubick Inches.

$28\frac{7}{8}$ = 1 Pint.
$57\frac{3}{4}$ = 2 = 1 Quart.
231 = 8 = 4 = 1 Gallon.
9702 = 336 = 168 = 42 = 1 Tierce.
14553 = 504 = 252 = 63 = $1\frac{1}{2}$ = 1 Hogshead.
19404 = 672 = 336 = 84 = 2 = $1\frac{1}{3}$ = 1 Puncheon.
29106 = 1008 = 504 = 126 = 3 = 2 = $1\frac{1}{2}$ = 1 Pipe.
58212 = 2016 = 1008 = 252 = 6 = 4 = 3 = 2 = 1 Tun.

13. Ale or Beer Measure.‡

2 Pints	make 1	Quart,	marked pts. qts.
4 Quarts		Gallon,	gal.
8 Gallons		Firkin of Ale in Lond.	A. fir.
$8\frac{1}{2}$Gallons		Firkin of Ale or Beer.	
9 Gallons		Firkin of Beer in Lond.	B. fir.
2 Firkins		Kilderkin,	kil.
2 Kilderkins		Barrel,	bar.
$1\frac{1}{2}$ Barrel, or 54 Gallons		Hogshead of Beer,	hhd.
2 Barrels		Puncheon,	pun.
3 Barrels or 2 Hogsheads		Butt,	butt.

* By Solid Meafure are meafured all things that have length, breadth and depth.

† All Brandies, Spirits, Perry, Cider, Mead, Vinegar, Honey and Oil, are meafured by Wine Meafure: Honey is commonly fold by the pound Avoirdupois.

‡ Milk is fold by the Beer quart.

A barrel of Mackarel, and other barrelled fifh, by law in Maffachufetts, is to contain not lefs than 30 gallons; in Connecticut and New York the Shad and Salmon Barrel muft contain 200 lb.

In England, a barrel of Salmon or Eels is 42 gallons, and a barrel of Herrings 32 gallons. The gallon, appointed to be ufed for meafuring all kinds of Liquors, in Ireland, is two hundred and feventeen cubick inches, and fix tenths.

Beer.

Cubick Inches.

35¼ = 1 Pint.
70½ = 2 = 1 Quart.
282 = 8 = 4 = 1 Gallon.
2538 = 72 = 36 = 9 = 1 Firkin.
5076 = 144 = 72 = 18 = 2 = 1 Kilderkin.
10152 = 288 = 144 = 36 = 4 = 2 = 1 Barrel.
15228 = 432 = 216 = 54 = 6 = 3 = 1½ = 1 Hogshead.
20304 = 576 = 288 = 72 = 8 = 4 = 2 = 1⅓ = 1 Puncheon.
30456 = 864 = 432 = 108 = 12 = 6 = 3 = 2 = 1½ = 1 Butt.

Ale.

Cubick Inches.

35¼ = 1 Pint.
70½ = 2 = 1 Quart.
282 = 8 = 4 = 1 Gallon.
2256 = 64 = 32 = 8 = 1 Firkin.
4512 = 128 = 64 = 16 = 2 = 1 Kilderkin.
9024 = 256 = 128 = 32 = 4 = 2 = 1 Barrel.
13536 = 384 = 192 = 48 = 6 = 3 = 1½ = 1 Hogshead.

16. Dry Measure.*

2 Pints	make 1	Quart, marked pts.	qts.
2 Quarts		Pottle,	pot.
2 Pottles		Gallon	gal.
2 Gallons		Peck,	pk.
4 Pecks		Bushel,	bu.
2 Bushels		Strike,	str.
2 Strikes		Coom,	co.
2 Cooms		Quarter,	qr.
4 Quarters		Chaldron,	ch.
4½ Quarters		Chaldron in London.	
5 Quarters		Wey,	wey.
2 Weys		Last,	last.

Cubick Inches.

268⅘ = 1 Gallon.
537⅗ = 2 = 1 Peck.
2150⅖ = 8 = 4 = 1 Bushel.
4300⅘ = 16 = 8 = 2 = 1 Strike.
8601⅗ = 32 = 16 = 4 = 2 = 1 Coom.
17203⅕ = 64 = 32 = 8 = 4 = 2 = 1 Quarter.
36016 = 320 = 160 = 40 = 20 = 10 = 5 = 1 Wey.
172032 = 640 = 320 = 80 = 40 = 20 = 10 = 2 = 1 Last.

* This measure is applied to all dry goods, as Corn, Seed, Fruit, Roots, Salt, Sand, Oysters and Coals.

A Winchester bushel, is 18½ inches diameter, and 8 inches deep.

COMPOUND ADDITION

IS the adding of several numbers together, having different denominations as Pounds, Shillings, Pence, &c. Tons, Hundreds, Quarters, &c.

RULE.*

I. Place the numbers so that those of the same denomination may stand directly under each other.

II. Add the first column or denomination together as in whole numbers; then divide the sum by as many of the same denomination as make one of the next greater, setting down the remainder under the column added, and carry the quotient to the next superiour denomination, continuing the same to the last, which add as in simple addition.

EXAMPLES.

1. FEDERAL MONEY.

1.

E.	D.	d.	c.	m.
7	3	8	9	5
	2	1	2	5
9	0	0	5	
	3	6	2	5
7	1	4	0	8
24	1	1	0	3

2.

D.	c.	m.
49	18	7
25	32	1
93	7	5
13	25	
	97	2

3.

D.	c.	m.
375		
29	13	
7	12	5
199	18	7
30	01	

2. ENGLISH MONEY.

1.

£	s.	d.
9	16	10
7	10	9
0	18	6
5	11	11
6	0	8
5	9	10
35	8	6

2.

£	s.	d.	qr.
47	17	6	2
3	9	10	3
75	13	9	1
4	11	11	0
0	16	8	2
17	6	2	1

3.

£	s.	d.	qr.
847	11	11	2
491	19	6	1
59	6	10	0
747	16	1	2
849	12	11	3
741	17	8	2

4.

£	s.	d.	qr.
915	10	10	2
64	8	9	1
5	16	11	3
419	2	10	2
491	19	11	3
762	17	6	1

As the denominations of Federal Money increase like whole numbers, in a ten fold ratio, the operation is the same as in whole

* The reason of this rule is evident from what has been said in Simple Addition: For, in addition of money, as 1, in the pence is equal to 4 in the farthings; 1, in the shillings, to 12 in the pence; and 1, in the pounds, to 20 in the shillings; therefore, carrying as directed, is the arranging the money, arising from each column, properly, in the scale of denominations; and this reasoning will hold good in the addition of compound numbers, of any denomination whatever.

numbers. But in denominations which do not increase in the same manner, the operations are somewhat different. Thus, in Ex. 1. of English Money, I find the sum of the pence to be 54. Now 54 pence are 4 shillings and 6 pence; therefore, I set down 6 under the pence, and carry 4 to the shillings, which I then find to be 68. But 68 shillings are 3 pounds and 8 shillings. I set down the 8 under the shillings, and carry 3 to the pounds, and the sum of the pounds is 35, which I set down. The sum of the whole is then 35 pounds, 8 shillings and 6 pence. The process is similar in each Example. In all sums of different denominations, the student should be careful to find the numbers by which the denominations in the Table increase, for by them he is to carry from one denomination to another.

3. Troy Weight.

1.

lb.	oz.	pwt.	gr.
767	10	17	22
39	6	9	17
417	11	16	18
935	9	17	19
478	10	17	22
387	9	16	15
3027	11	16	17

2.

lb.	oz.	pwt.	gr.
649	11	19	20
32	9	6	5
841	10	11	19
473	9	17	23
764	11	8	9
165	6	10	19
2028	11	4	23

3.

lb.	oz.	pwt.	gr.
859	9	15	20
437	10	17	22
640	11	6	0
738	9	12	18
49	0	16	17
584	10	0	9

In the 1st Ex. I find the sum of the grains to be 113. Now 113 grs. are 4 pwts. and 17 grs. because 24 is contained in 113, four times, and 17 is the remainder. Then I set down 17 under the grs. and carry 4 to the pwts. and their sum is 96. Now 96 pwts. are 4 oz. and 16 pwts. for 20 pwts. make 1 oz.; therefore I set 16 under the pwts. and carry 4 to the ounces, which makes their sum 59. But 59 oz. are 4 lbs. and 11 oz. because 12 oz. make a lb.; therefore I set down 11 oz. and carry 4 to the lbs. which makes their sum 3027. The answer, then is 3027 lbs. 11 oz. 16 pwts, and 17 grs.

4. Avoirdupois Weight.

1.

lb.	oz.	dr.
19	13	12
21	9	6
4	15	15
22	10	5
18	13	12
6	11	10
94	10	12

2.

Cwt.	qrs.	lb.
17	3	19
18	1	27
9	2	9
14	3	16
12	0	6
15	2	0

3.

T.	Cwt.	qrs.	lb.
59	13	2	17
6	17	1	21
45	11	3	25
57	16	2	19
75	17	3	17
6	19	0	26

4.

T.	Cwt.	qrs.	lb.	oz.	dr.
91	17	2	25	13	15
19	9	0	17	10	12
14	13	2	0	9	11
47	11	3	19	14	0
69	0	1	0	0	12
77	19	3	27	15	11

5. Apothecaries' Weight.

1.

ʒ	℈	gr.
9	1	17
3	2	19
6	1	17
4	0	6
5	2	12
8	1	10
33	2	1

2.

℥	ʒ	℈	gr.
10	7	2	19
6	3	0	12
7	6	1	17
9	5	2	12
6	1	0	16
9	3	2	18

3.

℔	℥	ʒ	℈	gr.
12	11	6	1	15
4	9	1	0	12
91	10	7	2	16
4	8	1	2	19
6	0	0	1	10
4	9	2	1	6

4.

℔	℥	ʒ	℈	gr.
5	9	3	2	13
4	8	6	0	19
9	10	5	2	12
6	5	6	1	17
8	9	4	0	0
7	1	0	1	17

6. Cloth Measure.

1.

Yd.	qr.	n.
76	2	3
3	3	1
42	3	3
57	2	2
16	3	3
49	2	2

2.

E.E.	qr.	n.
91	3	2
49	4	3
6	2	3
84	4	1
7	0	0
61	2	1

3.

E.Fl.	qr.	n.
75	2	1
7	1	3
84	0	2
76	2	3
48	2	2
9	2	3

4.

E.Fr.	qr.	n.
49	3	3
19	5	2
24	2	1
67	4	3
48	2	2
6	3	3

5.

Yds.	qr.	n.
914	2	3
49	2	1
561	3	0
84	0	2
549	3	1
617	1	3

7. Long Measure.

1.

Ft.	in.	bar.
9	11	2
6	9	1
7	0	2
8	10	0
9	6	2
7	10	2

2.

Yd.	ft.	in.
7	2	11
4	1	6
6	0	10
7	2	9
8	1	10
9	2	11

3.

Pol.	ft.	in.
12	11	10
9	10	9
8	12	11
7	15	6
4	14	9
5	11	11

4.

Mil.	fur.	pol.
9	7	36
7	3	19
4	1	24
6	5	12
4	6	9
5	1	10

5.

Deg.	mi.	fur.	pol.	ft.	in.	bc.
759	56	6	29	15	10	2
317	39	1	36	11	6	1
497	63	7	24	9	8	1
562	17	0	11	13	11	0
64	48	5	17	9	4	2
764	52	4	19	15	11	1

8. Time.

1.

W.	d.	h.	m.	s.
3	6	22	57	42
1	5	19	31	28
2	3	17	9	15
3	0	9	17	58
1	1	16	19	10
2	2	20	53	48

2.

Mo.	d.	h.	m.
5	24	19	43
4	27	21	35
9	18	0	12
4	19	23	19
8	11	12	13
9	19	8	29

3.

Y.	m.	d.
19	10	17
7	9	27
4	8	16
1	11	14
17	6	9
12	5	20

4.

Y.	mo.	w.	d.	h.	m.	s.
57	11	3	6	23	29	55
4	8	1	1	19	45	38
29	9	2	3	17	18	19
46	10	2	5	11	50	13
19	9	2	1	16	18	17
45	9	3	5	18	17	59

9. Motion.

1.

17°	55′	48″
1	37	51
29	19	45
19	19	37

2.

25°	49′	51″
5	21	36
19	47	18
25	25	39

3.

9s	29°	35′	53″
10	0	18	31
4	17	13	42
6	19	50	0

10. Land or Square Measure.

1.

Pol.	feet.	in.
36	179	137
19	248	119
12	96	75
18	110	122
9	269	24
25	221	143

2.

Yds.	ft.	in.
28	7	119
9	3	75
29	6	120
4	8	12
9	1	119
8	3	43

3.

Acres.	rood.	pol.	feet.	in.
756	3	37	245	128
29	1	28	93	25
516	3	31	128	119
37	1	19	218	20
61	0	0	92	103
191	1	25	129	136

11. Solid Measure.

1.

Ton.	feet.	in.
29	36	1229
12	19	64
18	11	917
19	8	1001
5	0	523
17	39	1119

2.

Yds.	feet.	in.
75	22	1412
9	26	195
3	19	1091
28	15	1110
49	24	218
18	17	1225

3.

Cord.	feet.	in.
37	119	1015
9	110	159
48	127	1017
8	111	956
21	9	27
9	28	1091

12. Wine Measure.

1.

Tier.	gal.	qts.	pts.
37	36	3	1
9	17	2	1
35	28	9	0
32	19	1	1
9	0	3	1
12	40	1	1

2.

Hhd.	gal.	qts.	pts.
51	58	1	1
27	39	3	0
9	18	0	1
9	9	2	1
16	24	1	1
5	0	3	0

3.

Ton.	hhd.	gal.	qts.
37	2	37	2
19	1	59	1
28	2	0	0
19	0	47	1
37	1	17	3
14	2	48	2

15. Ale and Beer Measure.

1.

A.B.	fi.	gal.
49	3	7
26	2	3
9	0	4
17	3	0
27	1	6
19	3	7

2.

B.B.	fir.	gal.
29	1	8
19	3	5
16	0	3
9	1	8
14	2	0
17	1	5

3.

Hhd.	gal.	qts.
379	53	3
19	0	1
121	37	2
467	19	1
561	16	0
75	0	2

16. Dry Measure.

1.

Qrs.	bu.	p.	qts.
64	7	3	7
9	4	1	5
19	6	2	1
4	0	2	0
17	3	0	6
9	5	3	4

2.

Bus.	p.	qts.	pts.
37	2	5	1
19	3	7	1
16	2	0	0
5	1	6	1
9	0	3	0
19	3	0	1

3.

Ch.	bu.	p.	qts.
37	27	3	7
6	29	1	5
15	30	0	0
4	11	3	[illegible]
5	0	1	0
2	0	2	1

COMPOUND SUBTRACTION

TEACHES to find the difference, inequality, or excess, between any two sums of divers denominations.

Rule.*

Place those numbers under each other, which are of the same denomination, the less being below the greater; begin with the least denomination, and, if it exceed the figure over it, add as many units as make one of the next greater; subtract it therefrom; and to the difference add the upper figure, remembering, always, to add one to the next superior denomination, for that which you added before.

Examples.

1. Federal Money.

	$	c.	m.
From	39	15	5
Take	23	17	2
Diff.	10	98	3

E.	$	c.	m.
21	8	1	2
10	7	5	

$	c.	m.
100		
48	87	5

	$	c.
Borrowed	100	
Paid	29	18
Remains to pay		

	$	c.
Lent	200	
Received	145	50
Due to me		

	$	c.	m.
Borrowed	3000		
Paid at several times.	195		
	1115	49	
	247	37	5
	995	12	5
Paid in all	2552	99	0
Remains to pay	447	01	0

	$	c.	m.
Lent	7159	12	8
Received at several times.	245	37	5
	3112	15	7
	2000		
	1092	92	0
Received in all			
Remains due			

2. English Money.

1.

	£	s.	d.	qr.
Borrowed	349	15	6	1
Paid	195	11	8	1
Rem. to pay	154	3	10	0
Proof				

2.

	£	s.	d.	qr.
Lent	791	9	8	1
Received	197	16	4	2
Due to me				

* The reason of this Rule will readily appear, from what was said in Simple Subtraction; for the adding depends upon the same principle, and is only different, as the numbers to be subtracted are of different denominations.

In the 1st Ex. of English Money, I take 1 qr. from 1 qr. and set down 0, the remainder. Because I cannot take eight from 6 pence, I add to 6, 12 pence which make a shilling, and from 18 take 8, and set down 10, the difference. As I added 12 pence = 1 shilling to the upper pence, I now carry 1 shilling to the lower shillings, and take 12 from 15, and set down 3, the remainder. The rest of the process is evident. It is obvious that a similar course must be pursued in the Examples under the several weights and measures.

3. Troy Weight.

	1.				2.				3.			
	lb.	oz.	pwt.	dr.	lb.	oz.	pwt.	gr.	lb.	oz.	pwt.	gr.
Bought	749	5	13	16	189	8	12	10	543	3	9	13
Sold	96	9	19	13	148	4	16	19	179	1	16	18
Rem.	652	7	14	03								

4. Avoirdupois Weight.

	1.			2.			3.				4.					
	lb.	oz.	dr.	C.	qr.	lb.	T.	cwt.	qr.	lb.	T.	cwt.	qr.	lb.	oz.	dr.
Bought	7	9	12	8	2	13	5	13	1	12	9	11	3	17	5	12
Sold	3	12	9	4	1	15	1	12	2	17	3	12	1	19	10	9
Rem.	3	13	3													

5. Apothecaries' Weight.

1.					2.					3.				
℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.	℔	℥	ʒ	℈	gr.
71	9	3	1	13	65	10	6	2	10	84	1	1	1	1
37	8	4	1	16	31	8	4	2	9	65	9	3	1	17
34	0	6	2	17										

6. Cloth Measure.

1.			2.			3.			4.		
Yds.	qr.	n.	E. E.	qr.	n.	E. Fl.	qr.	n.	E. Fr.	qr.	n.
35	1	2	467	3	1	765	1	3	549	4	2
19	1	3	291	3	2	149	2	1	197	4	3
15	3	3									

7. Long Measure.

1.			2.			3.			4.							
Yds.	ft.	in.	Pol.	ft.	in.	Mil.	fur.	pol.	Deg.	m.	fur.	p.	yds.	ft.	in.	bar.
28	2	10	21	11	9	76	3	11	38	41	3	29	2	1	7	2
17	2	11	9	13	8	27	3	21	19	35	5	31	3	1	9	1
10	2	11														

8. Time.

1.					2.				3.			4.						
Mo.	d.	h.	m.	s.	Mo.	w.	d.	h.	Y.	mo.	d.	Y.	mo.	w.	d.	h.	m.	s.
6	17	13	27	19	9	2	5	15	7	3	13	48	9	2	5	19	27	31
1	21	16	41	35	4	3	5	15	4	2	19	19	9	3	4	20	19	49
4	25	20	45	44														

9. Motion.

1.

79°	21′	31″
41	41	52

2.

6s	11°	12′	48″
3	8	39	29

3.

4s	19°	41′	22″
1	22	19	45

10. Land or Square Measure.

1.

A.	R.	Pol.
29	1	10
24	1	25

2.

A.	R.	Pol.
29	2	17
17	1	36

3.

A.	R.	Pol.	ft.	in.
56	3	19	27	110
29	0	21	210	129

11. Solid Measure.

1.

Tons.	ft.	in.
49	19	1100
38	36	1296

2.

Yds.	ft.	in.
79	11	917
17	25	1095

3.

Cords.	ft.	in.
349	97	1250
192	127	1349

12. Wine Measure.

1.

Hhd.	gal.	qts.	pts.
79	21	2	1
38	61	3	1

2.

Tier.	gal.	qts.
19	17	1
12	29	2

3.

Hhd.	gal.	qts.
375	41	2
197	36	3

4.

Tun.	hhd.	gal.
532	1	19
197	1	47

13. Ale and Beer Measure.

1.

A.B.	fir.	gal.	qts.
39	1	2	1
24	3	6	2

2.

B.B.	fir.	gal.	qts.	pts.
21	3	5	2	0
19	1	7	2	1

3.

Hhds.	gal.	qts.
769	17	1
391	42	3

14. Dry Measure.

1.

Qu.	bu.	pk.	qts.
56	2	2	1
39	3	1	2

2.

Bu.	pk.	qts.	pts.
91	1	3	2
29	2	1	1

3.

Chal.	bu.	pk.	qts.
39	12	2	1
24	25	3	2

PROBLEMS

RESULTING FROM A COMPARISON OF THE PRECEDING RULES.

PROB. 1. Having the sum of two numbers, and one of them given, to find the other.

Rule. Subtract the given number from the given sum, and the remainder will be the number required.

Let 288 be the sum of two numbers; one of which is 115, the other is required?

From 288 the Sum,
Take 115 the given number.
Rem. 173 the other.

PROB. 2. Having the greater of two numbers, and the difference between that and the less given, to find the less.

Rule. Subtract the one from the other.

Let the greater number be 325, and the difference between that and the other, 198: What is the other?

From 325 the greater,
Take 198 the difference.
Rem. 127 the less.

PROB. 3. Having the least of two numbers given, and the difference between that and a greater, to find the greater.

Rule. Add them together.

Given { 127 the less number.
  198 the difference.

Sum 325 the greater number required.

PROB. 4. Having the sum and difference of two numbers given, to find those numbers.

Rule. To half the sum add half the difference, and the sum is the greater, and from half the sum take half the difference, and the remainder is the less. Or, from the sum take the difference, and half the remainder is the least: to the least add the given difference, and the sum is the greatest.

What are those two numbers, whose sum is 48, and difference 14?

2)48 2)14 $24+7=31$ the greater, and $24-7=17$ the less.
$\frac{1}{2}$ sum$=24$ $\frac{1}{2}$diff.$=7$ Or $\overline{48-14}\div2=17$, & $17+14=31$.

This rule is obvious on considering any example in the following manner. Thus, let the two numbers be 32 and 46; their sum is $\overline{46+32}$, and their difference is $\overline{46-32}$. Now $\overline{46+32}\div2+\overline{46-32}\div2=\overline{46+32+46-32}\div2=\overline{46+46}\div2=46$, the greater. And $\overline{46+32}\div2-\overline{46+32}\div2=\overline{46+32-46+32}\div2=\overline{32+32}\div2=32$, the less. For, in the first case, the 32 to be subtracted from 32, leaves nothing; and, in the latter, the 46 is balanced by the other 46, and you have only $\overline{32+32}\div2=32$.

The preceding and following problems are evident from the rules of Addition and Subtraction, Multiplication and Division.

PROB. 5. Having the product of two numbers, and one of them given, to find the other.

Rule. Divide the product by the given number, and the quotient will be the number required.

Let the product of two numbers be 288 and one of them 8; I demand the other?

8)288
Answer, 36

PROB. 6. Having the dividend and quotient, to find the divisor.

Rule. Divide the dividend by the quotient.

COR. Hence we get another method of proving *Division.*

Given { 288 the Dividend. 36 the Quotient.
Required the Divisor.

36)288(8 Divisor.
288

PROB. 7. Having the Divisor and Quotient given, to find the Dividend.

Rule. Multiply them together.

Given { 8 the Divisor. 36 the Quotient.
Required the Dividend.

36
8
288 the Dividend.

By a due consideration and application of these Problems only, many questions (of which kind are some of the following) may be resolved in a short and elegant manner, although some of them are generally supposed to belong to higher rules.

APPLICATION OF THE PRECEDING RULES.

1. The least of two numbers is 19418, and the difference between them is 2384: What is the greater, and sum of both?

19418+2384=21802 greater, and 19418+21802=41220 sum.

2. Suppose a man born in the year 1743; when will he be 77 years of age? 1743+77=1820 *Answer.*

3. What number is that, which, being added to 19418, will make 21802? 2384 *Ans.*

4. Gen. Washington was born in 1732; what was his age in 1799? 67 *Ans.*

5. America was discovered by Columbus in 1492 and its independence declared in 1776: How many years elapsed between those two eras? 284 *Ans.*

6. The Massacre at Boston, by the British troops, happened March 5th, 1770, and the Battle at Lexington, April 19th, 1775: How long between?

April 19th, 1775—March 5th, 1770=5 y. 1 m. 14 d. *Ans.*

7. Gen. Burgoyne and his army were captured October 17th, 1777, and Earl Cornwallis and his army, October 19th, 1781: What space of time between? 4 years and 2 days. *Ans.*

8. The war between America and England commenced April 19th, 1775, and a general peace took place January 20th, 1783: How long did the war continue? 7 y. 9 m. 1 d. *Ans.*

9. A, B, C and D purchased a quantity of goods in partnership; A paid £12 10*s.* a dollar* and a crown† piece; B, 35*s.* C, 29*s.* 10*d.* and D, 79*d.*: What did the goods cost? *Ans.* £16 14 1.

* 6*s.* † 6*s.* 8*d.*

10. A man borrowed, at different times, these several sums, viz. £29 5*s*. £18 17*s*. 6*d*. £45 12*s*. £98, 3 dollars, one crown piece and an half: Pray how much was he in debt? *Ans*. £193 2 6.

11. There are four numbers; the first 317, the second 912, the third 1229, and the fourth as much as the other three, abating 97: What is the sum of them all? *Ans*. 4819.

12. Bought a quantity of goods for £125 10*s*. paid for truckage 45*s*. for freight 79*s*. 6*d*. for duties 35*s*. 10*d*. and my expences were 53*s*. 9*d*.: What did the goods stand me in? *Ans*. £136 4 1.

13. A Gentleman left his son £1725 more than his daughter, whose fortune was 15 thousand, 15 hundred and 15 pounds: What was the son's portion, and what did the whole estate amount to?

Ans. The son's fortune, £18240, and the whole estate £34755.

14. A merchant had 6 debtors, who together owed him £2917 10*s*. 6*d*. A, B, C, D and E, owed him £1675 13*s*. 9*d*. of it: What was F's debt? *Ans*. £1241 16 9.

15. What is the difference between £1309 7*s*. 1*d*. and the amount of £345 13*s*. 4*d*. and £571 4*s*. 8*d*.? *Ans*. £392 9 1

16. A merchant, at his first engaging in trade, owed £937 15*s*. he had in cash £1756 3*s*. 6*d*. in goods £459 12*s*. 3*d*. in good debts £197 16*s*. and he cleared the first year £249 19*s*. 10*d*. What was the neat balance at the year's end? *Ans*. £1724 16 7.

17. What sum of money must be divided between 12 men, so as that each may receive £155? £1860 *Ans*.

18. What number must I multiply by 9, that the product may be 675? 75 *Ans*.

19. A privateer of 175 men took a prize, which amounted to £59 per man, beside the owner's half: What was the value of the prize? £20650 *Ans*.

20. What is the difference between thrice five, and thirty, and thrice thirty five? 60 *Ans*.

21. The sum of two numbers is 750; the less 248: What is their difference and product? diff. 254, 124496 product.

22. What is the difference between six dozen dozen, and half a dozen dozen; and what is their product, and the quotient of the greater by the less?

Ans. 792 difference, 62208 product, and 12 quotient.

23. There are two numbers; the greater of them is 25 times 78, and their difference is 9 times 15; their sum and product are required.

Ans. 1950 the greater, 1815 the less. 3765 the sum, and 3539250 the product.

24. A merchant began trade with £25327; for six years together, he cleared £1253 per annum; the next 5 years, he cleared £1729 per annum; but, the last 4 years, had the misfortune to lose £3019 per annum: What was he worth at the 15 years' end? *Ans*. £29414.

25. If a man spends £192 in a year: What is that per calendar month? £16 *Ans*.

26. If the Federal Debt, which is 42 million dollars, be equally divided between the 13 States: What will be the share of each?
Ans. $3230769\frac{3}{13}$ dollars.

27. If 9000 men march in a column of 750 deep: How many march abreast? 12 *Ans.*

28. What number, deducted from the 32d part of 3072, will leave the 96th part of the same? 64 *Ans.*

29. What number is that, which, multiplied by 3589, will produce 92050672? 25648 *Ans.*

30. Suppose the quotient arising from the division of two numbers to be 5379, the divisor 37625: What is the dividend, if the remainder came out 9357? 202394232 *Ans.*

31. There is a certain number, which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient 20 being subtracted, and 30 added to the remainder, the half sum shall make 35: Can you tell me the number?
700 *Ans.*

32. A sheepfold was robbed three nights successively; the first night, half the sheep were stolen, and half a sheep more; the second half the remainder were lost, and half a sheep more; the last night they took half what were left and half a sheep more; by which time they were reduced to 30: How many were there at first?

Begin with 30, and, reckoning back from the last night to the first, you will find that 31 were stolen the 3d night, 62 the 2d, and 124 the first. *Ans.* 247.

33. Two boys, A and B, had 850 chesnuts between them; but A had 150 more than B: How many had each.
$850\div2=425$ half sum, and $150\div2=75$ half diff.; then $425+75=500$ A's, and $425-75=350$ B's.

34. What number added to the 27th part of 6615, will make 570?
325 *Ans.*

REDUCTION

TEACHES to bring numbers of one denomination to others of different denominations, retaining the same value.

It is of *two* sorts, viz. Descending and Ascending.

REDUCTION DESCENDING

Teaches to change numbers from a higher to a lower denomination. It is performed by multiplication.

Rule.*

Multiply the highest denomination given, by so many of the next less as make one of that greater, and thus continue until you have brought it down as low as your question requires.

Proof. Change the order of the question, and divide your last product by the last multiplier, and so on.

Note. From this rule and Case VI. of Simple Multiplication, it appears, that *Federal Money* is reduced from higher to lower denominations by annexing as many cyphers as there are places from the denomination given, to that required; or, if the given sum be of different denominations, by annexing the several figures of all the denominations in their order, and continuing with cyphers, (if necessary,) to the denomination required; or, what amounts to the same thing, by reading the whole number from the left to the required denomination, as one number in the required denomination.

Examples.

1. In 3 eagles 2 dollars, how many mills? *Ans.* 32000 *m.*
2. In 91 dollars 75 cents, how many cents? *Ans.* 9175 *c.*
3. In 50 eagles, how many dollars? *Ans.* 500 *D*
4. In 44 dollars, 1 cent, 4 mills, how many mills?
5. In 9 dollars, 31 cents, 7 mills, how many mills?
6. How many cents in 39 dollars 5 cents?
7. In 28 dollars 17 cents, 5 mills, how many mills?
8. In £27 15*s.* 9*d.* 2*qrs.* how many farthings?

£ *s.* 15 *d.* *qr.*
27 11 9 2
multiplied by 20=*shillings in a pound.*

555=*shillings.*
——*by* 12=*pence in a shilling.*

6669=*pence.*
——*by* 4=*farthings in a penny.*

Ans. =26678 farthings.

Note. In multiplying by 20, I added in the 15s. by 12, the 9d. and by 4, the 2qrs. which must always be done in like cases.

To prove the above question, change the order of it, and it will stand thus: In 26678 farthings how many pounds?

* The reason of this Rule is exceedingly obvious; for pounds are brought into shillings by multiplying them by 20; shillings into pence by multiplying them by 12; and pence into farthings by multiplying them by 4; and the contrary by division; and this will be true in the reduction of numbers consisting of any denomination whatever. The rule for Reduction ascending is simply the reverse of this, and equally evident.

4)26678

12)6669 2qrs.

2|0)55|5 9 d.

Answer, £27 15 9 2

9. In £36 12s. 10d. 1qr. how many farthings? Ans. 35177.
10. In £95 11s. 5d. 3qrs. how many farthings? Ans. 91751.
11. In £719 9s. 11d. how many half pence? Ans. 345358.
12. In 29 guineas, at 28s. how many pence? Ans. 9744.
13. In 37 pistoles, at 22s. how many shillings, pence, and farthings? Ans. 814s. 9768d. 39072qrs.
14. In 49 half johannes, at 48s. how many sixpences? Ans. 4704.
15. In 473 French crowns, at 6s. 8d. how many threepences? Ans. 12613⅓.
16. In 53 moidores, at 36s. how many shillings, pence and farthings? Ans. 1908s. 22896d. 91584qrs.
17. In £29 how many groats, threepences, pence, and farthings? Ans. 1740 groats, 2320 threepences, 6960d. 27840qrs.

18 Reduce 47 guineas and one fourth of a guinea into shillings, sixpences, groats, threepences, twopences, pence and farthings.

Ans. 1323 shillings, 2046 sixpences, 3969 groats, 5292 threepences, 7938 twopences, 15876 pence, and 63504 qrs.

REDUCTION ASCENDING

Teaches to change numbers from a lower to a higher denomination. It is performed by division.

Rule.

Divide the lowest denomination given, by so many of that name, as make one of the next higher, and thus continue till you have brought it into that denomination which your question requires.

Note. From this rule and the note under Case II. of Simple Division, it appears, that *Federal Money* is reduced from lower to higher denominations by cutting off as many places as the given denomination stands to the right of that required; the figures cut off belonging to their respective denominations.

Examples.

1. How many eagles in 42000 mills? Ans. 4 E. $ 2
2. In 3175 cents, how many dollars? Ans. $ 31 75 c.
3. In 500 dollars how many Eagles? Ans. 50
4. In 4414 mills, how many dimes?
5. In 9317 mills, how many dollars?
6. How many dollars in 28175 mills?
7. In 547325 farthings, how many pence, shillings, and pounds?

Farthings in a penny = 4)547325

Pence in a shilling = 12)136831 1 qr.

Shillings in a pound = 2|0)1140|2 7d.

£570 2s. 7d. 1 qr.

Ans. 136831d. 11402s. and £570.

Note. The remainder is always of the same name as the dividend.

8. Bring 35177 farthings into pounds.
9. Bring 91751 farthings into pence, &c.
10. Bring 345358 half pence into pence, shillings, and pounds.
11. Reduce 9744 pence to guineas, at 28s. per guinea.
12. In 39072 farthings, how many pistoles, at 22s.
13. In 4704 sixpences, how many half johannes?
14. In 12613⅓ threepences, how many French crowns, at 6s. 8d.?
15. In 91584 farthings, how many moidores, at 36s.?

16. In 27840 farthings, how many pence, threepences, groats, shillings and pounds?

17. In 63504 farthings, how many pence, twopences, threepences, groats, sixpences, shillings and guineas?

Note. The preceding questions may serve as proofs to those in Reduction descending.

REDUCTION DESCENDING AND ASCENDING.

1. Money.

1. In £97 how many pence and English or French crowns, at 6s. 8d.? Ans. 23280d. and 291 crowns.

2. In 947 English crowns, at 6s. 8d. how many shillings and English guineas? Ans. 6313s. 4d. and 225 guineas 13s. 4d.

3. In 519 English half crowns, how many pence and pounds? Ans. 20760d. and £86 10s.

4. In 1259 groats, how many farthings, pence, shillings, and guineas? Ans. 20144qrs. 5036d. 419s. 8d. and 14 guin. 27s. 8d.

5. In 75 pistoles, how many pounds? Ans. £82 10s.

6. In 735 French crowns, how many shillings and French guineas, at 26s. 8d.? Ans. 4900s. and 183 guin. 24s.

7. In 5793 pence, how many farthings, pounds, and pistoles? Ans. 23172qrs. £24 2s. 9d. and 21 pistoles, 20s. 9d.

8. In £99, how many shillings, and half johannes, at 48s.? Ans. 1980s. and 41 half joes. 12s.

9. In £179, how many guineas? Ans. 127 guin. 24s.

10. In £345 how many moidores? Ans. 191 moid. 24s.

11. In 59 half joes, 37 moidores, 45 guineas, 63 pistoles, 24 English crowns, and 19 dollars; how many pounds, half joes, moidores, guineas, pistoles, English crowns, dollars, shillings, pence, and farthings?

Ans. £354 4s. 147 half joes, 28s. 196 moidores, 28s. 253 guineas, 322 pistoles, 1062 English crowns, 4s. 1180 dollars, 4s. 7084 shillings, 85008d. and 340032qrs.

When it is required to know how many sorts of coin, of different values, and of equal number, are contained in any number of another kind; reduce the several sorts of coin into the lowest denomination mentioned, and add them together for a divisor; then reduce the money given, into the same denomination, for a dividend, and the quotient, arising from the division, will be the number required.

Note. Observe the same direction in weights and measures.

1. In 275 half johannes, how many moidores, guineas, pistoles, dollars, shillings and sixpences, of each the like number?

A moidore is 36s. that is } 72 sixpences.
A guinea is 28s. that is } 56 ditto.
A pistole is 22s. that is } 44 ditto.
A dollar is 6s. that is } 12 ditto
One shilling has 2 do.
1 do.

Divisor=187 sixpences.

```
  275 half joes.
   48 shil. in a johan.
 ----
 2200
1100
-----
13200 shillings.
    2 sixp. in a shill.
dividend=26400 sixpences.
```

187)26400(141 of each and 33 sixp. or 16s. 6d. over, the answer.

2. A Gentleman distributed £37 10s. between 4 poor persons; in the following manner, viz. that as often as the first had 20s. the second should have 15s. the third, 10s. and the fourth, 5s. What did each person receive? Ans. The first man £15, second £11 5s. third £7 10s. fourth £3 15s.

2. Troy Weight.

1. How many grs. in a silver bowl, that weighs 3lb. 10 oz. 12 pwt.?

```
  lb  oz.  pwt.
   3   10   12
  12 ounces in a pound.
  --
  46 ounces.
  20 pennyweights in an ounce.
 ---
 932 pennyweights.
  24 grains in one pwt.
 ----
 3728
1864
-----
Proof. 24)22368 grains, answer.

2|0)93|2
  12)46—12 pwt.
   lb 3—10 oz.
```

2. In 487ozs. how many pwts. and grs.?
Ans. 9740pwt. and 233760gr.

3. In 13 ingots of gold, each weighing 9oz. 5pwt. how many grains? Ans. 57720gr.

4. In 97397grs. how many pounds? Ans. 16lb 10oz. 18pwt. 5gr.

5. How many rings, each weighing 5pwt. 7gr. may be made of 3lb. 5oz. 16pwt. 2gr. of gold. Ans. 158.

3. Avoirdupois Weight.

Cwt. qrs. lb oz.
1. In 91 3 17 14 how many ounces?
4
367 quarters.
28
2943
735
10293 pounds.
16
61762
10294
164702 ounces.

Proof.
16)164702
28)10293 14oz.
4)367 17lb.
Cwt. 91 3qrs.

2. In 12 tons, 15cwt. 1qr. 19lb. 6oz. 12dr. how many drams?
Ans. 7323500dr.

3. In 24lb. 11oz. 9dr. how many drams? Ans. 6329dr.

4. In 44800 pounds, how many drams and tons?
Ans. 11468800dr. and 20 tons.

5. In 28lb. Avoirdupois how many pounds Troy?
28
7000 grains in 1 lb. Avoirdupois.
grs. in 1lb.tr. } =576|0)19600|0(34lb
1728
2320
2304
160
12
576|0)192|0(0oz.
20
576|0)3840|0)6pwt.
3456
3840 carried over.

6. In 47lb. 9oz. 13pwt. 17gr. Troy, how many pounds Avoirdupois?
47 9 13 17
12
573
20
11473
24
45899
22947
275369 carried over.

Brought over. 3840
24
1536
768
576|0)9216|0(16gr.
576
3456
3456

7|000)275|369(39℔ Br'ght over.
21
65
63
2369
16
14214
2369
7|000)37|904(5oz.
35
2904
16
17424
2904
7|000)46|464(6 $\frac{4464}{7000}$dr.
42
4464

4. Apothecaries' Weight.

1. How many grains are there in 37℔ 6℥?

℔ ℥
37 6
12
450 ounces.
8
3600 drams.
3
10800 scruples.
20

Ans. 216000 grains.

Proof.
2|0)21600|0
3)10800
8)3600
12)450
37 ℔ 6℥

2. In 9℔ 8℥ 1ʒ 2℈ 19gr. how many grains? Ans. 55799gr.
3. In 55799 grains, how many pounds, &c.?
Ans. 9℔ 8℥ 1ʒ 2℈ 19gr.

5. Cloth Measure.

1. In 127 yards, how many quarters and nails?

4
Ans. 508 qrs.
4
Ans. 2032 nails.

Proof.
4)2032
4)508
127 yards.

2. In 9173 nails, how many yards? Ans. 573yds. 1qr. 1n.

3. In 75 ells English, how many quarters and nails?
Ans. 375qrs. 1500n.

4. In 56 ells Flemish, how many quarters and nails?
Ans. 168qrs. 672n.

5. In 39 ells French, how many quarters and nails?
Ans. 234qrs. 936n.

6. In 7248 nails, how many yards, ells Flemish, ells English, and ells French?
Ans. 453yds. 604 ells Flem. 362 ells Eng. 2qrs. 302 ells French.

7. In 19 pieces of cloth, each 15 yards, 2 quarters, how many yards, quarters and nails? Ans. 294yds. 2qrs. 1178qrs. and 4712n.

6. Long Measure.

1. How many barley corns will reach from Newburyport to Boston, it being 43 miles?

43 miles.
8
344 furlongs.
40
13760 rods.
5½
68800
6880
75680 yards.
3
227040 feet.
12
2724480 inches.
3
8173440 Answer.

3)8173440 proof.
12)2724480
3)227040
11)75680
6880
2
4|0)1376|0
8)344
43

Here I divide by 11, and multiply the quotient by 2 because twice 5½ is 11; or I might first have multiplied by 2, and, then, have divided the product by 11.

2. How many barley corns will reach round the globe, it being 360 degrees? Ans. 4755801600.

3. How many inches from Newburyport to London, it being 2700 miles? Ans. 171072000.

4. How often will a wheel, of 16 feet and 6 inches circumference, turn round in the distance from Newburyport to Cambridge, it being 42 miles? Ans. 13440 times.

5. In 190080 inches, how many yards and leagues?
Ans. 5280yds. and 1 league.

7. Time.

1. In 20 years how many seconds?

```
 d.      h.
 365     6 in a year.
  24
 ----
1466
 730
 ----
8766 hours in 1 year.
  20
 ------
175320 hours in 20 years.
    60
 --------
10519200 minutes in ditto.
      60
 ---------
631152000 seconds in ditto.
```

Proof.

```
6|0)63115200|0
6|0)1051920|0
2|0)17532|0
4×6)8766
4)1461
365d. 6h.
```

2. Suppose your age to be 15y. 19d. 11h. 37m. 45s. how many seconds are there in it, allowing 365 days and 6 hours to the year? Ans. 475047465.

3. In 31536000 seconds how many years? Ans. 1 year.

4. How many minutes from the first day of January to the 14th day of August, inclusively? Ans. 325440.

5. How many days since the commencement of the Christian Æra?

6. How many minutes since the commencement of the American war, which happened on the 19th day of April, 1775?

7. How many seconds between the commencement of the war, April 19th, 1775, and the independence of the United States of America, which took place the 4th day of July, 1776*? Ans. 38188800.

8. Motion.

1. In 9 signs, 13° 25′, how many seconds?

```
9s  13°  25′
30
---
283 degrees.
 60
-----
17005 minutes.
   60
-------
1020300 seconds.
```

Proof.

```
6|0)102030|0
6|0)1700|5
3|0)28|3—25
9s 13° 25′
```

* 1776 was a leap year.

9. Land or Square Measure.

1. In 29 acres, 3 roods, 19 poles, how many roods and perches?

```
Acres. R. Poles.          Proof.
 29    3   19           4|0)477|9
  4                         ----
 ---                     4)119—19p.
 119 roods.                 ----
  40                        29ac. 3 roods.
 ----
```

Answer 4779 perches.

2. In 1997 poles how many acres? Ans. 12a. 1r. 37p.

3. In 89763 square yards how many acres, &c.?

Ans. 18a. 2r. 7p. 101ft. 36in.

4. How many square feet, square yards, and square poles, in a square mile?

Ans. 27878400 feet, 3097600 yards, and 102400 poles.

10. Solid Measure.

1. In 15 tons of hewn timber how many solid inches?

```
 15 tons.                      Proof.
 50                               5|0
 --                   1728)1296000(75|0
 750 feet.                 12096   ----
 1728                      -----   15 tons.
 ----                       8640
 6000                       8640
 1500
5250
 750
 -------
```

Ans. 1296000 inches.

2. In 9 tons of round timber how many inches? Ans. 622080.

3. In 25 cords of wood how many inches? Ans. 5529600.

Grindstones are usually sold by the solid foot, and the contents are found by the following Rule;—

Multiply the sum of the whole diameter and of the half of the diameter, by the half diameter, and this product by the thickness, and you have the contents in cubic inches.

4. What is the content of a grindstone, whose diameter is 32 inches and its thickness 3 inches?

```
 32 diameter.             1728)2304(1 foot.
 16 half diameter.             1728
 --                            ----
 48                             576
 16                               3
 ---                           ----
 768                           )1728( 1 third.
   3 thickness.                 1728
 ----
 2304 solid inches.
```

Ans. 1 foot and $\frac{1}{3}$ foot.

5. How many solid feet in a grindstone, whose diameter is 40 inches and thickness 4 inches? Ans. 2$\frac{7}{9}$ feet.

Note. This rule is not designed to give the solid contents with perfect accuracy. For the true rule, see Mensuration, Art. 30.

11. Wine Measure.

1. In 9hhds. 15galls. 3qts. of wine how many quarts?

```
hhds. gal. qts.                 Proof.
  9    15   3                  4)2331
 63                            ------
----                          63)582—3qts.
 32                            ------
 55                                9hhds.—15gals.
----
582 gallons.
  4
----
```

Ans. 2331 quarts.

2. In 12 pipes of wine how many pints? Ans. 12096.
3. In 9758 pints of brandy how many pipes? Ans. 9p. 1hhd. 22gal. 3qts.
4. In 1008 quarts of cyder how many tons? Ans. 1 ton.

12. Ale or Beer Measure.

1. In 29hhds. beer how many pints?

```
hhds.                           Proof.
  29                          2)12528
  54                           -------
----                           4)6264
 116                           -------
 145                          54)1566
----                           -------
1566 gallons                     29 hhds.
   4
----
6264 quarts.
   2
----
```

Ans. 12528 pints.

2. In 47bar. 18gal. of ale how many pints? Ans. 13680.
3. In 36 puncheons of beer how many butts? Ans. 24.

13. Dry Measure.

1. In 42 chaldrons of coals how many pecks?

```
Chaldrons.                      Proof.
   42                         4)5376
   32                        32)1344(42
 ----                           128
   84                           ---
  126                            64
 ----                            64
 1344 bushels.                   --
    4
 ----
```

Ans. 5376 pecks.

2. In 75 bushels of corn how many pints? Ans. 4800.
3. In 9376 quarts how many bushels? Ans. 293.

FRACTIONS.

Parts of a thing are expressed by figures, as well as whole things. When a *whole* is expressed by figures, the number is called an *integer*. But when a part, or some parts of a thing, are denoted by figures, as *one fourth, two thirds, four sevenths, three tenths*, &c. of a thing, the expressions of these parts by figures are called Fractions. The term, *fraction*, is derived from a Latin word, which signifies to *break*, as an integer or unity is supposed to be *broken* or divided into a certain number of equal parts, one or more of which parts are denoted by the fraction. Thus *one fourth* denotes *one* of the *four* equal parts, and *three tenths* denotes *three* of the *ten* equal parts, into which a thing is broken or an integer divided.

Fractions arise naturally from the operations of Division, when the divisor is not contained a certain number of times exactly in the dividend. For the *remainder* after the division is performed, is a part of the dividend which has not been divided; the *divisor* being the number of parts into which the integer is divided, and the *remainder* showing the number of those parts expressed by the fraction. Thus 4 is contained in 9, *two and one fourth times*, and, hence the quotient cannot be fully expressed in such cases, except by a whole number and a fraction.

Fractions are divided into two kinds, Vulgar, and Decimal.

VULGAR FRACTIONS.

Vulgar Fractions are expressions for any assignable parts of a unit, or whole number; and are represented by two numbers placed one above another, with a line drawn between them, thus: $\frac{5}{8}$, $\frac{4}{3}$, &c. signifying five eighths, four thirds.

The figure above the line is called the *numerator*, and that below it the *denominator*.

The denominator shews how many parts the integer is divided into; and the numerator shews how many of those parts are meant by the fraction.

Fractions are either proper, improper, single, compound, or mixed.

1. A *single* or *simple* fraction is a fraction expressed in a simple form; as $\frac{1}{2}$, $\frac{5}{8}$, $\frac{7}{16}$, &c.

2. A *compound* fraction is a fraction expressed in a compound form, being a fraction of a fraction; as $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{2}{7}$ of $\frac{5}{11}$ of $\frac{19}{20}$, which are read thus, one half of three fourths, two sevenths of five elevenths of nineteen twentieths, &c.

3. A *proper* fraction is a fraction whose numerator is less than its denominator; as $\frac{2}{5}$, $\frac{3}{8}$, &c.

4. An *improper* fraction is a fraction, whose numerator exceeds its denominator; as $\frac{5}{2}$, $\frac{8}{3}$, &c.

5. A *mixed number* is composed of a whole number and a fraction, as $7\frac{3}{5}$, $35\frac{4}{13}$, &c. that is, seven and three fifths, &c.

6. A fraction is said to be in its least, or lowest terms, when it is expressed by the least numbers possible.

7. The *common measure* of two, or more numbers, is that number which will divide each of them without a remainder: Thus, 5 is the common measure of 10, 20 and 30; and the *greatest* number, which will do this, is called the *greatest common measure.*

8. A number, which can be measured by two, or more numbers, is called their *common multiple:* And, if it be the least number, which can be so measured, it is called the *least common multiple;* thus, 40, 60, 80, 100, are multiples of 4 and 5: but their least common multiple is 20.

Note. The product of two or more numbers is a *common multiple* of those numbers. Thus, $3\times4\times5=60$, and 60, or $3\times4\times5$, is evidently divisible, without remainder, by each of those numbers. And the same must be true in every similar case.

9. A *prime number* is one, which can be measured only by itself or a unit, as, 3, 7, 23, &c.

10. A *perfect number* is equal to the sum of all its aliquot parts.* An aliquot part of a number is contained a certain number of times exactly in the number.

Problem I.†

To find the greatest common measure of two, or more, numbers.

Rule.

I. If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on, always dividing the

* The following perfect numbers are all which are, at present, known.

6	8589869056
28	137438691328
496	2305843008139952128
8128	2417851639228158837784576
33550336	9903520314282971830448816128

† This and the following problem will be found very useful in the doctrine of fractions, and several other parts of Arithmetick.

The truth of the rule may be shewn from the first example: For, since 108 measures 216, it also measures $216+108$, or 324.

Again, since 108 measures 216 and 324, it also measures $\overline{5\times324}+216$, or 1836. In the same manner it will be found to measure $\overline{2\times1836}+324$, or 3996, and so on.

It is also the greatest common measure; for suppose there be a greater, then, since the greater measures 1836 and 3996, it also measures the remainder 324; and since it measures 324 and 1836, it also measures the remainder 216; in the same manner it will be found to measure the remainder 108; that is, the greater measures the less, which is absurd; therefore, 108 is the greatest common measure.

In the same manner, the demonstration may be applied to one or more additional numbers.

last divisor by the last remainder, till nothing remain, then will the last divisor be the greatest common measure required.

II. When there are more than two numbers, find the greatest common measure of two of them, as before; then, of *that* common measure and one of the other numbers, and so on, through all the numbers, to the last; then will the greatest common measure, last found, be the answer.

III. If 1 happens to be the common measure, the given numbers are prime to each other, and found to be incommensurable, or in their lowest terms.

Examples.

1. What is the greatest common measure of 1836, 3996, and 1044?

```
1836)3996(2
     3672
     ----
      324)1836(5
          1620
          ----
           216)324(1
               216
               ---
Common meas.=108)216(2
                 216
```

So 108 is the greatest common measure of 3996 and 1836.

```
                    Hence 108)1044(9
                               972
                               ---
                                72)108(1
                                    72
                                    --
Last greatest com. meas.=36)72(2
                            72
                            --
```

Therefore, 36 is the answer required.

2. What is the greatest common measure of 1224 and 1080? Ans. 72.

3. What is the greatest common measure of 1440, 672 and 3472? Ans. 16.

Problem II.*

To find the least common multiple of two or more numbers.

Rule.

I. Divide by any number that will divide two, or more, of the given numbers without a remainder, and set the quotients, together with the undivided numbers, in a line beneath.

II. Divide the second line, as before, and so on, till there are no two numbers that can be divided; then, the continued product of the divisors and quotients will give the multiple required.

* The reason of this rule may also be shewn from the first example: Thus, it is evident that $6\times10\times16\times20=19200$ may be divided by 6, 10, 16 and 20, without a remainder; but 20 is a multiple of 5; therefore, $6\times10\times16\times4$, or 3840, is also divisible by 6, 10, 16 and 20. Also, 16 is a multiple of 4; therefore $6\times10\times4\times4=960$, is also divisible by 6, 10, 16 and 20. Also, 10 is a multiple of 2; therefore, $6\times5\times4\times4=480$, is also divisible by 6, 10, 16 and 20. Also, 6 is a multiple of 2; therefore, $3\times5\times4\times4=240$, is also divisible by 6, 10, 16 and 20, and is evidently the least number that can be so divided.

EXAMPLES.

1. What is the least common multiple of 6, 10, 16 and 20?

*5)	6	10	16	20
*2)	6	2	16	4
*2)	3	1	8	2
	*3	1	*4	1

* * * * *

$5 \times 2 \times 2 \times 3 \times 4 = 240$ Ans.

I survey my given numbers, and find that five will divide two of them, viz. 10 and 20, which I divide by 5, bringing into a line with the quotients the numbers which 5 will not measure: Again, I view the numbers in the second line, and find 2 will measure them all, and get 3, 1, 8, 2 in the third line, and find that two will measure 8 and 2, and in the fourth line get 3, 1, 4, 1, all prime; I then multiply the prime numbers and the divisors continually into each other, for the number sought, and find it to be 240.

2. What is the least common multiple of 6 and 8? Ans. 24.

3. What is the least number that 3, 5, 8 and 10 will measure? Ans. 120.

4. What is the least number which can be divided by the 9 digits, separately without a remainder? Ans. 2520.

REDUCTION OF VULGAR FRACTIONS

Is the bringing of them out of one form into another, in order to prepare them for the operations of Addition, Subtraction, &c.

CASE I.*

To abbreviate, or reduce fractions to their lowest terms.

RULE.

Divide the terms of the given fraction by any number, which will divide them without a remainder, and the quotients, again, in

* That dividing both the numerator and denominator of the fraction by the same number, will give another fraction of equal value, is evident, because both parts are diminished proportionally, and if both parts of the equal fraction be multiplied by the divisor, the original fraction will be formed again. Thus $\frac{288}{480} \div \frac{8}{8} = \frac{36}{60}$, and $\frac{36}{60} \times \frac{8}{8} = \frac{288}{480}$. And if the divisions be performed as often as can be done, or the common divisor be the greatest possible, the terms of the resulting fraction must be the least possible.

Note 1. Any number, ending with an even number or cypher, is divisible by 2.

2. Any number, ending with 5 or 0, is divisible by 5.

3. If the right hand place of any number be 0, the whole is divisible by 10.

4. If the two right hand figures of any number be divisible by 4, the whole is divisible by 4.

5. If the three right hand figures of any number be divisible by 8, the whole is divisible by 8.

6. If the sum of the digits, constituting any number, be divisible by 3 or 9, the whole is divisible by 3 or 9.

the same manner; and so on, till it appears that there is no number greater than 1, which will divide them, and the fraction will be in its lowest terms. Or,

Divide both the terms of the fraction by their greatest common measure, and the quotients will be the terms of the fraction required.

EXAMPLES.

1. Reduce $\frac{288}{480}$ to its lowest terms.

$$8\left\{\frac{288}{480}\right. \overset{(4)}{=} \frac{36}{60} \overset{(3)}{=} \frac{9}{15} = \frac{3}{5}$$ the answer.*

Or thus:

288)480(1
288
——
192)288(1
192
——
Com. meas. 96)192(2
192

Therefore 96 is the greatest common measure.

and $96\left\{\frac{288}{480}\right. = \frac{3}{5}$ the same as before.

2. Reduce $\frac{96}{544}$ to its lowest terms. Ans. $\frac{3}{17}$.
3. Reduce $\frac{384}{1152}$ to its lowest terms. Ans. $\frac{1}{3}$.
4. Reduce $\frac{57}{456}$ to its lowest terms. Ans. $\frac{1}{8}$.

7. If a number cannot be divided by some number less than the square root thereof, that number is a *prime*.

8. All *prime* numbers, except 2 and 5, have 1, 3, 7, or 9 in the place of units: and all other numbers are *composite*.

9. When numbers, with the sign of Addition or Subtraction between them, are to be divided by any numbers, each of the numbers must be divided: Thus $\frac{6+9+12}{3}=2+3+4=9$, or $\frac{6+9+12}{3}=\frac{27}{3}=9$.

10. But if the numbers have the sign of Multiplication between them; then only one of them must be divided: Thus, $\frac{4\times6\times10}{2\times5}=\frac{2\times6\times10}{1\times5}=\frac{2\times6\times2}{1\times1}=\frac{24}{1}=24$.

* Hence if both parts of a fraction be multiplied by the same number, its value is not altered. For $\frac{3}{5}=\frac{3}{5}\times\frac{3}{3}=\frac{9}{15}=\frac{9}{15}\times\frac{4}{4}=\frac{36}{60}\times\frac{8}{8}=\frac{288}{480}$, and so on. If fractions be multiplied together, in which equal terms occur in the numerator and denominator, these equal terms may be expunnged or cancelled, for their quotient would be 1, which as a factor would not alter the value of the fraction. Thus $\frac{4}{5}\times\frac{5}{8}=\frac{4\times5}{5\times8}=\frac{4}{8}=\frac{1}{2}$, and $\frac{1}{2}\times\frac{3}{4}\times\frac{2}{3}\times\frac{4}{1}=\frac{1}{2}\times\frac{3}{4}\times\frac{2}{3}\times\frac{4}{1}=\frac{1}{1}=1$. Arithmetical operations are often much shortened by observing what quantities may be expunged, and by omitting them in the operations. For the same object, expressions may be changed to equivalent ones, and quantities expunged. Thus $\frac{1}{2}\times\frac{30}{45}=\frac{1}{2}\times\frac{2\times15}{3\times15}=\frac{1}{3}$, and $\frac{8}{9}\times\frac{288}{480}=\frac{8}{9}\times\frac{9\times32}{8\times60}=\frac{4\times8}{4\times15}=\frac{8}{15}$.

5. Reduce $\frac{46}{184}$ to its lowest terms. Ans. $\frac{1}{4}$.
6. Reduce $\frac{1429}{2858}$ to its lowest terms. Ans. $\frac{1}{2}$.

Note. If the numerator of a fraction be multiplied, or its denominator divided, by a whole number, the value of the fraction will be so many times increased. Thus, $\frac{1}{9}$ multiplied by $3, = \frac{1 \times 3}{9} = \frac{3}{9} = \frac{1}{3} = \frac{1}{9 \div 3}$. Hence, to multiply a fraction by an integer, is to multiply the numerator, or divide the denominator of the fraction by the integer.

1. Multiply $\frac{5}{61}$ by 7. Ans. $\frac{35}{61}$.
2. Increase the value of $\frac{1}{114}$, nineteen times. Ans. $\frac{1}{6}$.
3. Increase the value of $\frac{3}{7}$, seven fold. Ans. 3.

If the numerator of a fraction be divided, or its denominator multiplied, by a whole number, the value of the fraction will be so many times diminished. Thus, $\frac{3}{9}$ divided by $3, = \frac{3 \div 3}{9} = \frac{1}{9} = \frac{3}{9 \times 3} = \frac{3}{27} = \frac{1}{9}$. Hence, to divide a fraction by an integer, is to divide the numerator, or multiply the denominator of the fraction, by the whole number.

1. Divide $\frac{5}{9}$ by 7. Ans. $\frac{5}{63}$.
2. Diminish the value of $\frac{1}{7}$, seven times. Ans. $\frac{1}{49}$.
3. Diminish the value of $\frac{7}{2}$, four times. Ans. $\frac{7}{8}$.

Note. The reason of many operations will be evident from an attention to the following *self-evident* truths.

1. If equals be added to equals, their sums will be equal. Thus, $3+4+9=8\times2$. Let 7 be added to each, and $3+4+9+7=8\times2+7=23$.

2. If equals be subtracted from equals, the remainders will be equal. Thus, $3+11=7\times2$. Let 3 be taken from each, and $3+11-3=7\times2-3=11$.

3. If equals be multiplied by the same quantity, the products will be equal. Thus, let $5+7=6\times2$, be multiplied by 6, and $\overline{5+7}\times6=6\times2\times6=72$.

4. If equivalent quantities be divided by the same quantity, the quotients will be equal. Thus, let $43+17=12\times5$ be divided by 5, and $\overline{43+17}\div5=12\times5\div5,=12$, or $\frac{43+17}{5}=\frac{12\times5}{5}=12$.

CASE II.

To reduce a mixed number to its equivalent improper fraction.

RULE.*

Multiply the whole number by the denominator of the fraction,

* All fractions represent a division of a numerator by the denominator, and are taken altogether as proper and adequate expressions of the quotient. Thus the quotient of 3, divided by 4, is $\frac{3}{4}$; from whence the rule is manifest; for if any number is multiplied and divided by the same number, it is evident the quotient must be the same as the quantity first given.

and add the numerator of the fraction to the product; under which subjoin the denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce $36\frac{5}{8}$ to its equivalent improper fraction.

$$\begin{array}{r} 36 \\ \times 8 + 5 \\ \hline \text{Ans. } 293 \\ \hline 8 \end{array}$$

I multiply 36 by 8, and adding the numerator 5 to the product, as I multiply, the sum 293 is the numerator of the fraction sought, and 8 the denominator: So that $\frac{293}{8}$ is the improper fraction, equal to $36\frac{5}{8}$.

Or, $\frac{\overline{36 \times 8} + 5}{8} = \frac{293}{8}$ Answer as before.

2. Reduce $127\frac{4}{17}$ to its equivalent improper fraction. Ans. $\frac{2163}{17}$.
3. Reduce $653\frac{3}{19}$ to its equivalent improper fraction. Ans. $\frac{12410}{19}$.

CASE III.†

To reduce a whole number to an equivalent fraction having a given denominator.

RULE.

Multiply the whole number by the given denominator: Place the product over the said denominator, and it will form the fraction required.

EXAMPLES.

1. Reduce 6 to a fraction, whose denominator shall be 8.
   $6 \times 8 = 48$, and $\frac{48}{8}$ the Ans.—Proof $\frac{48}{8} = 48 \div 8 = 6$.
2. Reduce 15 to a fraction, whose denominator shall be 12. Ans. $\frac{180}{12}$.
3. Reduce 100 to a fraction, whose denominator shall be 70. Ans. $\frac{7000}{70} = \frac{100}{1} = 100$.

A whole number is made a fraction by drawing a line under it, and putting unity or 1, for a denominator, as $\frac{9}{1} = \frac{9 \times 1}{1}$ by the rule, and 12 is $\frac{12}{1}$, &c.

CASE IV.‡

To reduce an improper fraction to its equivalent whole, or mixed number.

RULE.

Divide the numerator by the denominator: the quotient will be the whole number, and the remainder, if any, will be the numerator to the given denominator.

† Multiplication and Division are here equally used, and consequently the result is the same as the quantity first proposed.

‡ This case is, evidently, the reverse of case 2d, and has its reason in the nature of common division.

EXAMPLES.

1. Reduce $\frac{293}{8}$ to its equivalent whole, or mixed number.

$$\begin{array}{l} 8)293(36\frac{5}{8}\text{ Ans.} \\ 24 \\ \overline{53} \\ 48 \\ \overline{5} \end{array}$$

Or, $\frac{293}{8}=293\div8=36\frac{5}{8}$ as before.

2. Reduce $\frac{2163}{17}$ to its equivalent whole, or mixed number. Ans. $127\frac{4}{17}$.

3. Reduce $\frac{12410}{19}$ to its equivalent whole, or mixed number. Ans. $653\frac{3}{19}$.

4. Reduce $\frac{45}{5}$ to its equivalent whole number. Ans. 9.

CASE V.*

To reduce a compound fraction to an equivalent simple one.

RULE.

Multiply all the numerators continually together for a new numerator, and all the denominators, for a new denominator, and they will form the simple fraction required.

If part of the compound fraction be a whole or mixed number, it must be reduced to an improper fraction, by case 2d, or 3d.

If the denominator of any member of a compound fraction be equal to the numerator of another member thereof, these equal numerators and denominators may be expunged, and the other members continually multiplied, as by the rule, will produce the fractions required in lower terms.

EXAMPLES.

1. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

$$\frac{1\times2\times3\times4}{2\times3\times4\times5}=\frac{24}{120}=\frac{1}{5}\text{ the Answer.}$$

Or, by expunging the equal numerators and denominators, it will give $\frac{1}{5}$ as before.

2. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a simple fraction.

$\frac{3\times4\times5\times11}{4\times5\times6\times12}=\frac{660}{1440}=\frac{11}{24}$ Ans. Or, by expunging the equal numerators and denominators, it will be $\frac{3\times11}{6\times12}=\frac{33}{72}=\frac{11}{24}$ as before.

* That a compound fraction may be repreſented by a ſimple one is very evident; ſince a part of a part muſt be equal to ſome part of the whole. The truth of the rule for this reduction may be ſhown as follows.

Let the compound fraction to be reduced, be $\frac{3}{4}$ of $\frac{6}{10}$. Then $\frac{1}{4}$ of $\frac{6}{10}=\frac{6}{10}\div4=\frac{6}{40}$ and conſequently $\frac{3}{4}$ of $\frac{6}{10}=\frac{6}{40}\times3=\frac{18}{40}$ the ſame as by the rule.

If the compound fraction conſiſts of more numbers than two, the firſt two may be reduced to one, and that one and the third will be the ſame as a fraction of two numbers, and ſo on.

3. Reduce $\frac{5}{8}$ of $\frac{6}{7}$ of $\frac{15}{19}$ to a a simple fraction. Ans. $\frac{225}{532}$.

4. Reduce $\frac{3}{12}$ of $\frac{13}{15}$ of $\frac{8}{17}$ of 20 to a simple fraction. Ans. $\frac{624}{306}=2\frac{2}{51}$.

5. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of $12\frac{1}{2}$ to a simple fraction. Ans. $\frac{75}{64}=1\frac{11}{64}$.

CASE. VI.

To reduce fractions of different denominators to equivalent fractions having a common denominator.

Rule I.*

Multiply each numerator into all the denominators except its own, for a new numerator, and all the denominators into each other, continually, for a common denominator.

Examples.

1. Reduce $\frac{1}{4}$, $\frac{2}{5}$, and $\frac{5}{8}$ to equivalent fractions having a common denominator. $1\times5\times8=40$ the new numerator for $\frac{1}{4}$.

$2\times4\times8=64$ the new numerator for $\frac{2}{5}$.

$5\times4\times5=100$ ditto for $\frac{5}{8}$.

$4\times5\times8=160$ the common denominator.

Therefore the new equivalent fractions are $\frac{40}{160}$, $\frac{64}{160}$ and $\frac{100}{160}$, the answer.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to fractions having a common denominator. Ans. $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

3. Reduce $\frac{1}{2}$, $\frac{2}{3}$ of $\frac{5}{6}$, $7\frac{3}{4}$, and $\frac{3}{13}$, to a common denominator. Ans. $\frac{936}{1872}$, $\frac{1040}{1872}$, $\frac{14508}{1872}$, $\frac{432}{1872}$.

4. Reduce $\frac{11}{15}$, $\frac{3}{4}$ of $2\frac{1}{2}$, $\frac{7}{12}$, and $\frac{5}{8}$, to a common denominator. Ans. $\frac{8448}{11520}$, $\frac{21600}{11520}$, $\frac{6720}{11520}$, $\frac{7200}{11520}$.

Rule II.

To reduce any given fractions to others, which shall have the least common denominator.

1. By Problem 2, Page 73, find the least common multiple of all the denominators of the given fractions, and it will be the common denominator required.

2. Divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, and the product will be the numerator of the fraction required.

* By placing the numbers multiplied, properly under one another, it will be seen that the numerator and denominator of every fraction are multiplied by the very same number, and consequently their values are not altered. Thus, in the first example.

$$\frac{1}{4}\Big|\frac{\times5\times8}{\times5\times8}\qquad\frac{2}{5}\Big|\frac{\times4\times8}{\times4\times8}\qquad\frac{5}{8}\Big|\frac{\times4\times5}{\times4\times5}$$

In the second rule, the common denominator is a multiple of all the denominators, and consequently will divide by any of them: Therefore, proper parts may be taken for all the numerators as required.

Examples.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$ and $\frac{7}{8}$ to fractions having the least common denominator possible.

$$4)\underline{3\quad 4\quad 8}$$
$$\underline{3\quad 1\quad 2}$$

$4\times3\times2=24=$ least common denominator.

$\overline{24\div3}\times1=8$ the first numerator; $\overline{24\div4}\times3=18$ the second numerator; $\overline{24\div8}\times7=21$ the third numerator.

Whence, the required fractions are $\frac{8}{24}$, $\frac{18}{24}$, $\frac{21}{24}$.

2. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to fractions having the least common denominator. Ans. $\frac{30}{60}$, $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$.

CASE VII.

To reduce a fraction of one denomination to an equivalent fraction of a higher denomination.

Rule.*

Multiply the given denominator by the parts in the several denominations between *it* and *that* denomination to which it is to be reduced, for a new denominator, which is to be placed under the given numerator: Or, compare the given fraction with the several denominations between *it* and that denomination to which it is to be reduced, and then, by case 5th, reduce the compound fraction thus formed, to a single one, and the equivalent fraction of the required denomination will be obtained. Let this fraction be reduced to its lowest terms.

* The reaſon of the rule may be ſeen in the following manner. As there are 12 pence in a ſhilling, four-fifths of one penny can be only a *twelfth* part as much of 12 pence or a ſhilling, as it is of one penny. Hence, to reduce four fifths of a penny to the fraction of a ſhilling, the given fraction muſt be diminiſhed 12 times, or one twelfth of it will be the equivalent fraction of a ſhilling. A fraction is diminiſhed in value, according to the note to Caſe I. by multiplying the denominator by the whole number. Thus four fifths of a penny $=\frac{4}{5\times12}$ of a ſhilling $=\frac{4}{5}\times\frac{1}{12}=\frac{4}{5}$ of $\frac{1}{12}=\frac{4}{60}$ of a ſhilling. For the ſame reaſon, four ſixtieths of a ſhilling can be only one twentieth as much of a pound, or $\frac{4}{60}$ of a ſhilling, $=\frac{4}{60\times20}$ of a pound, $=\frac{4}{60}\times\frac{1}{20}=\frac{4}{60}$ of $\frac{1}{20}=\frac{4}{1200}=\frac{1}{300}$ of a pound. Put theſe two operations together, and you have four-fifths of a penny, $=\frac{4}{5\times12\times20}=\frac{4}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}=\frac{1}{300}$ of a pound, as before, which is the rule.

The ſame operation might have been performed thus. In a pound there are 240 pence. Then, four-fifths of a penny $=\frac{4}{5\times240}$ of a pound, $=\frac{4}{5}$ of $\frac{1}{240}$ $=\frac{1}{300}$ as before. And in general the fraction of one denomination muſt be as much diminiſhed to be an equivalent fraction of a higher denomination, as is indicated by the *number of parts* of the given denomination to make *one* of the higher denomination.

EXAMPLES.

1. Reduce $\frac{4}{7}$ of a cent to the fraction of a dollar.

By comparing it, it becomes $\frac{4}{7}$ of $\frac{1}{10}$ of $\frac{1}{10}$, which, reduced by case 5, will be $\frac{4\times1\times1}{7\times10\times10} = \frac{4}{700} = \frac{1}{175}$ D. Ans.

2. Reduce $\frac{3}{5}$ of a mill to the fraction of an eagle. Ans. $\frac{3}{50000}$ E.
3. Reduce $\frac{11}{15}$ of a mill to the fraction of a dollar. Ans. $\frac{11}{15000}$ D.
4. Reduce $\frac{3}{5}$ of a penny to the fraction of a pound. Ans. £$\frac{1}{400}$.
5. Reduce $\frac{3}{4}$ of a farthing to the fraction of a pound. Ans. $\frac{1}{1280}$.
6. Reduce $\frac{5}{8}$ of a penny to the fraction of a guinea. Ans. $\frac{5}{2016}$ guinea.
7. Reduce $\frac{12}{19}$ of a shilling to the fraction of a moidore. Ans. $\frac{1}{57}$ moidore.
8. Reduce $\frac{4}{7}$ of an ounce to the fraction of a lb. Avoirdupois. Ans. $\frac{1}{28}$ lb.
*9. Reduce $\frac{4}{5}$ of a pound to the fraction of a guinea. Ans. $\frac{4}{7}$ guin.
10. Reduce $\frac{7}{8}$ of a pwt. to the fraction of a pound Troy. Ans. $\frac{7}{1920}$ lb.
11. Reduce $\frac{8}{9}$ of a lb. Avoirdupois to the fraction of 1 Cwt. Ans. $\frac{1}{126}$ Cwt.
12. Express $5\frac{1}{2}$ furlongs in the fraction of a mile. Ans. $\frac{11}{16}$ mile.

CASE VIII.

To reduce a fraction of one denomination to an equivalent fraction of a lower denomination.

RULE.†

Multiply the given numerator by the parts in the denominations between *it* and *that* denomination you would reduce it to, for a

* $\frac{4}{5}$£ $=\frac{4}{5}$ of $\frac{20}{1}=\frac{80}{5}$, and $\frac{80}{5}$ of $\frac{1}{28}=\frac{80}{140}=\frac{4}{7}$ guinea.

† This rule is the reverse of the preceding, and the propriety of it may be seen in a similar manner. The fraction of a higher denomination is obviously less than the equivalent fraction of a lower denomination; for instance, $\frac{1}{4}$ of a pound is $\frac{20}{4}$ shillings or 5 shillings. Whence the value of the fraction must be increased, to render it an equivalent fraction of a lower denomination, so many times as there are parts of the less denomination in the higher. But, by the Note to Case I, the value of a fraction is increased by multiplying the numerator by a whole number. To reduce $\frac{1}{400}$£ to the fraction of a shilling, as there are 20 shillings in a pound, we have $\frac{1}{400}\times20=\frac{20}{400}$ of a shilling. And to reduce $\frac{20}{400}$ of a shilling to the fraction of a penny, we have $\frac{20}{400}\times12=\frac{240}{400}$ of a penny $=\frac{3}{5}$d. Put together these operations, and we have $\frac{1}{400}$£ $=\frac{1}{400}\times20\times12$, of a penny $=\frac{1}{400}$ of $\frac{20}{1}$ of $\frac{12}{1}=\frac{240}{400}=\frac{3}{5}$d. as before, which is the rule.

new numerator, which place over the given denominator: *Or*, only invert the parts contained in the integer, and make of them a compound fraction as before, then, reduce it to a simple one.

Examples.

1. Reduce $\frac{1}{175}$ of a dollar to the fraction of a cent.

By comparing the fraction it will be $\frac{1}{175}$ of $\frac{10}{1}$ of $\frac{10}{1}$; then

$$\frac{1}{175} \times \frac{10}{1} \times \frac{10}{1} = \frac{100}{175} = \frac{4}{7} \text{ c. Answer.}$$

2. Reduce $\frac{3}{50000}$ of an eagle to the fraction of a mill. Ans. $\frac{3}{5}$m.
3. Reduce $\frac{11}{15000}$ of a dollar to the fraction of a mill. Ans. $\frac{11}{15}$m.
4. Reduce $\frac{1}{400}$ of a pound to the fraction of a penny. Ans. $\frac{3}{5}$d.
5. Reduce $\frac{1}{1280}$ of a pound to the fraction of a farthing. Ans. $\frac{3}{4}$qr.
6. Reduce $\frac{5}{2688}$ of a guinea to the fraction of a penny. Ans. $\frac{5}{8}$d.
7. Reduce $\frac{1}{57}$ of a moidore to the fraction of a shilling. Ans. $\frac{12}{19}$s.
8. Reduce $\frac{1}{28}$ of a ℔ Avoirdupois to the fraction of an ounce. Ans. $\frac{4}{7}$oz.
9. Reduce $\frac{4}{7}$ of a guinea to the fraction of a pound. Ans. $\frac{4}{5}$£.
10. Reduce $\frac{7}{1920}$ of a ℔ Troy to the fraction of a pwt. Ans. $\frac{7}{8}$pwt.
11. Reduce $\frac{1}{126}$ of Cwt. to the fraction of a ℔ Avoirdupois. Ans. $\frac{8}{9}$℔.

CASE IX.

To find the value of a fraction in the known parts of the integer, as of coin, weight, measure, &c.

Rule.*

Multiply the numerator by the parts of the next inferior denomination, and divide the product by the denominator; and if any thing

* This rule follows from the preceding. Thus let $\frac{4}{5}$£ be the fraction, whose value is to be found. By the preceding rule, $\frac{4}{5}£ = \frac{4}{5}$ of $\frac{20}{1}$ of a shilling, $\frac{80}{5}$s. $=$ 16s. Again, $\frac{2}{3}£ = \frac{2}{3}$ of $\frac{20}{1}$ of a shilling $= \frac{40}{3}$s. $=$ by dividing, $13\frac{1}{3}$s. And on the same principle, $\frac{1}{3}$s. $= \frac{1}{3}$ of $\frac{12}{1}$ of a penny, $= \frac{12}{3}$d. $=$ 4d. Whence $\frac{2}{3}£ = 13\frac{1}{3}$s. $=$ 13s. 4d.

Again, $\frac{3}{7}£ = \frac{3}{7}$ of $\frac{20}{1}$ of a shilling, $= \frac{60}{7}$s. $= 8\frac{4}{7}$s. But $\frac{4}{7}$s. $= \frac{4}{7}$ of $\frac{12}{1}$ of a penny, $= \frac{48}{7}$d. $= 6\frac{6}{7}$d. and, therefore $8\frac{4}{7}$s. $=$ 8s. $6\frac{6}{7}$d. But $\frac{6}{7}$d. $= \frac{6}{7}$ of $\frac{4}{1}$ of a farthing, $= \frac{24}{7}$qr. $= 3\frac{3}{7}$qr. Therefore, $\frac{3}{7}£ = 8\frac{4}{7}$s. $=$ 8s. $6\frac{6}{7}$d. $=$ 8s. 6d. $3\frac{3}{7}$qr. The same process is obviously applicable to every similar case. Or, the process may be conducted thus; $\frac{3}{7}£ = \frac{3}{7}$ of $\frac{20}{1}$ of $\frac{12}{1}$ of $\frac{4}{1} = \frac{2880}{7}$qr. $= 411\frac{3}{7}$qrs. $=$ 102d. $3\frac{3}{7}$qrs. $=$ 8s. 6d. $3\frac{3}{7}$qrs.

remain, multiply it by the next inferior denomination, and divide by the denominator as before, and so on, as far as necessary; and the quotients placed after one another, in their order, will be the answer required; *or*, reduce the numerator, as if it were a whole number, to the lowest denomination, and divide the result by the denominator; the quotient will be the number of the lowest denomination, (which must be brought into higher denominations as far as it will go,) and the remainder will be a numerator to be placed over the given denominator for a fraction of the lowest denomination.

Note. From this rule, in connexion with what has been said of *Reduction of Federal Money*, it appears, that, annexing to the given numerator as many cyphers, as will fill all the places to the lowest denomination, and dividing the number so formed by the denominator, the quotient will be the answer in the several denominations, and the remainder a numerator to be placed over the given denominator, forming a fraction of the lowest denomination.

EXAMPLES.

1. What is the value of $\frac{5}{8}$ of a dollar?

By the general rule.

5
10
——
8)50(2
$6 10 Ans. 6d. 2c. 5m.
——
8)20(4 or 62c. 5m.
——
c. 2 10
——
8)40
——
m. 5

By the note.

$ d. c. m.
8) 5 0 0 0
——————
6 2 5

Or thus.

$5=5000m. and $\frac{5000}{8}$m.=625m.=62c. 5m. Ans. as before.

2. What is the value of $\frac{17}{64}$ of a dollar?

$ d. c. m.
64)17 0 0 0
128
——
420
384
——
360
320
——
40
—— = $\frac{5}{8}$
64

(2d. 6c. 5$\frac{5}{8}$m.
or 26c. 5$\frac{5}{8}$m. Ans.

Or, $17=17000m. And $\frac{17000}{64}$m.=265$\frac{5}{8}$m.= 26c. 5$\frac{5}{8}$m. Ans. as before.

3. What is the value of $\frac{9}{48}$ of an eagle? Ans. $1 87c. 5m.

4. What is the value of $\frac{7}{16}$ of a dollar ? Ans. 43c. $7\frac{1}{2}$m
5. What is the value of $\frac{5}{7}$ of a pound ? Ans. 14s. 3d. $1\frac{5}{7}$qr.
6. What is the value of $\frac{9}{24}$ of a shilling ? Ans. $4\frac{1}{2}$d.
7. What is the value of $\frac{7}{40}$ of a £ ? Ans. 3s. 6d.
8. What is the value of $\frac{27}{44}$ of a pistole ? Ans. 13s. 6d.
9. What is the value of $\frac{17}{29}$ of a Cwt. ?
Ans. 2 qrs. 9lb 10 oz. $7\frac{21}{29}$dr.
10. What is the value of $\frac{4}{5}$ of a lb Avoirdupois ?
Ans. 12oz. $12\frac{4}{5}$dr.
11. What is the value of $\frac{3}{5}$ of a lb Troy ? Ans. 7oz. 4pwt.
12. What is the value of $\frac{3}{13}$ of a ton ?
Ans. 4cwt. 2qrs. 12lb. 14oz. $12\frac{4}{13}$dr.
13. What is the value of $\frac{6}{9}$ of a yard ? Ans. 2qrs. $2\frac{2}{3}$n.
14. What is the value of $\frac{7}{8}$ of an ell English ? Ans. 4qrs. $1\frac{1}{2}$n.
15. What is the value of $\frac{5}{6}$ of a mile ? Ans. 6fur. 26p. 11ft.
16. What is the value of $\frac{9}{13}$ of a day ? Ans. 16h. 36m. $55\frac{5}{13}$s.
17. The value of $\frac{12}{17}$ of a Julian year is required ?
Ans. 257d. 19h. 45m. $52\frac{16}{17}$s.
18. The value of $\frac{9}{14}$ of a guinea is demanded ? Ans. 18s.
19. What is the value of $\frac{15}{16}$ of a dollar. Ans. 5s. $7\frac{1}{2}$d.
20. What is the value of $\frac{3}{5}$ of a moidore ? Ans. 21s. $7\frac{1}{5}$d.
21. What is the value of $\frac{6}{7}$ of an acre ? Ans. 3r. $17\frac{1}{7}$p.

CASE X.

To reduce any given quantity to the fraction of any greater denomination of the same kind.

Rule.*

Reduce the given quantity to the lowest term mentioned, for a numerator; then reduce the integral part to the same term for a denominator; which will be the fraction required.

Note. It appears from this rule and what has been said before, that, in *Federal Money*, where the given quantity contains no fraction of its lowest denomination, the annexing of as many cyphers to 1 of the required denomination, as will extend to the lowest denomination in the given quantity, will form a denominator, which placed under the given quantity used as one number for a numerator, will make the answer, which may be reduced to its lowest terms. Or, if there be a fraction of the lowest denomination, multiply the given whole numbers by its denominator, adding its numerator, for a numerator; and let the denominator itself at the left of as many cyphers as were mentioned above be a denominator; the fraction so formed will be the answer; which may be reduced to its lowest terms.

* This case is the reverse of the former, and the proof evident from that.

Note. If there be a fraction given with the said quantity, it must be farther reduced to the denominative parts thereof, adding thereto the numerator.

EXAMPLES.

1. Reduce 6d. 2c. 5m. to the fraction of a dollar.

By the general rule.

6d.	10d. int. pt.	
×10+2	10	
62	100	By the note.
×10+5	10	\$ d. c. m.
		6 2 5
625	1000	———— $=\frac{5}{8}$\$.
		1 0 0 0

And, $\frac{625}{1000}=\frac{5}{8}$\$ Ans. Ans. as before.

2. Reduce 26c. 5$\frac{5}{8}$m. to the fraction of a dollar.

By the general rule. By the note.

26c.	100c. int. pt.	\$ d. c. m.
×10+5	10	265×8+5=2 1 2 5
265	1000	———— $=\frac{17}{64}$\$.
×8+5	8	And \$1×8=8 0 0 0
2125	8000	Ans. as before.

And, $\frac{2125}{8000}=\frac{17}{64}$\$ Ans.

3. Reduce \$1 87c. 5m. to the fraction of an eagle. Ans. $\frac{3}{16}$E.
4. Reduce 43c. 7$\frac{1}{2}$m. to the fraction of a dollar. Ans. $\frac{7}{16}$\$.
5. Reduce 14s. 3$\frac{1}{4}$d. $\frac{5}{7}$ to the fraction of a pound. Ans. $\frac{4800}{6720}=\frac{5}{7}$£.
6. Reduce 4$\frac{1}{2}$d. to the fraction of a shilling. Ans. $\frac{3}{8}$s.
7. Reduce 3s. 6d. to the fraction of a pound. Ans. $\frac{7}{40}$£.
8. Reduce 13s. 6d. to the fraction of a pistole. Ans. $\frac{27}{44}$pistole.
9. Reduce 2qrs. 9lb. 10oz. 7$\frac{21}{29}$dr. to the fraction of a cwt. Ans. $\frac{17}{29}$cwt.
10. Reduce 12oz. 12$\frac{4}{5}$dr. to the fraction of a lb Avoirdupois. Ans. $\frac{4}{5}$lb.
11. Reduce 7oz. 4pwt. to the fraction of a lb Troy. Ans. $\frac{3}{5}$lb.
12. Reduce 4cwt. 2qrs. 12lb 14oz. 12$\frac{4}{13}$dr. to the fraction of a ton. Ans. $\frac{3}{13}$ton.
13. Reduce 2qrs. 2$\frac{2}{3}$n. to the fraction of a yard. Ans. $\frac{2}{3}$yd.
14. Reduce 4qrs. 1$\frac{1}{2}$n. to the fraction of an ell English. Ans. $\frac{7}{8}$E. E.
15. Reduce 6fur. 26po. 11ft. to the fraction of a mile. Ans. $\frac{5}{6}$m.
16. Reduce 16h. 36m. 55$\frac{5}{13}$s. to the fraction of a day. Ans. $\frac{9}{13}$day.
17. Reduce 257d. 19h. 45m 52$\frac{16}{17}$s. to the fraction of a Julian year. Ans. $\frac{12}{17}$J. Y.
18. Reduce 18s. to the fraction of a guinea. Ans. $\frac{9}{14}$G.
19. Reduce 5s. 7$\frac{1}{2}$d to the fraction of a dollar. Ans. $\frac{15}{16}$dol.
20. Reduce 21s. 7$\frac{1}{2}$d. to the fraction of a moidore. Answer $\frac{3}{5}$moidore.
21. Reduce 3r. 17$\frac{1}{7}$p. to the fraction of an acre. Ans. $\frac{6}{7}$acre.

ADDITION OF VULGAR FRACTIONS.

Rule.*

Reduce compound fractions to single ones; mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

Examples.

1. Add $7\frac{4}{5}$, $\frac{5}{7}$ of $\frac{3}{8}$, and 7 together.

First. $7\frac{4}{5}=\frac{39}{5}$, $\frac{5}{7}$ of $\frac{3}{8}=\frac{15}{56}$, and $7=\frac{7}{1}$.

Then the fractions are $\frac{39}{5}$, $\frac{15}{56}$, and $\frac{7}{1}$; therefore,

$$\begin{array}{r} 39\times56\times\ 1=2184 \\ 15\times\ 5\times\ 1=\quad 75 \\ 7\times\ 5\times56=1960 \\ \hline 4219 \\ \hline 5\times56\times1=280 \end{array}=15\frac{19}{280}$$

Or thus, $\dfrac{2184+75+1960}{280}=15\frac{19}{280}$.

2. Add $\frac{3}{7}$, $9\frac{1}{5}$, and $\frac{2}{3}$ of $\frac{1}{2}$ together. Ans. $9\frac{101}{105}$.
3. What is the sum of $\frac{4}{5}$, $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{1}{4}$, and $8\frac{4}{13}$? Ans. $9\frac{2319}{12480}$.
4. What is the sum of $\frac{7}{10}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{3}$, and $9\frac{1}{4}$? Ans. $12\frac{59}{80}$.
5. Add together $\frac{5}{7}$E. $\frac{3}{8}$\$ and $1\frac{1}{2}$c. Ans. \$7 53c. $2\frac{6}{7}$m.
6. Add together $\frac{1}{5}$\$ $\frac{5}{8}$c. $\frac{3}{16}$c. and $\frac{7}{8}$m. Ans. 20c. 9m.
7. Add £$\frac{1}{9}$ $\frac{3}{7}$s. and $\frac{4}{5}$d. together. Ans. 2s. $8\frac{64}{105}$d.
8. What is the sum of $\frac{3}{5}$ of £17, £$9\frac{5}{8}$ and $\frac{2}{3}$ of $\frac{1}{2}$ of £$\frac{4}{7}$? Ans. £16 12s. $3\frac{5}{7}$d.
9. Add $\frac{3}{4}$ of a yard, $\frac{1}{3}$ of a foot, and $\frac{5}{8}$ of a mile together. Ans. 1100yds. 2ft. 7inches.
10. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together. Ans. 2 days, 2 hours, 30 minutes, 45 seconds.

SUBTRACTION OF VULGAR FRACTIONS.

Rule.†

Prepare the fractions as in Addition, and the difference of the numerators, written above the common denominator, will give the difference of the fractions required.

* Fractions, before they are reduced to a common denominator, are entirely dissimilar, and therefore cannot be incorporated with one another; but when they are reduced to a common denominator, and made parts of the same thing; their sum, or difference, may then be as properly expressed by the sum or difference of the numerators, as the sum or difference of any two quantities whatever, by the sum or difference of their individuals; whence the reason of the rules, both for Addition and Subtraction, is manifest. If the given fractions have the same denominator and are of the same denomination, the sum of the numerators written over the given denominator, will be the sum of the fractions.

† In subtracting mixed numbers, when the fractions have a common denominator, and the numerator in the subtrahend is less than that in the minuend, the difference of the whole numbers will be a whole number, and the difference of the numerators a numerator to be placed over the given denominator; this

EXAMPLES.

1. From $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{8}$.

$\frac{2}{7}$ of $\frac{5}{8}=\frac{10}{56}=\frac{5}{28}$. Then the fractions are $\frac{3}{4}$ and $\frac{5}{28}$.

3×28 = 84
5× 4 = 20
4×28 = 112 com. den.

$\frac{3}{4}=\frac{84}{112}$, and $\frac{5}{28}=\frac{20}{112}$, therefore,
$\frac{84}{112}-\frac{20}{112}=\frac{64}{112}=\frac{4}{7}$ remainder.

2. From $\frac{49}{50}$ take $\frac{5}{9}$. *Ans.* $\frac{191}{450}$.
3. From $37\frac{1}{4}$ take $19\frac{4}{7}$. *Ans.* $17\frac{19}{28}$.
4. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. *Ans.* $2\frac{1}{12}$.
5. From \$$\frac{1}{2}$ take $\frac{7}{8}$c. *Ans.* 49c. $1\frac{1}{4}$m.
6. Take $3\frac{1}{3}$c. from $\frac{1}{5}$ of $2\frac{1}{3}$\$. *Ans.* $43\frac{1}{3}$c.
7. From $\frac{7}{8}$ of $\frac{4}{9}$ of \$5, take $\frac{5}{8}$ of 96c. added to $\frac{1}{3}$ of $1\frac{1}{8}$\$. *Ans.* 96c. $9\frac{4}{9}$m.
8. From $\frac{1}{4}$£ take $\frac{9}{10}$s. *Ans.* 4s. $1\frac{1}{5}$d.
9. From $\frac{5}{7}$oz. take $\frac{3}{4}$pwt. *Ans.* 13pwt. $12\frac{6}{7}$gr.
10. From $\frac{1}{2}$ of a league take $\frac{3}{8}$ of a mile. *Ans.* 1mi. 1fur
11. From 5 weeks take $19\frac{4}{5}$ days. *Ans.* 15da. 4ho. 48min:

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.*

Reduce compound fractions to simple ones, and mixed numbers to improper fractions; then the product of the numerators will be the numerator, and the product of the denominators, the denominator of the product required.

Note. Where several fractions are to be multiplied, if the numerator of one fraction be equal to the denominator of another, their equal numerators and denominators may be omitted.

EXAMPLES.

1. What is the continued product of $4\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{4}$ of $\frac{7}{8}$, and 6.

$$4\tfrac{1}{3}=\tfrac{13}{3},\ \tfrac{1}{4}\text{ of }\tfrac{7}{8}=\frac{1\times7}{4\times8}=\tfrac{7}{32},\text{ and }6=\tfrac{6}{1}.$$

whole number and the fraction thus formed will be the remainder: but, when the numerator in the subtrahend is greater than that in the minuend, subtract the numerator in the subtrahend from the common denominator, adding the numerator in the minuend, and carrying 1 to the integer of the subtrahend.

Hence, a fraction is subtracted from a whole number, by taking the numerator of the fraction from its denominator, and placing the remainder over the denominator, then taking one from the whole number.

EXAMPLES.

From	$12\frac{2}{5}$	$12\frac{2}{5}$	12
Take	$7\frac{3}{5}$	$7\frac{3}{5}$	$\frac{2}{5}$
Rem.	$5\frac{1}{5}$	$4\frac{4}{5}$	$11\frac{3}{5}$

* Multiplication of a fraction implies the taking of some part or parts of the multiplicand, and therefore may truly be expressed by a compound fraction. Thus $\frac{4}{5}$ multiplied by $\frac{3}{8}$ is the same as $\frac{4}{5}$ of $\frac{3}{8}$; and as the directions of the rule agree with the method already given, to reduce these fractions to simple ones, it is shown to be right.

Then $\frac{13}{3}\times\frac{1}{5}\times\frac{7}{32}\times\frac{6}{1}=\dfrac{13\times1\times7\times6}{3\times5\times32\times1}=\frac{546}{480}=1\frac{11}{80}$ the Answer.

2. Multiply $\frac{4}{17}$ by $\frac{5}{27}$. Ans. $\frac{20}{459}$.
3. Multiply $5\frac{1}{4}$ by $\frac{1}{6}$. Ans. $\frac{7}{8}$.
4. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. Ans. $\frac{5}{14}$.
5. Multiply $\frac{2}{7}$ of $\frac{5}{6}$ by $\frac{4}{5}$ of $\frac{1}{5}$ of $11\frac{3}{7}$. Ans. $\frac{64}{147}$.
6. Multiply $9\frac{3}{4}$, $\frac{1}{2}$ of $\frac{2}{3}$, and $12\frac{4}{7}$ continually together. Ans. $24\frac{18}{35}$.
7. What is the continual product of $\frac{3}{4}$ of $\frac{2}{3}$, $5\frac{1}{2}$, 7 and $\frac{1}{3}$ of $\frac{5}{8}$? Ans. $4\frac{1}{96}$.
8. What is the continual product of 7, $\frac{1}{2}$, $\frac{4}{7}$ of $\frac{3}{8}$, and $3\frac{1}{6}$? 2 Ans. $1\frac{11}{12}$.

Another method for the Multiplication of mixed Quantities.

CASE I.

To multiply a *whole number* by a *fraction*, or a *fraction* by a *whole number*.

Rule.

Multiply the whole number by the numerator of the fraction and divide the product by the denominator: But if the numerator be 1, divide by the denominator only.

	1.	2.	3.	4.	5.	6.	7.
Mult.	8	15	28	36	48	325	259
By	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{12}$
Prod.	2	$7\frac{1}{2}$	$9\frac{1}{3}$	3)72	4)144	8)1625	12)1813
			Prod.	24	36	$203\frac{1}{8}$	$151\frac{1}{12}$

CASE II.

To multiply a *whole number* by a *mixed one*.

Rule.

Multiply by the fraction as in Case 1st; then multiply by the whole number, and add the two products, as in the examples—or, to multiply a mixed number by a whole one, change the place of the factors, and proceed as the rule directs.—See example 6.

	1.	2.	3.	4.	5.	6.
Mult.	15	35	68	42	129	$1\frac{7}{15}$
By	$3\frac{1}{2}$	$5\frac{1}{3}$	$7\frac{11}{12}$	$9\frac{3}{7}$	$8\frac{5}{8}$	24
	$7\frac{1}{2}$	$11\frac{2}{3}$	748	126	645	Mult. 24
	45	175	$62\frac{4}{12}$	18	$80\frac{5}{8}$	By $1\frac{7}{15}$
Prod.	$52\frac{1}{2}$	$186\frac{2}{3}$	476	378	1032	15)168
		Prod.	$538\frac{4}{12}$	396	$1112\frac{5}{8}$	$11\frac{3}{15}$
						24
						Prod. $35\frac{3}{15}=\frac{1}{5}$.

CASE III.

To multiply a *mixed number* by a *mixed number.*

RULE.

Multiply the integral part of the multiplicand by the denominator of its fractional part, and add thereto its numerator: Then multiply by the mixed multiplier, by Case 2d, and divide the product by the denominator of the fractional part of the multiplicand, as in the following example.

Mult. $42\frac{3}{5}$	1st. $42\frac{3}{5}=213$	
By $8\frac{2}{3}$	which mult. by $8\frac{2}{3}$	
	3)426	
	142	
	1704	
	5)1846	
	Product $= 369\frac{1}{5}$	

After this manner may feet and inches be multiplied, calling 1 inch $\frac{1}{12}$ of a foot, 2 inches $\frac{1}{6}$, 3 inches $\frac{1}{4}$, 4 inches $\frac{1}{3}$, 5 inches $\frac{5}{12}$, 6 inches $\frac{1}{2}$, 7 inches $\frac{7}{12}$, 8 inches $\frac{2}{3}$, 9 inches $\frac{3}{4}$, 10 inches $\frac{5}{6}$, 11 inches $\frac{11}{12}$ of a foot.

DIVISION OF VULGAR FRACTIONS.

RULE.*

Prepare the fractions as before: then, invert the divisor and proceed exactly as in Multiplication: The products will be the quotient required.

EXAMPLES.

1. Divide $\frac{1}{3}$ of 17 by $\frac{2}{3}$ of $\frac{6}{8}$

$\frac{1}{3}$ of $17 = \frac{1}{3}$ of $\frac{17}{1} = \dfrac{1\times17}{3\times1} = \frac{17}{3}$ and $\frac{2}{3}$ of $\frac{6}{8} = \frac{12}{24} = \frac{1}{2}$; therefore, $\frac{17}{3} \div \frac{1}{2} = \dfrac{17\times2}{3\times1} = \frac{34}{3} = 11\frac{1}{3}$ the quotient required.

2. Divide $\frac{5}{7}$ by $\frac{3}{5}$. Ans. $1\frac{4}{21}$.
3. Divide $12\frac{1}{5}$ by $\frac{1}{3}$ of 7. Ans. $5\frac{8}{35}$.
4. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$. Ans. $\frac{41}{62}$.
5. Divide $\frac{3}{7}$ by 9. Ans. $\frac{1}{21}$.
6. Divide $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{3}$ by $\frac{1}{8}$ of $\frac{3}{4}$. Ans. $\frac{8}{9}$.
7. Divide 7 by $\frac{3}{8}$. Ans. $18\frac{2}{3}$.
8. Divide $4204\frac{1}{6}$ by $\frac{7}{8}$ of 112. Ans. $42\frac{529}{588}$.

* The reason of the rule may be shewn thus. Suppose it were required to divide $\frac{4}{5}$ by $\frac{2}{7}$. Now $\frac{4}{5} \div 2$ is manifestly $\frac{1}{2}$ of $\frac{4}{5}$ or $\frac{4}{2\times5}$; but $\frac{2}{7}=\frac{1}{7}$ of 2; therefore, $\frac{1}{7}$ of 2, or $\frac{2}{7}$, must be contained 7 times as often in $\frac{4}{5}$ as 2 that is $\frac{4\times7}{5\times2}=$ the answer, which is according to the rule.

DECIMAL FRACTIONS.

A DECIMAL FRACTION is a fraction whose denominator is a unit with so many cyphers annexed as the numerator has places of figures.

As the denominator of a decimal fraction is always 10, or 100, or 1000, &c. the denominators need not be expressed: For the numerator only may be made to express the true value: For this purpose it is only required to write the numerator with a point before it at the left hand, to distinguish it from a whole number, when it consists of so many figures as the denominator has cyphers annexed to unity, or 1: So $\frac{5}{10}$ is written ·5; $\frac{33}{100}$ ·33; $\frac{735}{1000}$ ·735, &c.

The point prefixed is called the *Separatrix*.

But if the numerator has not so many places as the denominator has cyphers, put so many cyphers before it, viz. at the left hand, as will make up the defect; so write $\frac{5}{100}$ thus, ·05; and $\frac{6}{1000}$ thus, ·006, &c. Thus do these fractions receive the form of whole numbers.

We may consider unity as a fixed point, from whence whole numbers proceed infinitely increasing toward the left hand, and decimals infinitely decreasing toward the right hand to 0, as in the following

TABLE.*

C Millions	X Millions	Millions	C Thousands	X Thousands	Thousands	Hundreds	Tens	Units	Tenth Parts	Hundredth Parts	Thousandth Parts	X Thousandth Parts	C Thousandth Parts	Millionth Parts	X Millionth Parts	C Millionth Parts
9	8	7	6	5	4	3	2	1	·2	3	4	5	6	7	8	9

* It will be very apparent to the learner from the nature of decimals, and what has been said of *Federal Money*, that this money is purely decimal; and, the dollar being the money unit, the lower denominations are plainly so many decimal parts of a dollar; thus 9 dollars and 8 dimes are expressed 9·8 = $9\frac{8}{10}$ doll.—12 dollars, 4 dimes, and 7 cents thus, 12·47 = $12\frac{47}{100}$ doll.—20 dollars, 3 dimes, 4 cents and 5 mills, thus 20·345 = $20\frac{345}{1000}$ doll.—100 dollars and 9 mills, thus 100.009 = $100\frac{9}{1000}$ doll. and 50 dollars, 5 cents, thus 50·05 = $50\frac{5}{100}$ doll. wherefore, it is, in all respects, added, subtracted, multiplied and divided, the same as decimals; and, of all coins, it is the most simple.

It may also be observed that the sum exhibits the particular number of each different piece of money contained in it, viz. 455997 mills = $45599\frac{7}{10}$ cents = $4559\frac{97}{100}$ dimes = $455\frac{997}{1000}$ dollars = $45\frac{5997}{10000}$ eagles = 4 5 5; 9 9 7.
E. D. d. c. m.

Also, the names of the coins, less than a dollar, are significant of their values. For the *mill*, which stands in the 3d place at the right hand of the separatrix

From this table it is evident, that in decimals, as well as in whole numbers, each figure takes its value by its distance from unit's place: If it be in the first place after units (or the separating point) it signifies tenths; if in the second, hundredths, &c. decreasing in each place in a tenfold proportion.

Every single figure expressing a decimal, has for its denominator a unit or 1, with so many cyphers as its place is distant from unit's place: Thus 2 in the decimal part of the table = $\frac{2}{10}$; 3 = $\frac{3}{100}$; 4 = $\frac{4}{1000}$, &c. And if a decimal be expressed by several figures, the denominator is 1, with so many cyphers as the lowest figure is distant from unit's place. So ·357 signifies $\frac{357}{1000}$, and ·0053 = $\frac{53}{10000}$, &c.

Cyphers, placed at the right hand of a decimal fraction, do not alter its value, since every significant figure continues to possess the same place: So ·5, ·50, and ·500, are all of the same value, and each = $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000} = \frac{1}{2}$.

But cyphers, placed at the left hand of a decimal, do alter its value, every cypher depressing it to $\frac{1}{10}$ of the value it had before, by removing every significant figure one place further from the place of units. So ·5, ·05, ·005, all express different decimals, viz. ·5, $\frac{5}{10}$; ·05, $\frac{5}{100}$; ·005, $\frac{5}{1000}$.

Hence may be observed the contrary effects of cyphers being annexed to whole numbers, and decimals.

It is likewise evident from the table, that since the places of decimals decrease in a tenfold proportion from units downwards, so they consequently increase in a tenfold proportion from the right hand toward the left, as the places of whole numbers do: For, ten hundredth parts make one tenth, ten tenths make 1; ten units, ten; ten tens, one hundred, &c. viz. $\frac{10}{100} = \frac{1}{10}$, $\frac{10}{10} = 1$, and $1 \times 10 = 10$, which proves that decimals are subject to the same law of Notation, and consequently of operation, as whole numbers are.

Decimal fractions of unequal denominators are reduced to one common denominator, when there are annexed to the right hand of those, which have fewer places, so many cyphers, as make them equal in places with that which has the most. So these decimals, ·5, ·06, ·455, may be reduced to the decimals, ·500, ·060, and ·455, which have, all, 1000 for their denominator.

Of Decimals, that is the greatest, whose highest figure is greatest, whether they consist of an equal or unequal number of places: Thus, ·5 is greater than, ·459, for if it be reduced to the same denominator with ·459, it will be ·500.

or place of thousandths, is contracted from *mille* the Latin for *thousand*: *Cent*, which occupies the second place, or place of hundredths, is an abbreviation of *centum*, the Latin for *hundred*: And *dime*, which is in the first place or place of tenths, is derived from *disme*, the French for *tenth*.

Such being the nature of *Federal Money*, its operations can in no other way be so well understood as in obtaining a good knowledge of decimals, and applying their several rules to the various cases of money matters.

In sums of Federal Money, it is customary to read it in dollars, cents and mills. Thus the above example is read 455 dolls. 99 cents and 7 mills.

A mixed number, viz. a whole number, with a decimal annexed, is equal to an improper fraction, whose numerator is all the figures of the mixed number, taken as one whole number, and the denominator, that of the decimal part. So 45·309 is equal to $\frac{45309}{1000}$, as is evident from the method given to reduce a mixed number to an improper fraction:

Thus, $45\times1000+\cdot309=\frac{45309}{1000}$ as above.

ADDITION OF DECIMALS.

Rule.

1. Place the numbers, whether mixed, or pure decimals, under each other, according to the value of their places.

2. Find their sum as in whole numbers, and point off so many places for decimals, as are equal to the greatest number of decimal places in any of the given numbers.

Examples.

1. Find the sum of 19·073+2·3597+223+·0197581+3478·1+12·358.

```
  19·073
   2·3597
 223·
    ·0197581
3478·1
  12·358
-------------
3734.9104581 the sum.
```

2. Required the sum of 429+21·37+355·003+1·07+1·7? Ans. 808·148.

3. Required the sum of 5·3+11·973+49+·9+1·7314+34·3? Ans. 103·2044.

4. Required the sum of 973+19+1·75+93·7164+·9501? Ans. 1088.4165.

SUBTRACTION OF DECIMALS.

Rule.

Place the numbers according to their value; then subtract as in whole numbers, and point off the decimals as in Addition.

Examples.

1. Find the difference of 1793·13 and 817·05693?

```
     From 1793·13
     Take  817·05693
     ---------------
Remainder  976·07307
```

2. From 171·195 take 125·9176. Ans. 45·2774.

3. From 219·1384 take 195·91. Ans. 23·2284.

4. From 480 take 245·0075. Ans. 234·9925.

MULTIPLICATION OF DECIMALS.

CASE I.

RULE.

1. Whether they be mixed numbers, or pure decimals, place the factors and multiply them as in whole numbers.

2. Point off so many figures from the product as there are decimal places in both the factors; and if there be not so many places in the product, supply the defect by prefixing cyphers.

The reason for pointing off the figures for decimals, is evident from substituting the equivalent vulgar fractions, as in the following example. Or, The reason of the rule for pointing off the figures for decimals, is evident from the notation of decimals. Thus $\cdot5\times\cdot5=\cdot25$; for $\cdot5\times1=\cdot5$, or once five tenths. But, as the multiplier is less than unity, or tenths, multiplying is only taking tenths of tenths, and so many tenths of tenths are evidently so many hundredths. So also, tenths of hundredths would be thousandths; hundredths of hundredths, would be ten thousandths, and so on.

EXAMPLES.

1. Multiply ·02345
by ·00163

```
      7035
     14070
     2345
 ----------
 ·0000382235 the product,
```

$$\cdot02345=\frac{2345}{100000}$$

$$.00163=\frac{163}{100000}$$

$$=\frac{382235}{10000000000}.$$

2. Multiply 25·238 by 12·17. Ans. 307·14646.
3. Multiply ·3759 by ·945. Ans. ·3552255.
4. Multiply ·84179 by ·0385. Ans. ·032408915.

To multiply by 10, 100, 1000, &c. remove the separating point so many places to the right hand, as the multiplier has cyphers.

So ·345 Multiplied by $\left\{\begin{matrix}10\\100\\1000\end{matrix}\right\}$ makes $\left\{\begin{matrix}3\cdot45\\34\cdot5\\345\end{matrix}\right.$

For $\cdot345\times10$ is 3·450, &c.

CASE II.

To contract the operation, so as to retain so many decimal places in the Product as may be thought necessary.

RULE.

1. Write the unit's place of the multiplier under that figure of the multiplicand, whose place you would reserve in the product; and dispose of the rest of the figures in a contrary order to what they are usually placed in.

2. In multiplying, reject all the figures which are on the right hand of the multiplying digit, and set down the products, so that

their right hand figures may fall in a straight line below each other; observing to increase the first figure of every line with what would arise by carrying 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. from the preceding figures, when you begin to multiply, and the sum will be the product required.

Examples.

1. It is required to multiply 56·7534916 by 5·376928, and to retain only five places of decimals in the product.

Contracted way.	Common way.	
56·7534916	56·7534916	
829673·5	5·376928	
28376746 . .	45	40279328
1702605 . . .	113	5069832
397274	5107	814244
34052	34052	09496
5108	397274	4412
113	1702604	748
45	28376745	30
305·15943	305·15943	80813048

By the operation in the *common way*, it is evident that all the figures which are cut off at the right hand, by the perpendicular line, are wholly omitted in the *contracted way*, and the last product here is the first there; consequently the reason of placing the multiplier in a reverse order, must appear very plain.

DIVISION OF DECIMALS.

Rule.*

1. The places of decimal parts in the divisor and quotient counted together, must always be equal to those in the dividend; therefore, divide as in whole numbers, and, from the right hand of the quotient, point off so many places for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places of the quotient be not so many as the rule requires, supply the defect by prefixing cyphers to the left hand.

3. If at any time there be a remainder, or the decimal places in the divisor be more than those in the dividend, cyphers may be annexed to the dividend, or to the remainder, and the quotient carried on to any degree of exactness.

* The reason of pointing off so many decimal places in the quotient, as those in the dividend exceed those in the divisor, will easily appear, for, since the number of decimal places in the dividend is equal to those in the divisor and quotient taken together, by the nature of multiplication: It therefore follows that the quotient contains so many as the dividend exceeds the divisor.

EXAMPLES.

1.		2.
219)·117841075(·000538087, &c.		·3719)38·0000(102·178, &c.
1095	In Example 1st, the divisor having no decimals, the quotient must have so many as there are in the dividend. In Example 2, the dividend being an integer must have at least so many cyphers annexed as there are decimals in the divisor, and so far the quotient will be whole numbers, then annexing more cyphers, the remaining figures in the quotient will be decimals, according to the Rule.	3719
834		8100
657		7438
1771		6620
1752		3719
1907		29010
1752		26033
1555		29770
1533		29752
22		18

3d. 133)5737(43·1353+ (4th.) 23·7)65321(2756·16+
5th. †72)918·217(12753+ (6th.) 25·17)315·6293(1253+
7th. ·317)29·417(92+ (8th.) 37·9)·0059374(156+
9th. ·375)·173948375(463862+

Having a multiplier, to find a divisor which shall give a quotient equal to the product by that multiplier.

RULE.

Divide unity by the given multiplier, and the quotient will be the divisor sought.

What divisor is that, by which dividing 5357, shall give a quotient equal to the product of the same number multiplied by 250 ?

250)1·000(·004 the Answer. And ·004)5357·000(1339250.
Proof. 5357 × 250=1339250.

Having a divisor, to find a multiplier which shall give a product equal to the quotient by that divisor.

RULE.

Divide unity by the given divisor, and the quotient will be the multiplier sought.

What multiplier is that, by which multiplying 5357, shall give a product equal to the quotient of the same number divided by ·004 ?

Ans. 250.

CASE II.

To contract Division, when there are many decimals in the dividend, and the divisor is large.

RULE.

1. Whatever place of the dividend corresponds with the unit's

† The following questions are left unpointed in the quotient to exercise the learner.

place of the divisor, at the first step of the division, the same place must the first figure of the quotient have.

2. In dividing reject the last right hand figure of the divisor, at every step, (instead of bringing down a figure, as is common,) and make the last remainder the dividend for the new divisor at every step: Thus continue the division until the divisor shall be exhausted.

```
99·5678)4·6789837568(·0469931 Quotient.
          3·982712
          --------
  99·567)696271
         597402
         ------
  99·56) 98869
         89604
         -----
   99·5) 9265
         8955
         ----
     99) 310
         297
         ---
      9) 13
          9
          -
Remainder 4
```

Here, the unit's place of the divisor in the first step falls under 7 in the place of hundredths in the dividend, therefore, I put 4, the first quotient figure, in the place of hundredths, by prefixing a cypher.

I have set down every divisor, to explain the work; but you need only put a dash over every figure rejected, as you proceed, to show it is omitted.

When decimals or whole numbers are to be divided by 10, 100, 1000, &c. (viz. unity with cyphers) it is performed by removing the separatrix, in the dividend, so many places toward the left hand as there are cyphers in the divisor.

EXAMPLES.

10	Dividing	7654	The Quot. is	765·4
100				76·54
1000				7·654
10000				·7654

REDUCTION OF DECIMALS.

CASE I.

To reduce a Vulgar Fraction to its equivalent Decimal.

RULE.*

Divide the numerator by the denominator, as in division of decimals, and the quotient will be the decimal required:—Or, so many cyphers as you annex to the given numerator, so many pla-

* By annexing one, two, three, &c. cyphers to the numerator, the value of the fraction is increased ten, a hundred, &c. times. After dividing, the quotient will of course, be ten, a hundred, &c. times too much: the quotient must therefore be divided by ten, a hundred, &c. to give the true quotient or fraction. In the first example, $\frac{1}{8}$, is made $\frac{1000}{8}=\frac{125}{1}$, which divided by 1000, is $\frac{125}{1000}=$ ·125, and is the rule.

ces must be pointed off in the quotient, and if there be not so many places of figures in the quotient, the deficiency must be supplied by prefixing so many cyphers before the quotient figures.

EXAMPLES.

1. Reduce $\frac{1}{8}$ to a decimal.

$$\begin{array}{r} 8)\underline{1.000} \\ \cdot125 \text{ Ans.} \end{array}$$

2. Reduce $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{2}{3}$ to decimals. Answers, ·375, 625, ·666+.
3. Reduce $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{4}{5}$, $\frac{5}{6}$ and $\frac{7}{8}$ to decimals.
Answers, ·25, ·5, 75, ·333+, ·8, ·833+, ·875.
4. Reduce $\frac{5}{19}$, $\frac{27}{39}$, $\frac{12}{480}$, and $\frac{9}{36}$ to decimals.
Answers, ·263+, ·692+, ·025, ·25.
5. Reduce $\frac{7}{375}$, $\frac{9}{1129}$, and $\frac{5}{1875}$ to decimals.
Answers, ·0186+, ·00797+, ·00266+.

CASE II.

To reduce numbers of different denominations, as of Money, Weight and Measure, to their equivalent decimal values.

RULE.*

I. Write the given numbers perpendicularly under each other, for dividends; proceeding orderly from the least to the greatest.

II. Opposite to each dividend, on the left hand, place such a number, for a divisor, as will bring it to the next superiour denomination, and draw a line perpendicularly between them.

III. Begin with the highest, and write the quotient of each division as decimal parts on the right hand of the dividend next below it, and so on, until they are all used, and the last quotient will be the decimal sought.

EXAMPLES.

1. Reduce 17s. 8¾d. to the decimal of a pound.

$$\begin{array}{r|l} 4 & 3\cdot \\ \hline 12 & 8\cdot75 \\ \hline 20 & 17\cdot729166, \text{\&c.} \\ \hline \end{array}$$

·886458, &c. the decimal required.

Here, in dividing 3 by 4, I suppose 2 cyphers to be annexed to the 3, which make it 3·00, and ·75 is the quotient, which I write against 8 in the next line; this quotient, viz. 8·75 being pence and decimal parts of a penny, I divide them by 12, which brings them to shillings and decimal parts, I therefore divide by 20, and, there being no whole number, the quotient is decimal parts of a pound.

* The reason of the rule may be explained from the first example: Thus, three farthings are $\frac{3}{4}$ of a penny, which, reduced to a decimal, is, ·75; consequently, 8¾d. may be expressed, 8.75d. but 8.75 is $\frac{875}{100}$ of a penny $= \frac{875}{1200}$ of a shilling, which reduced to a decimal, is, ·729166s.+ In like manner, 17·729166s.+ are $\frac{17729166}{1000000}$s. $= \frac{17729166}{20000000}$£ $=$ ·886458+ as by the rule.

2. Reduce 1, 2, 3, 4, and so on to 19 shillings, to decimals.

Shillings.	1	2	3	4	5	6	7	8	9	10
Answers.	·05,	·1,	·15,	·2,	·25,	·3,	·35,	·4,	·45,	·5,
Shillings.	11	12	13	14	15	16	17	18	19	
Answers.	·55,	·6,	·65,	·7,	·75,	·8,	·85,	·9,	·95.	

Here, when the shillings are even, half the number, with a point prefixed, is their decimal expression; but if the number be odd, annex a cypher to the shillings, and then halving them, you will have their decimal expression.

3. *Reduce 1, 2, 3, and so on to 11 pence, to the decimals of a shilling.

Pence.	1	2	3	4	5	6
Answers.	·083+,	·166,	·25,	·333+,	·416+,	·5,
Pence.	7	8	9	10	11	
Answers.	·583+,	·666+,	·75,	·833+,	·916+.	

4. Reduce 1, 2, 3, &c. to 11 pence, to the decimals of a pound.

Pence.	1	2	3	4	5	
Answers.	·00416+,	·0083+,	·0125,	·01666+,	·0208+,	
Pence.	6	7	8	9	10	11
Answers.	·025,	·02916+,	·0333+,	·0375,	0416+,	·04583+.

5. Reduce 1, 2 and 3 farthings to the decimals of a penny.
1qr.=·25d. 2qr.=·5d. and 3qr =·75d. Answers.

6. Reduce 1, 2 and 3 farthings to the decimals of a shilling.
Answers. 1qr.=·02083+s. 2qrs.=·04166+s. 3qrs.=·0625s.

7. Reduce 1, 2 and 3 farthings to the decimals of a pound.
Ans. 1qr.=·0010416+ £. 2qrs.=·002083+ £. 3qrs.=·003125 £.

8. Reduce 13s, 5¼d. to the decimal of a pound. Ans. ·6729+.

9. Reduce 7Cwt. 3qrs. 17lb. 10oz. 12dr. to the decimal of a ton.
Ans. ·39538+.

10. Reduce 10oz. 13pwt. 9gr. to the decimal of a pound Troy.
Ans. ·8890625.

11. Reduce 3qrs. 3n. to the decimal of a yard. Ans. ·9375.

12. Reduce 5fur. 12po. to the decimal of a mile. Ans. ·6625.

13. Reduce 55m. 37sec. to the decimal of a day.
Ans. ·03862+.

CASE III.

To find the decimal of any number of shillings, pence and farthings, by inspection.

RULE.†

I. Write *half* the greatest even number of shillings for the first decimal figure.

* The answers to this question are the same as the decimal parts of a foot.

† The invention of the rule is as follows: As shillings are so many 20ths of a pound, half of them must be so many tenths, and consequently take the place of tenths in the decimals; but when they are odd, their half will always consist of two figures, the first of which will be half the even number, next less, and the second a 5: Again, farthings are so many 960ths of a pound, and had it happened that 1000, instead of 960, had made a pound, it is plain any

II. Let the farthings in the given pence and farthings possess the second and third places: observing to increase the *second* place or place of hundredths, by 5 if the shillings be odd, and the third place by 1, when the farthings exceed 12, and by 2 when they exceed 36.

EXAMPLES.

1. Find the decimal of 13s. 9¾d. by inspection.

·6 . . =½ of 12s.
5 for the odd shilling.
39 =the farthings in 9¾d.
Add 2 for the excess of 36.

·691 = decimal required.

2. Find, by inspection, the decimal expressions of 18s. 3¼d. and 17s. 8½d. Ans. £·914 and £·885.

4. Value the following sums, by inspection, and find their total, viz. 15s. 3d.+8s. 11½d.+10s. 6¼d.+1s. 8½d.+½d.+2¾d.
Ans. £1·834 the total.

CASE. IV.

To find the value of any given decimal in the terms of the integer.

RULE.

I. Multiply the decimal by the number of parts in the next less denomination, and cut off so many places for a remainder, to the right hand, as there are places in the given decimal.

II. Multiply the remainder by the next inferiour denomination, and cut off a remainder as before.

III. Proceed in this manner through all the parts of the integer, and the several denominations, standing on the left hand, make the answer. This case is the reverse of Case II. and the reason of the rule is hence obvious.

EXAMPLES.

1. Find the value of ·73968 of a pound.

```
  ·73968
      20
 -------
 14·79360
      12
 -------
  9·52320
       4
 -------
  2·09280 Ans. 14s. 9½d.
```

number of farthings would have made so many thousandths, and might have taken their place in the decimal accordingly. But 960 increased by $\frac{1}{24}$ part, of itself, is=1000, consequently any number of farthings, increased by their $\frac{1}{24}$ part, will be an exact decimal expression for them: Whence, if the number of farthings be more than 12, $\frac{1}{24}$ part is greater than ½qr. and, therefore 1 must be added; and when the number of farthings is more than 36, $\frac{1}{24}$ part is greater than 1½qr. for which 2 must be added.

2. What is the value of ·679 of a shilling? Ans. 8·148d.
3. What is the value of ·9999£? Ans. 19s. 11¾d.$\frac{9}{10}$ or £1.
4. What is the value of ·617 of a Cwt.?
Ans. 2qrs. 13lb. 1oz. 10$\frac{6}{10}$dr.
5. What is the value of ·8593 of a lb. Troy?
Ans. 10oz. 6pwt. 5gr.
6. What is the value of ·397 of a yard? Ans. 1qr. 2·352n.
7. What is the value of ·8469 of a degree?
Ans. 58m. 6fur. 35po. 0ft. 11in.
8. What is the value of ·569 of a year?
Ans. 207da. 16ho. 26m. 24sec.
9. What is the value of ·713 of a day? Ans. 17h. 6m. 43sec.

CASE V.

To find the value of any decimal of a pound by inspection.

Rule.*

Double the first figure, or place of tenths, for shillings, and if the second figure be 5, or more than 5, reckon another shilling; then, after the 5 is deducted, call the figures in the second and third places so many farthings, abating 1 when they are above 12, and 2 when above 36, and the result will be the answer.

Note. When the Decimal has but 2 figures, if any thing remain after the shillings are taken out, a cypher must be annexed to the right hand, or supposed to be so.

Examples.

1. Find the value of ·876£ by inspection.

16s. = double of 8.
1s. for the 5 in the second place, which is to be taken out of 7.
And 6½d. = 26 farthings remain to be added.
Deduct ¼d. for the excess of 12.

17s. 6¼d. the Ans.

2. Find, by inspection, the value of ·49 £.

8s. - - = double of 4.
1s. - - for the 5 in the place of hundredths.
10d. = 40 farthings, a 0 being annexed to the remaining 4.
Ded. ½d. for the excess of 36.

9s. 9½d. the Answer.

3. Find the value of ·097£ by Inspection. Ans. 1s. 11¼d.

4. Value the following decimals by Inspection, and find their sum, viz. ·785£ + ·537£ + ·916£ + ·74£ + ·5£ + ·25£ + ·09£ + ·008£. Ans. £3 16s. 6d.

* As this rule is the reverse of the rule, Case III. the reason is apparent from the demonstration of that rule.

DECIMAL TABLES OF COIN, WEIGHT, AND MEASURE.

TABLE I.

Coin.

£ 1 the Integer.

Shil.	dec.	Shil	dec.
19	·95	9	·45
18	·9	8	·4
17	·85	7	·35
16	·8	6	·3
15	·75	5	·25
14	·7	4	·2
13	·65	3	·15
12	·6	2	·1
11	·55	1	·05
10	·5		

Pence.	Decimals.
11	·045833
10	·041666
9	·0375
8	·033383
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Farth's.	Decimals.
3	·003125
2	·0020833
1	·0010416

TABLE II.

Coin & Long Meas.

1 Shil. and 1 Foot the Integer.

Pence & Inches.	Decimals.
11	·916666
10	·833333
9	·75
8	·666666
7	·583333
6	·5
5	·416666
4	·333333
3	·25
2	·166666
1	·083333

Farth's.	Decimals.
3	·0625
2	·041666
1	·020383

TABLE III.

Troy Weight.

1lb. the Integer.

Ounces the same as Table II.

Pwts.	Decimals.
10	·041666
9	·0375
8	·033333
7	·029166
6	·025
5	·020833
4	·016666
3	·0125
2	·008333
1	·004166

Grains.	Decimals.
12	·002083
11	·00191
10	·001736
9	·001562
8	·001389
7	·001215
6	·001042
5	·000868
4	·000694
3	·000521
2	·000347
1	·000173

1 Oz. the Integer. Pennyweights the same as Shillings in the first Table.

Grains.	Decimals.
12	·025
11	·022916
10	·020833
9	·01875
8	·016666
7	·014583
6	·0125
5	·010416
4	·008333
3	·00625
2	·004166
1	·002083

TABLE IV.

Avoirdupois Wt.

112lb. the Integer.

Qrs.	Decimals.
3	·75
2	·5
1	·25

Pounds.	Decimals.
27	·241071
26	·232143
25	·223214
24	·214286
23	·205357
22	·196428
21	·1875
20	·178571
19	·169643
18	·160714
17	·151786
16	·142857
15	·133928
14	·125
13	·110671
12	·107143
11	·098214
10	·089286
9	·080357
8	·071428
7	·0625
6	·053571
5	·044643
4	·035714
3	·026786
2	·017857
1	·008928

Ounces.	Decimals.
15	·008370
14	·007812
13	·007254
12	·006696
11	·006138
10	·00558
9	·005022
8	·004464
7	·003906
6	·003348
5	·00279
4	·002232
3	·001674
2	·001116
1	·000558

qr.of ozs.	Decimals.
3	·000418
2	·000279
1	·000139

TABLE V.

Avoirdupois Wt.

1lb. the Integer.

Ounces.	Decimals.
15	·9375
14	·875
13	·8125
12	·75
11	·6875
10	·625
9	·5625
8	·5
7	·4375
6	·375
5	·3125
4	·25
3	·1875
2	·125
1	·0625

Drams.	Decimals.
15	·058593
14	·054687
13	·050781
12	·046875
11	·042968
10	·039062
9	·035156
8	·03125
7	·027343
6	·023437
5	·019531
4	·015625
3	·011718
2	·007812
1	·003906

TABLE VI.

Cloth Measuure.

1 Yard the Integer.

Qrs.	Decimals.
3	·75
2	·5
1	·25

Nails.	Decimals.
3	·1875
2	·125
1	·0625

TABLE VII.

Long Measure.

1 Mile the Integer.

Yards.	Decimals.
1000	·568182
900	·511364
800	·454545
700	·397727
600	·34
500	·284091
400	·227272
300	·170454
200	·113636
100	·056818

Carried over.

Yards.	Decimals.
90	·051136
80	·045454
70	·039773
60	·034091
50	·028409
40	·022727
30	·017045
20	011364
10	·005682
9	·005114
8	·004545
7	·003977
6	·003409
5	·002841
4	·002273
3	·001704
2	·001136
1	·000568

Feet.	Decimals.
2	·0003787
1	·0001892

Inches.	Decimals.
6	0000947
5	·000079
4	·0000632
3	·0000474
2	·0000316
1	·0000158

TABLE VIII.

LIQUID MEASURE.

1 Gal. the Integer.

Quarts the same as qrs. in TABLE VI.

1 Pint.	·125
3 Gills.	·09375
2	·0625
1	·03125

TABLE IX.

TIME.

1 Year the Integer.

Months the same as Pence in TABLE II.

Days.	Decimals.
365	1·000000
300	·821928
200	·547945
100	·273973
90	·246575
80	·219178
70	·191781
60	·164383
50	·136986
40	·109589

Days.	Decimals.
30	·082192
20	054794
10	·027397
9	·024657
8	·021918
7	019178
6	·016438
5	·013698
4	·010959
3	·008219
2	·005479
1	·002739

1 Day the Integer.

Hours.	Decimals.
23	·958333
22	·916666
21	·375
20	·833333
19	·791666
18	·75
17	·708333
16	·666666
15	·625
14	·583333
13	·541666
12	·5

Hours.	Decimals.
11	·458333
10	·416666
9	·375
8	·333333
7	·291666
6	·25
5	·208333
4	·166666
3	·125
2	·083333
1	·041666

minutes.	Decimals.
30	·020833
20	·013888
10	·006944
9	·00625
8	·005555
7	·004861
6	·004166
5	·003472
4	·002777
3	·002083
2	·001388
1	·000694

GENERAL RULE.

To find the value of goods in Federal Money.—Multiply the price and quantity together; point off in the product, for denominations lower than dollars, as many places as there are in the given price; or, if there be decimal places in the quantity also, according to the rule for multiplication of decimals. This is really multiplication of decimals, the dollar being considered the unit.

EXAMPLES.

1. Find the cost of 823 yards, at $1.29c. a yard.
   823×$1.29c.=$1061.67c. Ans.
2. Find the value of 56yds. 2qrs. at $3.11c. per yard.
   56yds. 2qrs.=56·5; and 56·5×3·11=$175.71c. 5m. Ans.
3. What must I pay for 6$\frac{1}{8}$yds. at $5.50c. per yard?
   Ans. $33.68c. 7m. ·5.
4. Bought 7$\frac{11}{15}$yds. at 34 cents per yard: what did I pay for the whole?
   Ans. 2.62\frac{14}{15}$c.

COMPOUND MULTIPLICATION*

IS extremely useful in finding the value of Goods, &c. And as in Compound Addition, we carry from the lowest denomination to the next higher, so we begin and carry in Compound Multiplication: One general rule being to multiply the price by the quantity. The reason of the following rules is obvious.

CASE I.

When the quantity does not exceed 12 *yards, pounds, &c*: Set down the price of 1, and place the quantity underneath the least denomination, for the multiplier, and, in multiplying by it, observe the same rules for carrying from one denomination to another as in Compound Addition.

Introductory Examples.

	1.			2.			3.				4.		
	£	s.	d.	£	s.	d.	D.	d.	c.	m.	£	s.	d.
Multiply	15	17	1	25	12	8	8	5	1	7	67	18	6¼
by			2			3				4			5
Prod.	31	14	2				34	0	6	8			

5.		6.			7.			8.				
D.	c.	£	s.	d.	£	s.	d.	E.	D.	d.	c.	m.
4	75	13	12	11	31	16	8¼	2	7	8	9	1
	6			7			8					9

9.			10.			11.			12.		
£	s.	d.	£	s.	d.	D.	c.	m.	£	s.	d.
4	13	4¾	8	15	11¾	35	87	5	14	17	8¼
		10			11			12			9
									133	19	2¼

In the last example, I say, 9 times 1 is 9 farthings=2¼d. I set down ¼ and carry 2, saying, 9 times 8 is 72, and 2 I carry makes 74 pence,=6s. 2d. I set down 2 in the pence and carry 6; then, 9 times 7 (the unit figure in the shillings) is 63, and 6 I carry is 69,

* The product of a number, consisting of several parts or denominations, by any simple number whatever, will be expressed by taking the product of that simple number, and each part by itself, as so many distinct questions: Thus £33 15s. 9d. multiplied by 5, will be 165 75s. 45d.=(by taking the shillings from the pence, and the pounds from the shillings, and placing them in the shillings and pounds respectively,) £168 18s. 9d. and this will be true when the multiplicand is any compound number whatever.

I set down 9 under the unit figure of the shillings, and carry 6, saying, 9 times 1 is 9, and 6 I carry is 15, then I halve 15, which is 7 and 1 over, which I set in the ten's place in the shillings, and that, with the 9, makes 19 shillings; then I carry the 7 as pounds: Lastly, 9 times 4 is 36, and 7 I carry, are 43 pounds: I set down 3 and carry 4, saying, 9 times 1 is 9, and 4 I carry makes 13, which I set down, and the product is £133 19s. 2½d.

Practical Questions.

Note. The facility of reckoning in the *Federal Money* compared with *pounds, shillings,* &c. may be seen from the examples in this and the following cases; where the same questions are given in both the currencies, as near as can be, avoiding small fractions.

In the following examples in this and the succeeding cases, the price in pounds, or shillings, &c. is in the currency of New Jersey, Pennsylvania, Delaware, and Maryland, where the dollar is 7s. 6d. in the last example in each case; in the last example but one, the price is in the currency of New York and North Carolina, where the dollar is 8s.; and in the other examples, in the currency of New England, where the dollar is 6s. Thus in the 3d example, the price, 9s. 10d. in the currency of New York, &c. is equal to 122c. 9m.; and in example 4th, the price 13s. 7½d.=181c. 7m.

1. What will 9 yards of cloth at {5s. 4d. / 88c. 9m.} per yard, come to?

	£0	5s.	4d.	price of one yard,	·88c.	9m.	
Multiplied by			9	yards.		9	
Ans.	£2	8	0	price of 9 yds.	$8	·00	1

2. 3 yards at	15s. 4d. / $2 55c. 6m.	per yard	=	£2 6s. / $7 66c. 8m.
3. 6 - - - -	9s. 10d. / $1 22c. 9m.	- - - -	=	£2 19s. / $7 37c. 4m.
4. 9 - - - -	13s. 7½d. / $1 81c. 7m.	- - - -	=	£6 2s. 7½d. / $16 35c. 3m.

CASE II.

When the multiplier, that is, the quantity, is above 12: You must multiply by two such numbers, as, when multiplied together, will produce the given quantity.

Examples.

1. What will 144 yards of cloth cost at {3s. 5¼d. / 57c. 6⅘m.} per yard?

	£	s.	d.		c. m.	
	0	3	5½	price of 1 yard.	·5764	Or, ·5764
Multiply by			12		144	12
Produces	2	1	6	price of 12 yards.	23056	69168
Multiplied by			12		23056	12
					5764	
Answer	£24	18	0	price of 144 yards.		83·0016
					Ans. $83·0016	

Questions.				Answers.
2. 24 yards at	6s. 3¾d. / $1 5c. 2m.	per yard =		£7 11s. 6d. / $25 24c. 8m.
3. 27 - - - -	9s. 10d. / $1 22c. 9m.	- - - - -		£13 5s. 6d. / $33 18c. 3m.
4. 44 - - - -	13s. 7½d. / $1 81c. 7m.	- - - - -		£29 19s. 6d. / $79 94c. 8m.

CASE III.

When the quantity is such a number, as that no two numbers in the table will produce it exactly: Then multiply by two such numbers as come the nearest to it; and for the number wanting, multiply the given price of one yard by the said number of yards wanting, and add the products together for the answer; but if the product of the two numbers exceed the given quantity, then find the value of the overplus, which subtract from the last product, and the remainder will be the answer.

EXAMPLES.

1. What will 47 yards of cloth, at { 17s. 9d. / $2 95c. 8m. } per yard, come to?

	£	s.	d.		
	0	17	9	price of 1 yard.	$2·958
Multiplied by			5		47
Produces	4	8	9	price of 5 yards.	20706
Multiplied by			9		11832
Produces	39	18	9	price of 45 yards.	Ans. $139·026
Add	1	15	6	price of 2 yards.	
Ans.	£41	14	3	price of 47 yards.	

Note. This may be performed by first finding the value of 48 yards, from which if you subtract the price of 1, the remainder will be the answer as above.

Questions.			Answers.
2. 75 yards, at	5s. 7½d. / 93c. 7½m.	per yard =	£21 1s. 10½d. / $70 31c. 2½m.
3. 67½ - - - -	16s. 4d. / $2 4c.	- - - - =	£55 2s. 6d. / $137 81c.
4. 59 - - - -	10s. 0d. / $1 33c⅓.	- - - - =	£29 10s. 0d. / $78 66c. 6m.

CASE. IV.

When the quantity is any number above the Multiplication Table: Multiply the price of 1 yard by 10, which will produce the price of 10 yards: *This* product, multiplied by 10, will give the price of 100 yards; then, you must multiply the price of one hundred by the number of hundreds in your question; the price of ten, by the number of tens; and the price of unity, or 1, by the number of

units: Lastly, add these several products together, and the sum will be the answer.

EXAMPLES.

1. What will 359 yards of cloth, at { 4s. 7½d. / 77c. 1m. } per yard, amount to?

£ s. d.		c. m.
0 4 7½	price of 1 yard.	·771
		359
10		
		6939
2 6 3	price of 10 yards.	3855
10		2313
23 2 6	price 100 yds. Ans.	$276·789
3		
69 7 6	price of 300 yards.	
5 times the price of 10 yds. = 11 11 3	price of 50 yards.	
9 times the price of 1 yd. = 2 1 7½	price of 9 yards.	
Answer £83 0 4½	price of 359 yards.	

2. 297 yards at	{ 17s. 8½d. / $2 88c. 2m. }	per yard =	{ £256 15s. 7½d. / $855 95c. 4m. }
3. 473 . . .	{ 9s. 11¼d. / $1 65c. 6¼m. }	. . . . . =	{ £235 0s. 5¼d. / $783 40c. 6¼m. }
4. 512 . . .	{ 5s. 10d. / $0 72½c. }	. . . . . =	{ £149 6s. 8d. / $373 33⅓c. }
5. 765 . . .	{ 18s. 9d. / $2 50c. }	. . . . . =	{ £717 3s. 9d. / $1912 50c. }

CASE V.

To find the value of one hundred weight: As 112 is the gross hundred, so 112 farthings are =2s. 4d. and 112 pence =9s. 4d.; therefore, if the price be farthings, or not more than 3d. multiply 2s. 4d. by the farthings in the price of 1 lb. or, if the price be pence, multiply 9s. 4d. by the pence in the price of 1 lb. and in either case, the product will be the answer.

EXAMPLES.

1. What will 1 Cwt. of chalk come to at { 1½d. / 2c. 1m. } per pound?

112 farthings =	0 2 4	price of 1 Cwt. at ¼ per lb.	·021
1½d. =		6 farthings in the price.	112
Answer	£0 14 0	price of 1 Cwt. at 1½ per lb.	42
			21
			21
			Ans. 2·352

2. 1 Cwt. of tin at $2\frac{1}{4}$d. per lb. ?

	s.	d.	
	2	4	price of 1 Cwt. at $\frac{1}{4}$d. per lb.
		9	farthings in the price of 1 lb.
Ans. £1	1	0	price of 1 Cwt. at $2\frac{1}{4}$ per lb.

$$\begin{array}{r} \cdot 03125 \\ 112 \\ \hline 6250 \\ 3125 \\ 3125 \\ \hline \end{array}$$

Ans. \$3·50000

3. 1 Cwt. of lead at $\left\{ \begin{array}{c} 7d. \\ 9c.\ 8m. \end{array} \right\}$ lb. ?

	s.	d.	
	9	4	pr. of 1 Cwt. at 1d. per lb.
		7	pence in the price of 1 lb.
£3	5	4	pr. of 1 Cwt. at 7d. per lb.

Ans. \$10·976.

Questions.	Answers.
4. 1 Cwt. of copper at $0\frac{3}{4}$d. per lb. =	£0 7s.
5. 1 - - - - - - - $\left\{ \begin{array}{c} 3d. \\ 3\frac{1}{8}c. \end{array} \right\}$ per lb. =	$\left\{ \begin{array}{c} £1\ 8s. \\ \$3\ 50c. \end{array} \right\}$
6. 1 - - - - - - - $\left\{ \begin{array}{c} 4\frac{1}{2}d. \\ 5c. \end{array} \right\}$ - - - -	$\left\{ \begin{array}{c} £2\ 2s. \\ \$5\ 60c. \end{array} \right\}$

CASE VI.

To find the value of a hundred weight, when the price of 1lb. *is any number of pounds and shillings; or shillings, pence and farthings:* Multiply the price of 1 lb. by 7, its product by 8, and *this* product by 2; which last product will be the answer required: for 7×8×2=112.

Examples.

1. What will 1 cwt. of tobacco cost at $\left\{ \begin{array}{c} 5s.\ 7\frac{1}{2}d. \\ 93c.\ 7\frac{1}{2}m. \end{array} \right\}$ per lb.

£.	s.	d.	
0	5	$7\frac{1}{2}$	price of 1 lb.
		7	
1	19	$4\frac{1}{2}$	price of 7 lb.
		8	
15	15	0	price of 56 lb. or $\frac{1}{2}$ cwt.
		2	
Ans. £31	10	0	price of 112lb. or 1 cwt..

$$\begin{array}{r} D. \\ \cdot 9375 \\ 112 \\ \hline 18750 \\ 9375 \\ 9375 \\ \hline \end{array}$$

\$105· Ans.

Questions.	Answers.
2. 1 Cwt. at $\left\{ \begin{array}{c} 3s.\ 10\frac{1}{2}d. \\ 64c.\ 6m. \end{array} \right\}$ per lb. =	$\left\{ \begin{array}{c} £21\ 14s. \\ \$72\ 35c.\ 2m. \end{array} \right\}$
3. 1 - - - $\left\{ \begin{array}{c} 9s.\ 6d. \\ \$1\ 58\frac{1}{3}c. \end{array} \right\}$ - - - -	$\left\{ \begin{array}{c} £53\ 4s. \\ \$177\ 33\frac{1}{3}c. \end{array} \right\}$

4. 1 Cwt. at	{ 16s. 11½d. / \$2 82c. 6m. }	per lb.	=	{ £94 19s. 4d. / \$316 51c. 2m. }
5. 1 - - -	{ 13s. 4d. / \$1 77$\frac{7}{9}$c. }	- - -	=	{ £74 13s. 4d. / \$186 66⅔c. }
6. 1 - - -	{ 22s. 6d. / \$3 00c. }	- - -	=	{ £126 0s. 0d. / \$336 00c. }

Practical Questions in Weights and Measures.

1. What is the weight of 4 hogsheads of sugar, each weighing 7cwt. 3qrs. 19lb.? Ans. 31cwt. 2qrs. 20lb.

2. What is the weight of 6 chests of tea, each weighing 3cwt. 2qrs. 9lb.? Ans. 21cwt. 1qr. 26lb.

3. If I am possessed of 1½ dozen of silver spoons, each weighing 3oz. 5pwt.—2 dozen of tea spoons, each weighing 15pwt. 14gr. —3 silver cans, each 9oz. 7pwt.—2 silver tankards, each 21oz. 15pwt. and 6 silver porringers, each 11oz. 18pwt. pray what is the weight of the whole? Ans. 18lb. 4oz. 3pwt.

4. In 35 pieces of cloth, each measuring 27¾ yards, how many yards? Ans. 971¼ yards.

5. How much brandy in 9 casks, each containing 45gal. 3qts. 1pt.? Ans. 412gal. 3qts. 1pt.

6. If I have 9 fields, each of which contains 12 acres, 2 roods and 25 poles; how many acres are there in the whole?

Ans. 113A. 3r. 25p.

COMPOUND DIVISION*

IS the dividing of numbers of different denominations: In doing which, always begin at the highest, and when you have divided that, if any thing remain, reduce it to the next lower denomination, and so on, till you have divided the whole, taking care to set down your quotient figures under their respective denominations.

Introductory Examples.

	1.				2.					3		
	£	s.	d.		D.	d.	c.	m.		£	s.	d.
Divide	549	17	9 by 5	3)	14	1	9	6	4)	731	5	10½
Quot.	£109	19	6½									

* To divide a number consisting of several denominations by any simple number whatever, is the same as dividing all the parts or members of which that number is composed, by the same number. And this will be true when any of the parts are not an exact multiple of the divisor; for, by conceiving the number, by which it exceeds that multiple, to have its proper value by being placed in the next lower denomination, the dividend will still be divided into parts, and the true quotient found as before: Thus £41 17s. 6d. divided by 6, will be the same as £36 117s. 42d. divided by 6, which is equal to £6 19s. 7d. as by the rule.

4.	5.	6.	7.
£ s. d.	£ s. d.	D. c. m.	£ s. d.
2)97 19 10½	6)37 11 4¾	7)25 49 4	8)739 12 1¼
£48 19 11¼			

8.	9.	10.	11.
D. c.	£ s. d.	£ s. d.	E. D. d. c. m.
10)37 50	10)79 13 9½	11)58 19 11½	12)3 9 8 7 5

In the first example, having divided the pounds, the 4, which remains, is 4 pounds, which are equal to 80 shillings, and 17 in the shillings make 97; I then find 5 is contained 19 times in 97, and 2 over: I set down 19 under the shillings, and reduce the 2 shillings, which remain, into pence, and they make 24, and the 9 pence, in the question, added, make 33: Then how often 5 in 33; I find it 6 times, and 3 over: I set down 6 under the pence, and reduce the 3 pence, which remain, to farthings, and they make 12; then, how often 5 in 12; I find it to be twice: I therefore set down ½d. and the 2 which remains, is ⅖ of a farthing, which I make no account of.

12	13	14	15
T. cwt. qr. lb oz. dr.	T. cwt. qr. lb	cwt. qr. lb	lb oz. dr.
3)29 13 2 25 12 13	4)6 11 3 19	5)14 1 12	6)10 13 9

16.	17.	18.	19.
lb oz	lb oz. pwt. gr.	lb oz. pwt. gr.	lb oz. pwt. gr.
7)20 13	8)7 10 15 2	9)56 9 13 19	10)849 11 12 14

20.	21.	22.
M. w. d. h. m.	M. d. h. m.	
6)6 3 5 10 29	7)9 21 12 45	8)3s. 25° 55′ 25″

23.	24.
9)19° 45′ 38″	12)189° 37′ 29″

25. Suppose that two places lie east and west of each other, and it is found by observation that it is noon at the former 2 hours, 6′ 30″ sooner than at the latter; how many degrees are they asunder?

4′)2h. 6′ 30″

31° 37′ 30″ Ans.

Reduce the hours and minutes to minutes, then, minutes divided by 4′ give degrees in the quotient.*

* Because 360°, the whole circumference of the earth, divided by 24, the hours in a day, give 15° for one hour or 60 minutes: and 60÷15=4′ for one degree.

26. The longitude of Cambridge is 4h. 44′ 17″, and that of Philadelphia, 5h. 0′ 43″; how many degrees difference.

4° 6′ 30″ Ans.

Practical Questions.

CASE I.

Having the price of any number of yards, pounds, &c. which is within the pence table, or is a composite number, to find the price of one yard, pound, &c. use the following rule. If the quantity do not exceed 12, proceed by setting down the price, and dividing it by the quantity; which quotient will be the price of one yard, required; but if the quantity exceed 12, then divide by such numbers, as, when multiplied together, will produce the quantity, and the last quotient will be the value of 1 yard, required.

Examples.

1. If 9 yards of cloth cost £4 3s. 7½d. what is it per yard?

	£	s.	d.
9)	4	3	7½
	0	9	3½ Ans.

2. If 7 ells cost £5 17s. 5d. what cost 1 ell? Ans. 16s. 9¼d.
3. If 11 sheep cost £6 5s. 9d. what did each cost? Ans. 11s 5¼d.
4. If 12 gallons of rum cost £8 11s. 9½d. what is it per gallon?
Ans. 14s. 3¾d.
5. If 84 cows cost £253 13s. what is the price of each?
Ans. £3 0s. 4¾d.
6. If 132 bushels of corn cost £20 12s. 6d. what is that per bushel?
Ans. 3s. 1½d.
7. If 11 sheep cost $25.63c. what is the price of each?
Ans. $2.33c.
8. If 84 cows cost $863.52c. what is the price of one?
Ans. $10.28c.
9. If 132 bushels of corn cost $66 what is the price of a bushel?
Ans. 50c.

Note. If there be a remainder after the division by one of the parts of a composite number before the last, that remainder must be divided according to the rule for division of fractions, as in the following example.

10. If 35 yards cost £37 11s what is the price of one yard?

		£	s.		
35=5×7	5)	37 ..	11		
	7)	7 ..	10 ..	2 ..	$1\frac{3}{5}$qr.
		1	1	5	$1\frac{33}{35}$qr. Ans.

In dividing the farthings by 7, there is a remainder of $6\frac{3}{5}$qrs. which is to be divided by 7 to obtain the whole answer. Now $6\frac{3}{5}=\frac{33}{5}$, and $\frac{33}{5}\div 7=\frac{33}{35}$, which must be annexed to farthing in the last quotient.

11. If 42 bushels of wheat cost \$48.57c. what costs one bushel?
Ans. \$1.15$\frac{9}{14}$c.

12. What do I pay a pound for cotton, when 99lbs. cost £6 2s. 4d.
Ans. 1s. 2$\frac{82}{99}$d.

CASE II.

Having the price of a hundred weight, to find the price of 1 lb:

Divide the given price by 8, that quotient by 7, and this quotient by 2, and the last quotient will be the price of 1 lb required.

EXAMPLES.

1. If 1 cwt. of flax cost £2 7s. 8d. what is that per lb?

```
8)£2  7s. 8d.
  ---------
7)0   5  11½
  ---------
2)0   0  10d.0 6/7qr.
  ---------
  0   0   5 3/28d. price of 1 pound.
```

2. At £3 10s. per cwt. what cost 1lb? Ans. 7$\frac{1}{2}$d.
3. At £6 6s. per cwt. what cost 1lb? Ans 1s. 1$\frac{1}{2}$d.
4. At £42 11s. 8d. per cwt. what cost 1 lb? Ans. 7s. 7$\frac{1}{4}$d.
5. At \$5.60 per cwt. what cost 1 lb? Ans. 5c.
6. At \$3.33c.3m. per cwt. what cost 1 lb? Ans. 2c. 9$\frac{85}{112}$m.
7. If 1 cwt. cost \$156, what is the price of 1 lb? Ans. \$1.39$\frac{2}{7}$c.

CASE III.

Having the price of several hundred weight, to find the price per lb:
Divide the whole price by the number of hundreds, which will give the price per cwt. and then proceed as in the last Case.

EXAMPLES.

1. If 5 cwt. of sugar cost £13 8s. 4d. what is that per lb?

```
5)£13  8s. 4d.
  ----------
8)  2 13   8   price of 1 cwt.
  ----------
7)  0  6   8½d. price of 14 lb or 1/8 cwt.
  ----------
2)  0  0  11½d. price of 2 lb or 1/56 cwt.
  ----------
    0  0   5¾ price of 1 lb.
```

2. If 8 cwt. cocoa cost £15 17s. 4d. what is that per lb? Ans. 4$\frac{1}{4}$d.
3. If 3$\frac{1}{4}$ cwt. of sugar cost £9 17s. 2d. what is that per lb?
Ans. 6$\frac{1}{2}$d.
4. If 1$\frac{3}{4}$ cwt. of cotton wool cost £6 10s. 8d. what is that per lb?
Ans. 8d.

5. If 3 cwt. of raisins cost $50.52c. what cost 1 ℔?

$
3)50.52
8)16.84
7) 2.105
2) .300$\frac{5}{7}$
.15c.0$\frac{5}{14}$m.

6. If 3$\frac{3}{4}$ cwt. cost $17.56c. what is one ℔?
7. If 11$\frac{1}{4}$ cwt. cost $87.33c. what cost 1 ℔?
8. If 3$\frac{1}{4}$ cwt cost $3.64, what cost 1 ℔? Ans. 1c.
Note. This Case proves the 6th in Compound Multiplication.

CASE IV.

Having the price of any number of yards, &c. to find the price of 1 yard: Divide the price by the quantity, beginning at the highest denomination, and, if any thing remain, reduce it into the next, and every inferiour denomination, and, at each reduction, divide as before, remembering each time, to add the odd shillings, pence, &c. if there be any, and you will have the value of unity required.

Note. If there be $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of a yard, pound, &c. multiply both the price and quantity by 4, and then proceed as above directed; or, in federal money, work by decimals.

Examples.

1. If 95$\frac{1}{2}$℔ of figs cost { £16 13s. 6$\frac{3}{4}$d. / $55 59c. 3$\frac{3}{4}$m. } what are they per ℔?

	℔		£	s.	d.
Quantity =	95$\frac{1}{2}$	Price =	16	13	6$\frac{3}{4}$
Mult. by	4				4

Produces 382 for a divisor. Product £66 14 3 for a dividend.

382)66 14 3 (0 3 5$\frac{3}{4}$ $\frac{250}{382}$ per ℔.
20
382)1334(3
1146
188
12
382)2259(5
1910
349
4
382)1396(3
1146
250

$ c.m.dec. c. m.
95$\frac{1}{2}$=95·5)55·59375(·58 2+ Ans.
4775
7843
7640
2037
1910
1275

2. If 147 bushels of rye cost £47 12s. 6d. what is it per bushel? Ans. 6s. $5\frac{3}{4}$d.

3. If $33\frac{1}{4}$ yards of baize cost £25 13s. $9\frac{1}{2}$d. what is it per yard? Ans. 15s. $5\frac{1}{4}$d. $\frac{25}{133}$.

4. If 147lbs. cost $158 76c. what is the price of 1 pound? Ans. $1 8c.

5. Bought $33\frac{1}{4}$yds. of cloth for $85 63c. 2m.; what did I pay a yard? Ans. $2 57c. 5m.

Note. This proves the 3d and 4th Cases in Multiplication.

Practical Questions in Money.

1. Divide { £273 9s. 4d. / $911 55c. 5m. } among 5 men and 4 women, and give the men twice as much as the women.

```
   Men. Women.                    £ s. d.  £   s. d.
      5 and 4  Divide by 14)273 9 4(19  10  8=1 woman's share.
Mult. by 2                   14               4 women.
      —                      ——     ————————
      10 shares.             133     78   2  8=women's share.
Add    4 women's shares.     126    ————————
      —                      ——   £19  10  8
      14 the number of equal  7             2
         shares in the whole 20     ————————
         =Divisor.           —    £39   1  4=1 man's share.
                          14)149(10          5 men.
                             14     ————————
                             — £195   6  8=men's share.
                              9    78   2  8=women's share.
                             12     ————————
                             — £273   9  4 Proof.
                          14)112(8  ————————
                             112

     D.
   14)911·555(65.111+  =1 woman's share.
      84             4 women.
      ——        ————————
       71     260·444=women's share.
       70       ————————
       ——          65·111+
        15               2
        14          ————————
        ——        130·222+  =1 man's share.
         15              5
         14         ————————
         ——      651·111+  =men's share.
          15     260·444+  =women's share.
          14      ————————
          —      911·555+ Proof.
           1
```

2. Divide £120 17s. 4d. among 7 men and 7 women, and give the women 3 times so much as the men.

Ans. £4 6s 4d.=a man's share. £12 19s.=a woman's share.

3. Divide £39 12s. 5d. among 4 men, 6 women, and 9 boys: Give each man double to a woman, and each woman double to a boy.

Ans. £1 1s. 5d.=a boy's share. £2 2s. 10d.=a woman's share. £4 5s. 8d.=a man's ditto.

4. Divide 5 guineas among 8 men:—Give A 8d. more than B, and B 8d. more than C, &c. Ans. H's share=15s. 2d.

REDUCTION OF COINS.

RULES for reducing the Federal Coin, and the currencies of the several United States; also English, Irish, Canada, Nova-Scotia, Livres Tournois and Spanish milled Dollars, each to the *par* of all the others.*

I. *To reduce New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency:*

1. *To Federal Money.*

Rule.—Reduce the shillings, pence and farthings, to decimals, by Inspection (Case 3d, Dec. Frac.) divide the whole by 3, putting the comma one figure further to the right hand in the quotient, than in the pounds of the dividend, and the quotient will be the answer in dollars, cents and mills.

1. Reduce £349 19s. 1d. to dollars.

·9 = ½ the shillings.
·05 = odd shillings.
·004 = qrs. in 1d.
——
·954 = decimal.
3)349·954 D. c. m.
1166·513=1166 51 3 Ans.

2. Reduce 19s. 1¾d. to dollars.
$3 19c. Ans.

3. Reduce 1s. to cents.
1s. = ·05 then
3)0 5 c. m.
0· 166⅔ = 16 6⅔.

* The Rules for the reduction of money depends upon the relative value of the currency of different States, &c. This value is given in the several rules. Thus 4 pounds of the currency of New York and North Carolina, are equal to 3 pounds of New England and Virginia; 4 of New England and Virginia, are equal to 3 of England; 4 of New England &c. are equal to 5 of Pennsylvania, New Jersey, &c.; and 1 pound of New England, &c. is equal to seven ninths of a pound of South Carolina and Georgia, and thus of the others. The rules are therefore obvious. In some cases, the process is contracted, and the contraction is given for the rule, because the operation is simplified. Thus the first rule is equivalent to multiplying the pounds and decimals by 20 and dividing the product by 6, the shillings in a dollar. But as $\frac{20}{6}=\frac{10}{3}$, the sum is multiplied by 10 in removing the separatrix one figure to the right, before the division by 3 is made. The relative value of a £, depends on the number of shillings reckoned to the dollar,—the greater the number of shillings in a dollar, the less the £, and the reverse.

4. Reduce 1d.

Ans. 1c. $3\frac{1}{3}$m.

5. Reduce 1 qr.

1qr. = ·001041 and

3)·0 01 041

0·00 347 = $3\frac{47}{100}$ mills.

2. *To New York and North Carolina currency.*

Rule.—Add one third to the New Hampshire, &c. sum, and the sum total will be the New York, &c. currency.

Reduce £100 New Hampshire, &c. to New York, &c.

$$\begin{array}{r} £ \\ 3)100 \\ +\ 33\ \ 6\ \ 8 \\ \hline £133\ \ 6\ \ 8 \text{ Ans.} \end{array}$$

3. *To Pennsylvania, New Jersey, Delaware and Maryland currency.*

Rule.—Add one fourth to the New Hampshire, &c. sum.

Reduce £100 New Hampshire, &c. to Pennsylvania, &c.

$$\begin{array}{r} 4)100 \\ +\ 25 \\ \hline £125 \text{ Ans.} \end{array}$$

4. *To South Carolina and Georgia currency.*

Rule.—Multiply the New Hampshire, &c. sum by 7, and divide the product by 9, and the quotient is the answer.

Reduce £100 New Hampshire, &c. to South Carolina, &c.

$$\begin{array}{r} 100 \\ 7 \\ \hline 9)700 \\ £77\ \ 15\ \ 6\frac{2}{3} \text{ Ans.} \end{array}$$

5. *To English Money.*

Rule.—Deduct one 4th from the New Hampshire, &c. sum.

Reduce £100 New Hampshire, &c. to English Money.

$$\begin{array}{r} 4)100 \\ -\ 25 \\ \hline £75 \text{ Answer.} \end{array}$$

6. *To Irish Money.*

Rule.—Multiply the New Hampshire, &c. sum by 13, and divide the product by 16.

Reduce £100 New Hampshire, &c. to Irish Money.

$$\begin{array}{r} 100 \\ \hline 4\times3+\text{the given sum.} \\ \hline 400 \\ 3 \\ \hline 1200 \\ +100 \\ \hline 16=4\times4)1300 \\ \hline 4)325 \\ \hline £81\ \ 5 \text{ Ans.} \end{array}$$

7. *To Canada and Nova Scotia currency.*

Rule.—Multiply the New Hampshire, &c. sum by 5, and divide the product by 6.

Reduce £100 New Hampshire, &c. to Canada, &c.

$$\begin{array}{r} 100 \\ 5 \\ \hline 6)500 \\ \hline £83\ \ 6\ \ 8 \text{ Answer.} \end{array}$$

8. *To livres Tournois.*

Note. 12 deniers, or pence, make 1 sol, or shilling, 20 sols, or sous, 1 livre, or pound.

Rule.—Multiply the New Hampshire, &c. pounds, by $17\frac{1}{2}$, and the product will be livres: Or, multiply the sum in shillings by 7: Divide the product by 8, and the quotient will be livres, sous, &c.

Reduce £100 New Hampshire, &c. to Livres Tournois.

```
   100         Or, 100
   17½              20
   ———             ———
   700            2000
   100               7
    50            ————
   ———        8)14000
Ans. 1750 liv.   ————
   ———     Ans. 1750 livres.
```

1d.=1sou. 5⅓deniers.=17½sous.
£1=17½ livres.

II. *To reduce Federal Money, to New England and Virginia currency.*

Rule.—Multiply the Federal money by 3, and if it consist of dollars only, cut off 1 figure, if of cents also, cut off 3, and if of mills, 4 figures at the right hand; then reduce the figures so cut off to farthings each time cutting off as at first and the left hand figures are pounds, shillings, &c. Or, reduce them by inspection.

1. Reduce $1166 51c. 3m. to New England currency.

```
     $  c. m.
  1166·51 3
          3
  ———————————
  £349·953 9
         20
  ———————————
  s.19·0780
         12
       ————
      ·936 =1d. nearly.
Or, 18s.   = double of 9.
    1s.    =5 in the 2d place.
      1d. = 3·9 or 4qrs. that
  ————              [remain.
  19s. 1d. Ans.
```

2. Reduce 45 dollars. £13 10s. Ans.

3. Reduce $12 7c. to N. E money. £3 12s. 5 0¼.

III. *To reduce New Jersey, Pennsylvania, Delaware and Maryland currency.*

1. *To New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency.*

Rule. Deduct one fifth from the New Jersey, &c. sum, and the remainder will be New-Hampshire, &c. currency.

Reduce £100 New-Jersey, &c. to New-Hampshire, &c.

```
    5)100
    — 20
    ——
   £80 Answer.
```

2. *To New York and North Carolina currency.*

Rule. Add one fifteenth to the New Jersey, &c. sum.

Reduce £100 New Jersey, &c. to New York, &c.

```
15=3×5)100
       ———
       3)20
       ———
       + 6 13 4+giv. sum.
       ——————————
    £106 13 4 the Answer.
```

3. *To South Carolina and Georgia currency.*

Rule: Multiply the New Jersey, &c. sum by 28, and divide the product by 45, and the quotient is South Carolina &c.

Reduce £100 New Jersey, &c. to South Carolina, &c.

```
            100
           ————
           4×7=28
         ————
          400
            7
         ————
45=5×9)2800
         ————
        5)311 2 2⅔
         ——————————
        £62 4 5⅓ Ans.
```

4. *To English Money.*

Rule. Multiply the New Jersey, &c. by 3, and divide the product by 5.

Reduce £100 New Jersey, &c. to English money.

```
    100
      3
   ----
  5)300
   ----
   £60 Answer.
```

5. *To Irish Money.*

Rule. Multiply the New Jersey, &c. by 13, and divide the product by 20.

Reduce £100 New Jersey, &c. to Irish.

```
          100
          ---
          4×3+the giv. sum.
          ---
          400
            3
         ----
         1200
         +100
         ----
20=4×5)1300
         ----
        4)260
         ----
         £65 Answer.
```

6. *To Canada and Nova Scotia currency.*

Rule. Deduct one third from the New Jersey, &c.

Reduce £100 New Jersey, &c. to Canada, &c.

```
   3)100
   — 33  6  8
   ----------
   £66 13 4 Ans.
```

7. *To Livres Tournois.*

Rule. Multiply the New Jersey, &c. pounds by 14, and the product will be Livres Tournois,—or multiply the sum in shillings by 7; divide the product by 10, and the quotient will be livres, sous, &c.

Reduce £100 New Jersey, &c. to Livres Tournois.

```
        100      Or, 100 ⎫ 1d.=1⅙sous.
         14           20 ⎬ 1s.=14sous.
        ---         ---- ⎭ £1=14liv.
        400         2000
        100            7
        ---         ----
Ans. 1400liv. 10)14000
                    ----
                    1400
```

8. *To Spanish milled dollars*

Rule. Multiply the New Jersey, &c. pounds by $2\frac{2}{3}$ and the product will be dollars.—Or, multiply them by 8: Divide the product by 3, and the quotient will be dollars.—If there be shillings in the given sum, for every 7s. 6d. add 1 dollar to the quotient.

Reduce £100 10s. New Jersey, &c. to dollars.

```
       100           Or 100
         8                2
       ---              ---
     3)800              200
       ---     100×⅔=   66⅔
      266⅔     10s.=     1⅓
  10s.=1⅓               ---
      ----              268 as be-
Ans. 268 dol.                [fore.
```

IV. *To reduce New York and N. Carolina currency.*

1. *To New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency.*

Rule. Deduct one fourth from the New York, &c.

Reduce £100 New York, &c. to New Hampshire, &c.

```
   4)100
    —25
     ---
    £75 Answer.
```

2. *To New Jersey, Pennsylvania, Delaware, and Maryland currency.*

Rule. Deduct one sixteenth from the New York, &c. sum.

Reduce £100 New York, &c. to New Jersey, &c.

16=4×4)100

4)25

—£6 5

£93 15 Answer.

3. *To South Carolina and Georgia currency.*

Rule. Multiply the New York, &c. sum by 7, and divide the product by 12: The quotient is South Carolina, &c.

Reduce £100 New York, &c. to South Carolina, &c.

100
7

12)700

£58 6 8 Answer.

4. *To English Money.*

Rule. Multiply the New York, &c. sum by 9: Divide the product by 16, and the quotient is English.

Reduce £100 New York, &c. to English Money.

100
9

16=4×4)900

4)225
£56 5 Answer.

5. *To Irish Money.*

Rule. Multiply the New York, &c. sum by 39: Divide the product by 64, and the quotient is Irish.

Reduce £100 New York, &c. to Irish money.

100

6×6+ thrice the giv. sum,

600
6

3600
+300=100×3

64=8×8)3900

8)487 10

£60 18 9 Ans.

6. *To Canada and Nova Scotia currency.*

Rule. Multiply the New York, &c. sum by 5, and divide the product by 8.

Reduce £100 New York, &c. to Canada, &c.

100
5

8)500

£62 10 Ans.

7. *To Livres Tournois.*

Rule. Multiply the New York, &c. sum in shillings by 21: Divide the product by 32, and the quotient will be livres, sous, &c.

Reduce £100 New York, &c. to Livres Tournois.

100
20

2000
21

2000
4000

32=4×8)42000

4)5250

Ans. 1312½ livres.

Note.
1d.=1 3/32 sou.
1s.=13⅛ sou.
1l.=13⅛ liv.

8. *To Spanish milled Dollars*

Rule. If the New York sum be pounds only, annex a cypher to them, then divide by 4, and the quotient is dollars: But if it be pounds and shillings, annex half the shillings to the pounds, and divide as before, and the quotient is dollars.

Reduce £100 New York, &c. to Dollars.

4)1000

250 Doll. Ans.

Reduce £100 8s. to Dollars.

4)1004

251 Dolls. Ans.

V. *To reduce South Carolina and Georgia currency.*

1. *To New Hampshire, Massachusetts, Rhode Island, Connecticut and Virginia currency.*

Rule. Multiply the South Carolina, &c. sum by 9, and divide the product by 7.

Reduce £100 South Carolina, &c. to New Hampshire, &c.

100
9

7)900

£128 11 5$\frac{1}{7}$ Ans.

2. *To New Jersey Pennsylvania, Delaware and Maryland currency.*

Rule. Multiply the South Carolina, &c. sum by 45, and divide the product by 28.

Reduce £100 South Carolina &c. to New Jersey, &c.

100
9×5=45

900
5

28=4×7)4500
4)642 17 1$\frac{1}{7}$

£160 14 3$\frac{2}{7}$ Ans.

3. *To New York and North Carolina currency.*

Rule. Multiply the South Carolina, &c. sum by 12, and divide the product by 7.

Reduce £100 South Corolina, &c. to New York, &c.

100
12

7)1200

£171 8 6$\frac{6}{7}$ Ans.

4. *To English Money.*

Rule. From the South Carolina, &c. sum, deduct one twenty-eighth.

Reduce £100 South Carolina, &c. to English Money.

28=4×7)100
4)14 5 8$\frac{4}{7}$

— 3 11 5$\frac{1}{7}$ from 100.

£96 8 6$\frac{6}{7}$ Ans.

5. *To Irish Money.*

Rule. Multiply the South Carolina, &c. sum by 117, and divide the product by 112.

Reduce £100 South Carolina, &c. to Irish.

100

12×9+9 times [the giv. sum.

1200
9

10800
+100×9=900

112=4×4×7)11700

4)1671 8 6$\frac{6}{7}$

4)417 17 1$\frac{5}{7}$

£104 9 3$\frac{3}{7}$ Ans.

6. *To Canada and Nova Scotia currency.*

Rule. Multiply the South Carolina, &c. sum by 15, and divide the product by 14.

Reduce £100 South Carolina, &c. to Canada, &c.

100
———
5×3
———
500
3
———
14=2×7)1500
———
2)214 5 8$\frac{4}{7}$
———
£107 2 10$\frac{2}{7}$ Answer.

7. *To Livres Tournois.*

Rule. Multiply the South Carolina, &c. pounds by 22$\frac{1}{2}$, and the product will be livres.

Reduce 100l. South Carolina, &c. to Livres.

100 Note. 1d.=1$\frac{7}{8}$ sous.
22$\frac{1}{2}$ 1s.=1$\frac{1}{8}$ livre.
——— 1l.=22$\frac{1}{2}$ livres.
200
200
50
———
Ans. 2250 livres.

8. *To Spanish milled Dollars.*

Rule. Multiply the South Carolina, &c. pounds by 30, and divide the product by 7, and if there be shillings, turn them into dollars, and add them.

Reduce £100 South Carolina, &c. to Dollars.

100
———
10×3=30
———
1000
3
———
7)3000
———
Dollars 428$\frac{4}{7}$. Note. $\frac{1}{7}$=8d.

VI. *To reduce English Money.*

1. *To New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency.*

Rule. To the English sum, add one third.

Reduce £100 English to New Hampshire, &c.

3)100
+ 33 6 8
———
£133 6 8 Ans.

2. *To New Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. Multiply the English money by 5, and divide the product by 3.

Reduce £100 English, to New Jersey, &c.

100
5
———
3)500
———
£166 13 4

3. *To New York and North Carolina currency.*

Rule. Multiply the English money by 16, and divide the product by 9.

Reduce £100 English, to New York, &c.

100
———
4×4
———
400
4
———
9)1600
———
£177 15 6$\frac{2}{3}$ Answer.

4. *To South Carolina and Georgia currency.*

Rule. To the English money add one twenty seventh.

Reduce £100 English, to South Carolina, &c.

27=3×9)100

3)11 2 2⅔

+ 3 14 0⅞

£103 14 0⅞ Ans.

5. *To Irish Money.*

Rule. To the English sum add one twelfth.

Reduce £100 English money to Irish Money.

12)100
+ 8 6 8

£108 6 8 Ans.

6. *To Canada and Nova Scotia currency.*

Rule. To the English sum add one ninth.

Reduce £100 English, to Canada, &c.

9)100
+ 11 2 2⅔

£111 2 2⅔ Answer.

7. *To Livres Tournois.*

Rule. Multiply the English pounds by 23⅓, and the product will be livres.

Reduce £100 English to Livres Tournois.

100 Note. 1d.=1$\frac{17}{18}$sous.
23⅓ 1s =1⅙livre.
£1=23⅓livres.

300
200
33⅓

Liv. sou. den.
Ans. 2333⅓ Liv.=2333 6 8.

8. *To Federal Money.*

Rule. Multiply the pounds, or pounds and decimals of a pound by 40 and divide the product by 9, and the quotient will be the dolls. or dollars and cents.

1. Reduce £50 sterling to Federal money.

50
40

9)2000

$222.22⅔cts. Ans.

2. Reduce £36 10s. 9d. sterling to dollars and cents.

£36 10s. 9d.= £36.525, and

$$\frac{36.525\times40}{9}=\$162.33\tfrac{1}{3}\text{cts. Ans.}$$

3. Reduce £1 sterling to Federal money. Ans. $4.44⅘c.

4. Reduce £1003.5 sterling to Federal money.

Ans. 4460

VII. *To reduce Irish Money.*

1. *To New Hampshire, Massachusetts, Rhode Island, Connecticut and Virginia currency.*

Rule. Multiply the Irish sum by 16, and divide the product by 13.

Reduce £100 Irish, to New Hampshire, &c.

100

4×4

400
4

13)1600

£123 1 6$\frac{6}{13}$ Ans.

2. *To New Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. Multiply the Irish sum by 20, and divide the product by 13.

Reduce £100 Irish to New Jersey, &c.

100

4×5=20

400
5

13)2000(153 16 11$\frac{1}{13}$ Answer.

3. *To New York and North Carolina currency.*

Rule. Multiply the Irish sum by 64, and divide the product by 39.

Reduce £100 Irish to New York, &c.

100

$8 \times 8 = 64$

800

8

£ s.

39)6400(164 $2\frac{2}{39}$ Answer.

4. *To South Carolina and Georgia currency.*

Rule. Multiply the Irish sum by 112, and divide the product by 117.

Reduce £100 Irish to South Carolina, &c.

100

$7 \times 4 \times 4 = 112$

700

4

2800

4

£ s. d.

117)11200(95 14 $6\frac{42}{117}$ Ans.

5. *To English Money.*

Rule. From the Irish sum deduct one thirteenth.

Reduce £100 Irish to English money.

£ s. d.

13)100(7 13 $10\frac{2}{13}$

100 0 0

—7 13 $10\frac{2}{13}$

92 6 $1\frac{11}{13}$ Ans.

6. *To Canada and Nova Scotia currency.*

Rule. To the Irish sum add one thirty ninth.

Reduce £100 Irish to Canada, &c.

£ s. d.

39)100(2 11 $3\frac{5}{13}$

100

+2 11 $3\frac{5}{13}$

102 11 $3\frac{5}{13}$ Ans.

7. *To Livres Tournois.*

Rule. Multiply the Irish sum, in pence, by 70; divide that product by 39, and the quotient will be sous, which, divided by 20, will be livres.

Reduce £100 Irish to Livres Tournois.

$100 \times 20 \times 12 = 24000$d.

70

2·0

39)1680000(4307|6

sou.

Ans. Livres. 2153 $16\frac{12}{13}$

1d.$=1\frac{31}{39}$sous. 1s.$=21\frac{7}{13}$sous.

£1=21 liv. $10\frac{10}{13}$sous.

VIII. *To reduce Canada and Nova Scotia currency.*

1. *To New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency.*

Rule. To the Canada, &c. sum add one fifth.

Reduce £100 Canada, &c. to New Hampshire, &c.

5)100

+ 20

£120 Answer.

2. *To New York and North Carolina currency.*

Rule. Multiply the Canada, &c. sum by 8, and divide the product by 5.

Reduce £100 Canada, &c. to New York, &c.

100

8

5)800

£160 Answer.

3. *To New Jersey, Pennsylvania, Delaware, and Maryland currency.*

Rule. To the Canada, &c. sum add one half.

Reduce £100 Canada, &c. to New Jersey, &c.

```
 2)100
 + 50
 ----
 £150 Answer.
```

4. *To South Carolina and Georgia currency.*

Rule. From the Canada, &c. sum deduct one fifteenth.

Reduce £100 Canada, &c. to South Carolina, &c.

```
15=3×5)100
       ---
     3)20
       --
    — 6 13 4
    --------
    £93  6 8 Answer.
```

5. *To English Money.*

Rule. From the Canada, &c. deduct one tenth.

Reduce £100 Canada, &c. to English money.

```
10)100
 — 10
   --
 £90 Answer.
```

6. *To Irish Money.*

Rule. From the Canada, &c. deduct one fortieth.

Reduce £100 Canada, &c. to Irish money.

```
40)100
 — 2 10
 ------
 £97 10 Answer.
```

7. *To Livres Tournois.*

Rule. Multiply the Canada, &c. pounds by 21, and the product will be livres.

Reduce £100 Canada, &c. to livres Tournois.

```
      100
      ---
     7×3=21
      --
      700        1d.=1¾sous.
        3        1s.=21sous.
      ---        £1=21livres.
Ans. 2100
```

8. *To Spanish milled Dollars.*

Rule. Reduce the Canada, &c. sum to shillings: Divide them by 5, and the quotient is dollars. Or, Multiply the pounds by 4, and the product is dollars: And if there be shillings turn them into dollars, and add them to the product.

Reduce £100 Canada, &c. to dollars.

```
   100          155 15
    20            4
  ----          ---
5)2000          620
  ----         + 3=15s.
 $400  Ans.    ---
               $623 Ans.
```

IX. *To reduce Livres Tournois.*

1. *To New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency.*

Rule. Multiply the livres by 2: Divide the product by 35, and the quotient will be pounds. Or, Multiply the livres by 8: Divide the product by 7, and the quotient will be shillings.

Reduce 1750 livres to New-Hampshire, &c. currency.

```
   1750          Or, 1750
      2                 8
   ---- £              ----
35)3500(100Ans.7)14000
   35              -----
   ---            2|0)200|0
    00              ----
                 £100 as bef.
```

2. *To New York and North Carolina currency.*

Rule. Multiply the livres by 32: Divide the product by 21, and the quotient will be shillings.

Reduce 1312½ livres to New York, &c. currency.

1312·5
32
———
26250
39375
———2|0
21)42000(200|0
£100 Answer.

3. *To New Jersey, Pennsylvania, Delaware and Maryland currency.*

Rule. Divide the livres by 14, and the quotient will be pounds. Or, multiply the livres by 10: Divide the product by 7, and the quotient will be shillings.

Reduce 1400 livres to New Jersey, &c. currency.

1400
10
———
7)14000
———
20)200|0
———
£100 Ans.

Or,
14)1400(100l.
14
——
00

4. *To South Carolina and Georgia currency.*

Rule. Multiply the livres by 2, divide the product by 45, and the quotient will be pounds. Or, deduct one ninth, and the remainder will be shillings.

Reduce 2250 livres to South Carolina, &c. currency.

2250
2
——— £
45)4500(100 Ans.
45
———
00

Or,
9)2250
— 250
———
2|0)2000|0
———
£100 as bef.

5. *To English Money.*

Rule. Multiply the livres by 6: Divide the product by 7, and the quotient is shillings: Or, deduct one seventh from the livres, and the remainder will be shillings.

Reduce 2333⅓ livres to English money.

2333⅓
6
———
7)14000
———
2|0)200|0
——
Ans. £100

Or,
7)2333⅓
— 333⅓
———
2|0)200|0
——
£100 as bef.

6. *To Irish Money.*

Rule. Reduce the livres to sous, then multiply them by 39: divide this product by 70, and the quotient will be pence.

Reduce 2153 liv. 16$\frac{12}{13}$ so. to Irish money.
20
———
43076$\frac{12}{13}$
39
———
387720
129228
———
7|0)168000|0
12)24000
———
2|0)200|0
———
£100 Answer.

7. *To Spanish milled Dollars, or to Federal Dollars.*

Rule. Multiply the livres by 4: Divide the product by 21, and the quotient will be Spanish or Federal Dollars.

Reduce 1000 livres to dollars.

1000
4
——
21)4000(190 { Spa. Dol.
21
——
190=190$\frac{10}{21}$
\$189
——
10
6
—s. d. q.
21)60(2 10 1

Or, 1000
4
——
21)4000(190 { Fed. Dol.
21
——
190
189
——
10
10
—d. c. m.
21)100)4 7 6$\frac{4}{21}$

X. *To reduce Spanish milled Dollars.*

1. *To New Hampshire, Massachusetts, Rhode Island, Connecticut, and Virginia currency.*

Rule. Multiply the Dollars by 3, and double the right hand figure of the product, for shillings; the left hand figures are pounds.

Reduce 529 dollars to New Hampshire, &c.

```
 529
   3
 ---
£158 14 Answer.
```

2. *To New York and North Carolina currency.*

Rule. Multiply the number of dollars by 4: Double the right hand figure of the product for shillings, and the left hand figures are pounds.

Reduce 529 dollars to New-York, &c.

```
 529
   4
 ---
£211 12 Answer.
```

3. *To New Jersey Pennsylvania, Delaware and Maryland currency.*

Rule. Multiply the number of dollars by 3, and divide by 8.

Reduce 529 dollars to New Jersey, &c.

```
  529
    3
  ---   £   s. d.
8)1587(198  7  6 Answer.
  Or, 8)1587
        ------
```

£198$\frac{3}{8}$ Ans.

4. *To South Carolina and Georgia currency.*

Rule. Multiply the number of dollars by 7, and divide by 30.

Reduce 529 dollars to South Carolina, &c.

```
   529
     7
  -----
3|0)370|3
  -----
```

£123$\frac{13}{30}$ Answer.

*5. *To English Money, at 4s. 6d. per dollar.*

Rule. Multiply the dollars by 9, and divide by 40.

Reduce 529 dollars to English money.

```
    529
      9
   -----
4|0)476|1
   -----
```

£119$\frac{1}{40}$ Answer.

6. *To Canada and Nova Scotia currency.*

Rule. Divide the dollars by 4.

Reduce 529 dollars to Canada, &c. 4)529

£132$\frac{1}{4}$ Answer.

7. *To Livres Tournois.*

Rule. Multiply the dollars by $5\frac{1}{4}$, and the product will be livres. Or, multiply them by 21: divide by 4, and the quotient will be livres.

Reduce 100 Spanish dollars to livres.

100	Or,
$5\frac{1}{4}$	100
—	21
500	—
$100\times\frac{1}{4}=25$	4)2100
—	—
Ans. 525 livres.	525 *as bef.*

* Note, that in England dollars are Bullion, that is, they are bought and sold by weight, and their value varies as other articles of merchandize.

Note. { 1 Cent = $1\frac{1}{20}$ Sous. 1 Dime = $10\frac{1}{2}$ Sous. 1 Dollar = $5\frac{1}{4}$ Livres. }

DUODECIMALS,

OR CROSS MULTIPLICATION,

IS a Rule, made use of by workmen and artificers in casting up the contents of their works.

Dimensions are generally taken in feet, inches and parts.

Inches and parts are sometimes called primes, seconds, thirds, &c. and are marked thus; inches or primes ($'$), seconds ($''$), thirds ($'''$), fourths ($''''$), &c.

This method of multiplying is not confined to *twelves*; but may be greatly extended: For any number, whether its inferiour denominations decrease from the integer in the same ratio, or not, may be multiplied crosswise; and, for the better understanding of it, the learner must observe, that if he multiplies any denomination by an integer, the value of an unit in the product will be equal to the value of an unit in the multiplicand; but if he multiplies by any number of an inferiour denomination, the value of an unit in the product will be so much inferiour to the value of an unit in the multiplicand as an unit of the multiplier is less than an integer.

Thus, pounds, multiplied by pounds, are pounds; pounds, multiplied by shillings, are shillings, &c. shillings, multiplied by shillings are twentieths of a shilling; shillings, multiplied by pence, are twentieths of a penny; pence, multiplied by pence, are 240ths of a penny, &c.

Rule.*

1. Under the multiplicand write the corresponding denominations of the multiplier.

* The reason of this rule is evident by considering the denominations below the integer, as fractional parts of the integer, and multiplying as in Vulgar Fractions. Thus inches or primes are 12ths of a foot, seconds are 12ths of an inch, or 144ths of a foot, and so on. Then feet multiplied by inches would give *inches*, for 2 feet $\times \frac{3}{12} = \frac{6}{12} =$ 6 inches; inches by inches give *seconds*, for $\frac{3}{12} \times \frac{6}{12} = \frac{18}{144} = \frac{18}{12 \times 12} = \frac{12+6}{12 \times 12} = \frac{12}{12 \times 12} + \frac{6}{12 \times 12} = \frac{1}{12} + \frac{6}{144} =$ 1 inch and $6''$; inches into thirds gives *fourths*, for $\frac{6}{12} \times \frac{3}{144} = \frac{18}{12 \times 144} = \frac{12}{12 \times 144} + \frac{6}{12 \times 144} = \frac{1}{144} + \frac{6}{1728}$ $= 1'''$ and $6''''$, and so on.

A similar process will show the correctness of the Rule, when the denominations do not decrease uniformly by 12 or any one number, as in pounds, shillings and pence, where 1 shilling would be $\frac{1}{20}$ of a pound, and 1 penny $\frac{1}{240}$, of a pound, and so on.

Note. It is evident that when the denominations decrease by any one number, as 12, the denomination of the product is the sum of the denominations of the factors. Thus primes into primes give seconds, 2 being the sum of 1+1, the denominations of the factors; seconds into thirds give fifths, 2+3=5; seconds into fourths give sixths, and so on.

2. Multiply each term in the multiplicand, beginning at the lowest, by the highest denomination in the multiplier, and write the result of each under its respective term, observing, in duodecimals, to carry an unit for every 12, from each lower denomination to its next superiour, and for other numbers accordingly.

3. In the same manner multiply all the multiplicand by the primes or second denomination in the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand.

4. Do the same with the seconds in the multiplier, setting the result of each term two places to the right hand of those in the multiplicand.

5. Proceed in like manner with all the rest of the denominations, and their sum will be the answer required.

EXAMPLES.

1. Multiply $2\frac{1}{2}$ feet by $2\frac{1}{2}$ feet.

```
   F. '
   2  6
   2  6
   ----
   5  0
   1  3  0
   -------
Ans. 6  3
```

Or thus.

```
 2½
 2½
 ---
 5
 1¼
 ---
```

Ans. $6\frac{1}{4}$ square feet = 6ft. 36in.
So that the 3 is not 3 inches, but 36 inches, or $\frac{1}{4}$ of a square foot.

Or thus.

```
  2·5
  2·5
  ---
  125
  50
  ---
```

Ans. 6.25 square feet.

2. Multiply 9f. 8′ 6″ by 7f. 9′ 3″

```
   F.  '  "
   9   8  6
   7   9  3
   --------
  67  11  6          = Product by the feet in the multiplier.
   7   3  4  6'''    = ditto by the primes.
       2  5  1  6''''  = ditto by the seconds.
  ----------------
  75   5  3  7  6 Answer.
```

3. How many square feet in a board 17 feet 7 inches long, and 1 foot 5 inches wide? Ans. 24ft. 10′ 11″

4. How many cubick feet in a stick of timber 12 feet 10 inches long, 1 foot 7 inches wide, and 1 foot 9 inches thick?
Ans. 35ft. 6′ 8″ 6‴

5. How many cubick feet of wood in a load 6 feet 7 inches long, 3 feet 5 inches high, and 3 feet 8 inches wide?
Ans. 82ft. 5′ 8″ 4‴

6. There is a house with 4 tiers of windows, and 4 windows in a tier; the height of the first tier is 6ft. 8′; of the second, 5ft. 9′; of the third, 4ft. 6′; and of the fourth, 3ft. 10′; and the breadth of each is 3ft. 5′; how many square feet do they contain in the whole? Ans. 283ft. 7′

The two following questions are Sexcesimals.

7. If 2 places differ in longitude 2° 12′; what is their difference of time?

Mult. 2° 12′ 00′′ 00′′′
by 3′ 59′′ 20′′′ the time in which the sun passes through 1°

8′ 46′′ 32′′′ Answer.

8. Two places differ in longitude 31° 27′ 30′′; What is the difference, in time, of the sun's coming to the meridian of those places, the sun passing through 15° in an hour?

31° 37′ 30′′
4′ 00′′ In 4′ of a solar day, or day of 24 hours, the sun passes 1°

2° 6′ 30′′ 00′′′ Answer.

9. Bought a load of wood, which was 3 feet wide, 2 feet 8 inches high, and 8 feet long; what part of a cord of wood did it contain? Ans. Half a cord.

10. A load of wood was 4 feet 6 inches wide, 3 feet 10 inches high, and 7 feet 8 inches long; how many feet more than a cord did it contain? Ans. $4\frac{1}{4}$ feet.

11. A stick of timber is 1 foot 8 inches in depth, and 2 feet 3 inches in width, and 42 feet 8 inches long; how many solid feet of timber does it contain? Ans. 160.

12. Multiply £3 6 8 by £2 5 7.

		£	s.	d.
		3	6	8
		2	5	7
£3×£2=£6	=	6	0	0
6s.×£2=12s.	=	0	12	0
8d.×£2=16d.	=	0	1	4
£3×5s. =15s.	=	0	15	0
6s.×5s. =$\frac{30}{20}$s.	=	0	1	6
8d.×5s. =$\frac{40}{20}$d.	=	0	0	2
£3×7d. =21d.	=	0	1	9
6s.×7d. =$\frac{42}{20}$d.	=	0	0	$2\frac{1}{10}$
8d.×7d. =$\frac{56}{240}$d.	=	0	0	$0\frac{7}{30}$
Ans.		7	11	$11\frac{1}{3}$

13. A, B and C bought a drove of sheep in company; A paid £14 5s. B, £13 10s. and C, £11 5s. They agreed to dispose of them at the market; that each man should take 18s. as pay for his time, &c. and that the remainder should be divided in proportion to their several stocks: At the close of the sale, they found themselves possessed of £46 5s. what was each man's gain, exclusive of the pay for his time, &c.

£14 5+£13 10+£11 5=£39, and £46 5—£39=£7 5, and £7 5—18s.×3=£4 11s. whole gain, and £4 11÷39=2s. 4d. gain in the pound.

£14 5 0	£13 10 0	£11 5 0				
× 2 4	× 2 4	× 2 4		£	s.	d.
1 8 6	1 7 0	1 2 6	Proof.	1	13	3
4 9	4 6	3 9		1	11	6
				1	6	3
A. £1 13 3	B. £1 11 6	C. £1 6 3		£4	11	0

THE SINGLE RULE OF THREE,

IS so called, because three numbers are given to find a fourth, which shall have the same ratio to one of the given numbers, as there is between the other two. It is usually distinguished into Direct and Inverse. The reason of this distinction, and the particular rules, will be given hereafter. It will be more easy however, for the student to proceed according to the following General Rule for stating and working questions in the Rule of Three.

General Rule.*

1. Place that number, which is of the same name or quality as the answer sought, for the second term.

2. Consider whether the answer should be greater or less than the second term. If it must be greater, place the greater of the two remaining numbers in the question on the right for the third

* This Rule, on account of its great and extensive usefulness, is sometimes called the *Golden Rule of Proportion:* For, on a proper application of it and the preceding rules, the whole business of Arithmetick, as well as every mathematical enquiry depends. The rule itself is founded on this obvious principle, that the magnitude or quantity of any effect varies constantly in proportion to the varying part of the cause: Thus, the quantity of goods bought, is in proportion to the money laid out; the space gone over by an uniform motion, is in proportion to the time, &c.

As the idea, annexed to the term, *proportion*, is easily conceived, the truth of the rule, as applied to ordinary inquiries, may be made evident by attending to principles, already explained.

It has been shewn, in Multiplication of Money, that the price of one, multiplied by the quantity, is the price of the whole; and in Division, that the price of the whole, divided by the quantity, is the price of one: Now, in all cases of valuing goods, &c. where one is the first term of the proportion, it is plain that the answer found by this rule, will be the same as that, found by Multiplication of Money; and, where one is the last term of the proportion, it will be the same as that found by Division of Money.

In like manner, if the first term be any number whatever, it is plain, that the product of the second and third terms will be greater than the true answer, required, by as much as the price in the second term exceeds the price of one, or as the first term exceeds a unit; consequently, this product, divided by the first term, will give the true answer required.

Note 1. When it can be done, multiply and divide as in Compound Multiplication, and Compound Division.

2. If the first term, and either the second or third can be divided by any number without a remainder, let them be divided and the quotient used instead of them.

The following methods of operation, when they can be used, perform the work in a much shorter manner than the general rule.

1. Divide the second term by the first: Multiply the quotient into the third, and the product will be the answer.

2. Divide the third term by the first; multiply the quotient into the second, and the product will be the answer.

3. Divide the first term by the second, and the third by that quotient, and the last quotient will be the answer.

4. Divide the first term by the third, and the second by that quotient, and the last quotient will be the answer.

h; but if the answer must be less, place the less of the two nbers on the right for the third term, and, in each case, place remaining number on the left for the first term.

3. Divide the product of the second and third terms by the first n, and the quotient will be the fourth term or answer sought.

Note. As all questions in the Rule of Three, are readily solved this process, all the statements, unless specially mentioned, will made according to this rule.

The method of proof is by inverting the question.

But, that I may make the method of working this excellent Rule intelligible as possible to the learner, I shall divide it into the eral cases following:

1. The fourth number is always found in the same name in ich the second is given, or reduced to; which, if it be not the hest denomination of its kind, reduce to the highest when it can done.

2. When the second number is of divers denominations, bring it the lowest mentioned, and the fourth will be found in the same ne to which the second is reduced, which reduce back to the hest possible.

3. If the first and third be of different names, or one or both of ers denominations, reduce them both to the lowest denomination ntioned in either.

4. When the product of the second and third is divided by the t; if there be a remainder after the division, and the quotient not the least denomination of its kind; then multiply the remainder by that number, which one of the same denomination with quotient contains of the next less, and divide this product again the first number; and thus proceed till the least denomination found, or till nothing remain.

5. If the first number be greater than the product of the second l third; then bring the second to a lower denomination.

6. When any number of barrels, bales, or other packages or ces are given, each containing an equal quantity, let the cont of one be reduced to the lowest name, and then multiplied by given number of packages or pieces.

7. If the given barrels, bales, pieces, &c. be of unequal conts, (as it most generally happens) put the separate content of ch properly under one another, then add them together, and u will have the whole quantity.

Examples.

1. If 6lb of sugar cost 9s. what will 30lb cost at the same rate?

Here the answer must be money, refore 9s. is the second term; as lb must cost more than 6lb, 30lb st be placed on the right of 9s. for third term, and 6lb on the left t term.

```
      lb  s.   lb
   As 6 : 9 :: 30 : the Answer.
               9
              ---
            6)270
              ---
              45s. = £2 5s. Ans.
```

Again, By inverting the order of the question, it will be,

2. If 9s. buy 6℔ of sugar, how much will £2 5s. buy at that rate?

s. ℔ s.
As 9 : 6 : : 45 : the Ans.
6

9)270(30℔ Ans.

Again, 3. If 30℔ of sugar be worth £2 5s. how much may I buy for 9s.

s. ℔ s.
As 45 : 30 : : 9 : the Ans.
9

45)270(6℔ the Ans.
270

Again, 4. Suppose £2 5s. will buy 30℔ of sugar: What will 6℔ of the same sugar cost?

℔ s. ℔
As 30 : 45 : : 6 : the Ans.
6

3|0)27|0

9s. Ans.

N. B. The last three questions are only the first *varied*, being put merely to show how any question, in this Rule, may be inverted.

5. If 5 yds. of cloth cost $10 what will 20 yds. come to?

yds. $ yds.
As 5 : 10 : : 20
10÷5=2

$40 Ans.

Here I divide the 2d term by the 1st, and multiply the quotient into the 3d, for the answer.

yds. $ yds.
As 5 : 10 : : 20
4=20÷5

$40 Ans.

Here I divide the 3d term by the 1st, and multiply the quotient into the 2d, for the answer.

7. If 20yds. cost $120, how many yards may I have for $30?

$ yds. $
As 120 : 20 : : 30
120÷20=6 quot. and 30÷6=5 yards, Answer.

Here I divide the 1st term by the 2d, and then, the 3d term by the quotient for the answer.

$ yds. $
Again, 8. As 120 : 20 : : 30
120÷30=4 quot. and, 20÷4=5 yards, Ans.

Here I divide the 1st term by the 3d, and then, the 2d term by that quotient for the answer.

9. If 1cwt. of tobacco cost £5 12 9½; what will 8cwt. ditto cost?

cwt. £ s. d. cwt.
As 1 : 5 12 9½ : : 8
8

Ans. £45 2 4

Here there is no need of reducing the middle term, because it can be performed by compound multiplication, the first term being an unit.

10. If 8cwt. of tobacco cost £45 2 4d; what is that per cwt.?

£ s. d.
8)45 2 4

Ans. 5 12 9½

Here there is no need of reducing the middle term, because it may be performed by compound division only, the 3d term being an unit.

11. If 9cwt. 3qrs. sugar cost £27 17s. 6d. what will 2cwt. 1qr. 1lb cost?

9cwt. 3qrs.	£ s. d.	2cwt. 1qr. 1lb
4	27 17 6	4
39	20	9
28	557	28
312	12	73
78	6690	19
1092		263

lb d. lb
As 1092 : 6690 : : 263 : the answer.
263

2007
4014
1338
——12
1092)1759470(1611
1092
——2|0)13|4 3d.
6674 £6 14s. 3d. Answer.
6552

1227
1092

1350 carried over.

Brought over 1350
1092

258
4

1092)1032(0qr.

Note 1. If you look at the stating, you will see that the first and third terms are of the same kind, but of different denominations, and therefore are reduced to the same name or denomination, and that the demand of the question lies on the 3d term.

2. That the middle term, being given in pounds, shillings and pence, is reduced to pence. But,

3. If the second term were in federal money, it would be sufficient to proceed according to decimals. Thus: if the price were $92 91c. 7m,

lb. D. c. m. lb.
As 1092 : 92·917 : : 263 : the Ans.
263

278751
557502
185834

D. c. m.
1092)24437·171(22·378+, Ans.
2184

2597
2184

4131
3276

8557
7644

9131
8736

395

12. If 57yds. cost £69 what will 9yds. cost at that rate?

yds. £ yds.
As 57 : 69 : : 9
9

57)621(£10 17s. 10d. 2$\frac{6}{57}$qrs. Ans.

Here, all the terms being whole numbers, there is no need of reducing the middle one until after stating.

The same in Federal money would stand thus:

yds. D. c. yds.
As 57 : 172·50 : : 9
9 D. c. m.
57)1552·50(27·23·7

13. If my income be 109 guineas per annum, I desire to know what I may spend per day, so that I may lay up £45 at the year's end? Ans. £0 5 $10\frac{3}{4}$ $\frac{1}{365}$ per day.

Note 1. You must subtract £45 from the value of 109 guineas.

2. There being 365 days in a year, your question must next be stated thus:

D.	Guin.	£	D.	s. d. qr.	
As 365	: 109—45	::	1	: 5 10 $3\frac{1}{365}$	the Ans.

14. If my salary be £43 12s. 5d. per annum, what does it amount to per week?

Ans. £0 16s. $9\frac{17}{52}$d.

The Stating.

W. £ s. d. W.

As 52 : 43 12 5 :: 1 : the Ans.

Note. As there are 52 weeks and 1 day in a year, you will get the true answer to the above question by the following ratio.

D.	£ s. d.		D.	
As 365	: 43 12 5	::	7	: 16s. $8\frac{283}{365}$d.

15. Suppose my income to be 16s. $8\frac{283}{365}$d. per week, what is it per annum? Ans. £43 13s. $7\frac{1}{4}$ $\frac{51}{365}$d.

Note 1. You must first reduce the middle term to pence.

2. You must multiply by 365 (the denominator of the fraction) and add to the product the 283 which remains; and remember always to do so in similar cases.

3. You must divide by 7, the first term and the quotient will be the answer in 365ths of a penny, which (in all similar cases) must be first divided by the denominator, and then brought into pounds.

16. If I am to pay 1s. 7d. per week for pasturing a cow; what must I give per week for 37 cows? £2 18s. 7d. Ans.

17. How many yards of cloth may be bought for \$195 75c. of which $9\frac{1}{2}$yds. cost \$11 2c.? 168yds. 3qrs. Ans.

18. If I buy 57 yards of cloth for 49 guineas; what did it cost per ell English? £1 10s. $1\frac{13}{228}$d. Ans.

19. A merchant, failing in trade, owes in all £3475, and has in money and effects but £2316 13 4: Now, supposing his effects are delivered up, pray, what will each creditor receive on the pound?

£	£ s. d.		£	
As 3475	: 2316 13 4	::	1	: £0 13s. 4d. Ans.

20. A owes B £3475, but B compounds with him for 13s, 4d. on the pound; pray, what must he receive for his debt?

£2316 13s. 4d. Ans.

21. If the distance from Newburyport to York be 31 miles; I demand how many times a wheel, whose circumference is $15\frac{1}{2}$ feet will turn round in performing the journey?

10560 times, Answer.

22. Bought 9 chests of tea, each weighing 3cwt. 2qrs. 21lb. at £4 9s. per cwt. what came they to?

£147 13s. $8\frac{1}{4}$d. Ans.

23. What will $37\frac{1}{2}$ gross of buttons come to at 13 cents per dozen? \$58 50c. Ans.

24. A farm, containing 125 A. 3r. 27p. is rented at \$11 50c. per acre; what is the yearly rent of that farm?

\$1447 6c. 5$\frac{5}{8}$m. Ans.

25. If a ship cost £537 what are $\frac{3}{8}$ of her worth?

Eigh. £ Eigh. £ s. d.
As 8 : 537 :: 3 : 201 7 6 Ans.

26. If $\frac{7}{16}$ of a ship cost \$1163 what is the whole worth?

\$2658 28c. 5m. Ans.

27. Bought a cask of wine at 76c. 5m. per gallon, for \$125; How much did it contain? 163gal. 1qt. 1$\frac{3}{17}$pt. Ans.

28. What comes the insurance of £537 15s. to, at £4$\frac{1}{2}$ per centum?

£ £ £ s. £ s. d.
As 100 : 4$\frac{1}{2}$ :: 537 15 : 24 3 11$\frac{1}{2}$ $\frac{8}{16}$ Ans.

29. What come the commissions of £785 to at 3$\frac{1}{2}$ guineas per cent? £38 9s. 3$\frac{1}{2}$ $\frac{4}{16}$d. Ans.

30. A merchant bought 9 packages of cloth, at 3 guineas for 7 yards: each package contained 8 parcels, each parcel 12 pieces, and each piece 20 yards; how many dollars came the whole to, and how many per yard?

Yds. guin. pack. \$
As 7 : 3 :: 9 : 34560 Ans. for the whole cost.
Yds. guin. yd. \$
As 7 : 3 :: 1 : 2 Ans. per yard.

31. A merchant bought 49 tuns of wine for \$910; freight cost \$90; duties \$40; cellar \$31 67c.; other charges \$50 and he would gain \$185 by the bargain; what must I give him for 23 tuns?

Tuns. \$ \$ \$ \$ c. \$ \$ Tuns. \$
As 49 : 910+90+40+31 67+50+185 :: 23 : 613 33c Ans.

32. If \$100 gain \$6 in a year, what will \$475 gain in that time?

Ans. \$28 50c.

33. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43d parallel of latitude; the degree of longitude, in this latitude, being about 51 statute miles? H. D. M. M. M.

As 24 : 360 × 51 :: 1 : 12$\frac{3}{4}$ Ans.

34. Shipt for the West Indies 225 quintals of fish, at 15s. 6d. per quintal; 37000 feet of boards, at 8$\frac{1}{3}$ dolls. per 1000; 12000 shingles, at $\frac{1}{2}$ guin. per 1000; 19000 hoops at \$1$\frac{1}{3}$ per 1000, and 53 half joes; and in return, I have had 3000 galls. of rum, at 1s. 3d. per gallon; 2700 gallons of molasses, at 5$\frac{1}{4}$d. per gallon; 1500lb of coffee, at 8$\frac{1}{2}$d. per lb; and 19cwt. of sugar, at 12s. 3d. per cwt. and my charges on the voyage were £37 12s. pray, did I gain or lose, and how much by the voyage? Ans. lost £134 9s. 9d.

35. If a staff, 4 feet long, cast a shade (on level ground) 7 feet; what is the height of that steeple, whose shade, at the same time, measures 198 feet?* F.sh. F.hei. F.sh. F.hei.

As 7 : 4 :: 198 : 113$\frac{1}{7}$ Ans.

* As the rays of light from the sun may be considered parallel, the *lengths* of the shadows must be proportioned to the *heights* of the objects. Hence the reason of the statement of this question.

*36. Suppose a tax of $755 be laid on a town, and the inventory of all the estates in the town amounts to $9345, what must A pay whose estate is $149?

	$		$		$		$ c. m.	
As	9345	:	755	::	149	:	12 12 7	Ans.

* It may not be amiss to show the general method of assessing town or parish taxes. First, then, an inventory of the value of all the estates, both real and personal, and the number of polls for which each person is rateable, must be taken in separate columns: The most concise way is then to make the total value of the inventory the first term, the tax to be assessed the second, and $1 the third, and the quotient will show the value on the dollar: 2dly, make a table, by multiplying the value on the dollar by 1, 2, 3, 4, &c.—3dly, From the inventory take the real and personal estates of each man, and find them separately in the table, which will shew you each man's proportional share of the tax for real and personal estates.

Note. If any part of the tax is averaged on the polls, or otherwise, before stating to find the value on the dollar, you must deduct the sum of the average tax from the whole sum to be assessed: for which average you must have a separate column, as well as for the real and personal estates.

Example.

Suppose the General Court should grant a tax of $500000, of which the town of Newburyport is to pay $5312 50c. and, of which the polls, being 1550, are to pay $1 25c. each:—The town's inventory amounts to $450000, what will it be on the dollar, and what is A's tax, whose estate (as by the inventory) is as follows, viz. real $1376, personal $1149, and he has 3 polls?

	Pol.		$ c.		Pol.		$ c.	
First, As	1	:	1 25	::	1550	:	1937 50	the average part of the tax to be deducted from $5312 50c. and there will remain $3375.

	$		$		$		
Secondly, As	450000	:	3375	::	1	:	7½m. on the dollar.

TABLE.

$	$	c.	m.	$	$	c.	m.	$	$	c.
1 is	0	0	7½	20 is	0	15	0	200 is	1	50
2 —	0	1	5	30 —	0	22	5	300 —	2	25
3 —	0	2	2½	40 —	0	30	0	400 —	3	00
4 —	0	3	0	50 —	0	37	5	500 —	3	75
5 —	0	3	7½	60 —	0	45	0	600 —	4	50
6 —	0	4	5	70 —	0	52	5	700 —	5	25
7 —	0	5	2½	80 —	0	60	0	800 —	6	00
8 —	0	6	0	90 —	0	67	5	900 —	6	75
9 —	0	6	7½	100 —	0	75		1000 —	7	50
10 —	0	7	5							

Now, to find what A's rate will be,
His real estate being $1376, I find by the table, that $1000 is - $7 50c.
that $300 is - - - 2 25
that $70 is - - - 52 5m.
and that $6 is - - - 4 5
for his real estate $10 32

Real. $ c. m.	Personal. $ c. m.	Polls. $ c. m.	Total. $ c. m.
10 32 0	8 61 7½	3 75	22 68 7½

In like manner I find his tax for personal estate to be $8 61 7½ }
His 3 polls at $1 25c. each are 3 75 } +$10 32=$22 68c. 7½m.

or, $22 69c. Ans.

37. If 50 gallons of water, in one hour, fall into a cistern, containing 230 gallons, and by a pipe in the cistern 35 gallons run out in an hour; in what time will it be filled? Ans. $15\frac{1}{3}$h.

38. A butcher went with £416, to buy cattle: Oxen, at £22 each, cows at £4, steers at £3 10s. and calves at £2 10s. and of each a like number; how many of each could he purchase with that sum?

Ans. 13 each.

39. Said Harry to Dick, my purse and money are worth $3\frac{1}{4}$ guineas, but the money is worth eleven times as much as the purse; pray, how much money is there in it? Ans. £4 3s. 5d.

40.* If $\frac{3}{8}$ of a yard cost $\frac{4}{5}$ of a £, what will $\frac{7}{11}$ of a yard cost?

As $\frac{3}{8} : \frac{4}{5} :: \frac{7}{11} : \frac{4}{5}\times\frac{7}{11}\div\frac{3}{8}=\frac{224}{165}$£ Answer.

Or $\frac{8}{3} : \frac{4}{5} :: \frac{7}{11} : \frac{8}{3}\times\frac{4}{5}\times\frac{7}{11}=\frac{224}{165}$£ = £1 7s. $1\frac{27}{33}$d.

41. There is a cistern, having four cocks; the first will empty it in ten minutes; the second in 20 minutes; the third in 40, and the fourth in 80 minutes; in what time will all four, running together empty it?

$$\text{As }\left\{\begin{matrix}10\\20\\40\\80\end{matrix}\right\}\begin{matrix}\text{Cist. Min.}\\ : 1 :: 60 :\end{matrix}\left\{\begin{matrix}6\\3\\1\frac{1}{2}\\\frac{3}{4}\end{matrix}\right\}\begin{matrix}\text{Cist. Min. Cist. Min.}\\ \text{As } 11\frac{1}{4} : 60 :: 1 : 5\frac{1}{3}\text{ Ans.}\\ \text{that is } \frac{45}{4} : 60 :: 1 : \frac{60\times4}{45}=5\frac{1}{3}.\end{matrix}$$

$11\frac{1}{4}$ Cist.

42. A and B depart from the same place, and travel the same road; but A goes 5 days before B, at the rate of 20 miles per day; B follows at the rate of 25 miles per day: In what time and distance will he overtake A?

M. M. D. M. D. D. D. M. D. M.

As 25—20 : 1 :: 20×5 : 20. And, As 1 : 25 :: 20 : 500

43. If the earth revolves 366 times in 365 days, in what time does it perform one revolution?

Ans. 23h. 56′ 3″ 56‴+=1 Sidereal day.†

44. If the earth makes one complete revolution in 23h. 56′ 3″+, in what time does it pass through one degree?

Ans. 3′ 55″ 20‴

45. If the earth performs its diurnal revolution in a solar day,‡ or 24 hours; in what time does it move one degree? Ans. 4′

46. Sold a cargo of flax seed in Ireland, for £1795 10s. Irish money; what does that amount to, in Massachusetts currency, £81 5s. Irish being equal to £100 Massachusetts.

Ans. £2209 16s. 11d.

* If the first term of the statement be a Vulgar Fraction, whether the other terms are or not, after the first and third terms are reduced to the same denomination, invert the first term as in division of Vulgar Fractions, and the product of the three terms will of course be the answer.

The student should work the questions in Vulgar, or Decimal Fractions, according as the rules for fractions require.

† A sidereal day is the space of time which happens between the departure of a star from, and its return to the same meridian again.

‡ The solar day is that space of time which intervenes between the sun's departing from any one meridian, and its return to the same again.

47. My correspondent in Maryland purchased a cargo of flour for me, for £437 that currency; how much Massachusetts money must I remit him, £125 Maryland being equal to £100 Massachusetts, or 5 Mar.=4 Mass. Ans. £349 12s.

48. A bill of exchange was accepted at Newburyport for the payment of £345 10, for the like value delivered in New York, at £133⅓ New York currency, for £100 Massachusetts ditto; how much money was paid in New York? Ans. £460 13s. 4d.

49. When the exchange from Massachusetts to Georgia is £83⅓ Georgia per £100 Massachusetts, how much Massachusetts money must be paid in Boston to balance £457 Georgia currency?

Ans. £548 8s. Mass.

50. A merchant delivered at Boston £320 Massachusetts currency, to receive £400 in Philadelphia; what was the Massachusetts pound valued at? Ans. £1 5s. Penn.

51. If I draw a bill of exchange for £537 10s. 6d. Massachusetts, to be paid in Ireland, at £123$\frac{1}{13}$ Massachusetts, per £100 Irish, or 16 Mass. for 13 Irish; for how much Irish money must I draw the bill? Ans. £436 14s. 9¼d Irish.

52. Suppose a bill is drawn in Ireland, and payable in Boston, for £673 12s. 6d. Irish; how much Massachusetts money comes it to, the exchange at £81¼ Irish, per £100 Massachusetts?

Ans. £829 1s. 6$\frac{6}{13}$d. Mass.

The value of any quantity of silver in any of the currencies of the United States may be found by the following proportion.

As the number of grains, contained in £1, is to £1; so are the grains, in any given quantity, to its value.

53. What is the value of 1℔ of silver in Massachusetts currency; the pound, or 20 shillings, containing 1393½ grains? £ s. d.

As 1393½ : 1 :: 5760 : 4 2 8.

54. If ⅗yd. cost \$⅞ what will 40½yds. come to?

Ans. \$59 6c. 2½m.

55. If 70 bushels of corn cost £12⅔, what is it per bushel?

Ans. 3s 7⅓d.

56. If $\frac{7}{16}$ of a ship cost £51, what are $\frac{3}{32}$ of her worth?

Ans. £10 18s. 6¾d. $\frac{3}{7}$.

57. At \$3⅝ per cwt. what will 9⅔℔ come to? Ans. 31c. 3m.—

58. A person having ⅘ of a vessel, sells ⅔ of his share for \$1080⅔; what is the whole vessel worth? Ans. \$2026 25c.

59. A merchant sold 5½ pieces of cloth, each containing 12⅔yds. at 12⅔c. per yard; what did the whole amount to?

Ans. \$8 82$\frac{4}{9}$c.

60. A buys of B £560¾ bank stock, at £85⅔ per cent. what comes it to? Ans. £480 7s. 6⅓d.

61. A merchant makes insurance upon a vessel and cargo, valued at £3750 16s at 15½ guineas per cent. what does the premium amount to? Ans £813 18s. 5½d.

62. A merchant in Holland draws a bill upon his correspondent in Boston for 3750 ducats at 3s. 4½d.: How much Massachusetts currency must he receive? Ans. £1565 12s. 6d.

63. A gentleman from Boston being in England, where the price of silver is to that of gold, as 1 to $15\frac{1}{14}$, exchanged $158\frac{1}{4}$ ℔ of silver for gold; on his return to Massachusetts, where the price of silver is to that of gold, as 1 to $15\frac{15}{31}$, a friend, wanting his gold, gave him the value thereof in silver; what weight of silver did he gain by the exchange?

℔ S. G. ℔ S. ℔ G. G. S. G. ℔ S.

As $15\frac{1}{14}$: $\frac{1}{1}$ :: $158\frac{1}{4}$: $10\frac{1}{2}$ As $\frac{1}{1}$: $15\frac{15}{31}$ :: $10\frac{1}{2}$: $162\frac{36}{62}$. Ans. $4\frac{41}{124}$ ℔.

64. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of \$7 for 5 yards, and sold them out at the rate of \$11 for 7 yards; and gained \$200 by the bargain; how many bales were there?

Yds. \$ Yds. \$
As 7 : 11 :: 5 : $7\frac{6}{7}$

Sold 5 yards for $7\frac{6}{7}$ Dollars.
Bought 5yds. for 7 Dollars.
In 5 yards gained $\frac{6}{7}$ Dollar.

\$ Yds. \$ Yds. Yds. B. Yds. B.

As $\frac{6}{7}$: 5 :: 200 : $1166\frac{2}{3}$, and, As $129\frac{17}{27}$: $\frac{1}{1}$ :: $1166\frac{2}{3}$: 9 Ans.

Although the method before laid down be universally applicable, yet there are other methods more ready and expeditious in some particular cases.

Rule I.

If the first and third terms be fractions, and the second a whole number, reduce the first and third to one common denominator, then, rejecting the denominators, make the numerator of the first, the first term, and the numerator of the third, the third term, and work as in whole numbers.

If $\frac{5}{8}$ of a yard cost 9s. what cost $\frac{7}{12}$ yard at that rate?

$\frac{5}{8}=\frac{15}{24}$ and $\frac{7}{12}=\frac{14}{24}$. Now, as 15 : 9s. :: 14 : 8s. $4\frac{4}{5}$d. Ans.

Rule II.

If of the first and third terms, one be 1, and the other a fraction: put the denominator of the fraction instead of 1, and the numerator in the place of the fraction, and work as in whole numbers, as before.

If 1 acre of land cost £12, what will $\frac{5}{8}$ of an acre cost at that rate?

	Den.		£		Num.		£ s.
As	8	:	12	::	5	:	7 10 Ans.

If the question were wrought with the fractions, it is evident that the denominator would belong both to the dividend and divisor, and thus destroy each other. Then in the example under Rule I. the statement would be,

$$\text{As } \frac{15}{24} : 9 :: \frac{14}{24} : \text{the answer}=\frac{24}{15}\times9\times\frac{14}{24}=\frac{9\times14}{15}.$$

And under Rule II. the statement would be,

$$\text{As } \frac{8}{8} : 12 :: \frac{5}{8} : \text{answer}=\frac{8}{8}\times12\times\frac{5}{8}=\frac{12\times5}{8}.$$

Whence the reason of the rules.

65. If ·625 of a yard cost £·25, what will 4·75yds. come to?

Yds. £ Yds. £ £ s.

$$\text{As } \cdot625 : \cdot25 :: 4\cdot75 : \frac{4\cdot75\times\cdot25}{\cdot625}=1\cdot9=1\ 18 \text{ Ans.}$$

66. If 9·75yds. cost \$11 25c. what will ·5yds. cost?

Ans. 57c. $6\frac{12}{13}$m.

67. There is a cistern, which has 3 cocks; the first will empty it in ·25 hour, the second in ·75 of an hour, and the third in 1·5 hour: in what time will it be emptied if all three run together?

$$\begin{array}{llllllll} & & \text{H.} & \text{Cist.} & & \text{H.} & & \text{Cist.} \\ \text{As} & \left\{\begin{array}{l} \\ \\ \\ \end{array}\right. \!\!\!\!\!\! & \begin{array}{l}\cdot 25\\ \cdot 75\\ 1\cdot 5\end{array} & \begin{array}{l}: 1\\ : 1\\ : 1\end{array} & \begin{array}{l}::\\ ::\\ ::\end{array} & \begin{array}{l}1\\ 1\\ 1\end{array} & \begin{array}{l}:\\ :\\ :\end{array} & \begin{array}{l}4\\ 1\cdot 333+\\ 0\cdot 667-\end{array} \\ & & & & & & & \overline{6 \text{ Cist.}} \end{array}$$

Cist. H. Cist. H. H. m.

$\text{As } 6 : 1 :: 1 : \frac{1}{6} = 0\cdot 167- = 10$ Ans.

68. A conduit has a cock, which will fill a cistern in ·2 of an hour: this cistern has 3 cocks; the first will empty it in 1·25 hour, the second in ·625 of an hour, and the third in ·5 hour. In what time will the cistern be filled, if all four run together?

Ans. 1h. 40m.

69. If 19yds. cost \$25 75c. what will 435·5yds. come to.

Ans. \$590 21c. $7\frac{2}{19}$m.

70. If 345yds. of tape cost \$5 17c. 5m. what will one yard cost?

Ans. 0c. 1m. ·5.

71. If I give \$12 82c. 5m. for 675 tops, how many tops will 19 mills buy?

Ans. 1 top.

72. If when wheat is \$1 per bushel, the two penny loaf weigh 9·6oz. what ought it to weigh when wheat is \$1 25c. per bushel?

Ans. 7oz. 13pwt. 14grs.

73. How much in length, that is $8\frac{5}{7}$ inches broad, will make a foot square?

Ans. $16\frac{32}{61}$ inches.

74. What number of men must be employed to finish in 9 days, what 15 men would perform in 30 days?

Ans. 50 men.

75. If 9 men can build a wall in 5 months by working 14 hours a day, in what time will the same men do it, when they work only 10 hours a day?

Ans. 7 months.

76. How many yards of carpet, $2\frac{3}{4}$ feet wide, will cover a floor, which is 18 feet long and 16 feet wide?

Ans. $34\frac{10}{11}$yds.

77. If 745 soldiers are to be clothed, and each suit is to contain $3\frac{1}{2}$yds. of cloth $1\frac{3}{8}$yd. wide, and to be lined with shalloon $\frac{7}{8}$yd. wide: how many yards of shalloon will be necessary?

Ans. $4097\frac{1}{2}$yds.

78. If a man count 100 cents in a minute for 10 hours in a day; in how many days will he count a million of cents?

Ans. $16\frac{2}{3}$ days.

79. Proceeding to count at the same rate as in the last question; how many men must be employed for 100 years of 365 days each, to count one trillion?.

Ans. $456621004\frac{124}{219}$ men.

80. The number of inhabitants on the earth is computed to be 750000000; suppose they had each counted one for every second from the creation to this time or 6000 years of 365 days each; how many would they have counted?

Ans. 141912000 billions.

81. In a certain school, $\frac{1}{20}$th of the pupils study Greek, $\frac{1}{10}$ study Latin, $\frac{2}{5}$ study Arithmetick, $\frac{1}{4}$ read and write, and 20 attend to other things; what is the number of pupils?

$\frac{1}{20}+\frac{1}{10}+\frac{2}{5}+\frac{1}{4}=\frac{8}{10}$, then $20=\frac{2}{10}$ and $\frac{2}{10} : 20 :: \frac{10}{10} : 100$. Ans.

To find the value of Gold in Massachusetts currency.

PROB. 1. Given the weight of any quantity of gold, to find its value.

THEOREM 1. As 1 Oz. : $5\frac{1}{3}$ £ :: 12 Oz. : 64 £ :: 1 pwt. : $5\frac{1}{3}$ s. :: 1 gr. : $2\frac{2}{3}$ d. (Case 1.) $=\frac{2\frac{2}{3}}{1}$ (Case 2.) $=\frac{5\frac{1}{3}}{2}$ (Case 3.) $=\frac{8}{3}$, Therefore,

Rule 1.—*If the given quantity be in grains; say, As the denominator is to the number of grains; so is the numerator to their value in pence.*

1. What is the value of 18 grains of gold?

By Case 1.	By Case 2.	By Case 3.
Gr.	Gr.	Gr.
As 1 : 18 :: $2\frac{2}{3}$	As 2 : 18 :: $5\frac{1}{3}$	As 3 : 18 :: 8
$2\frac{2}{3}$	$5\frac{1}{3}$	8
36	90	3)144
12	6	48d.=4s.
12)48(4s. Ans.	2)96(48d.=4s.	

Rule 2.—*If the given quantity consist of ounces, pennyweights, and grains, halve the grains, and then proceed as in multiplication of pounds, shillings and pence, making the numerator in Case 2d, the multiplier.*

1. What is the value of 7oz. 8pwt. 16gr. of gold?

```
Gr.          gr.    oz. pwt. gr.
16 ÷ 2 = 8, then,   7   8   8
                            5⅓
                    ----------
                   37   3   4
                    2   9   6⅔
                    ----------
                  £39  12  10⅔ Ans.
```

Rule 3.—*If the given quantity consist of pounds only, multiply by 64, and the product will be the answer; but, if it consist of pounds, ounces, &c. it will be most convenient to reduce the pounds to ounces, and proceed by Rule 2.*

1. What is the value of 36lb. of gold, at £64 per lb.?

```
    64
  ----
   144
  216
  ----
 £2304 Ans.
```

2. What is the value of 15lb. 9oz. 12pwt. 18gr. of gold?

```
        12
        —— pwt. gr.   gr.
   oz. 189   12   9 = 18 ÷ 2
                  5⅓
       ——————————————
       948    3   9
        63    4   3
      ——————  ————
   £1011    8   0 Ans.
```

Prob. 2. To ascertain the value of any given quantity of gold in Spanish milled dollars, or federal money.

Theorem 2. 1pwt. of gold = $5\frac{1}{3}$s. 1 dollar = 6s. And,

$$\frac{5\frac{1}{3}}{6}=\frac{16}{18}=\frac{8}{9}.$$ Therefore,

Rule. Reduce the given quantity of gold to pennyweights; then, as the denominator is to the given quantity; so is the numerator to the answer in dollars. Or,

Divide by the denominator, and multiply the quotient by the numerator. Or,

Divide by the denominator and subtract the quotient from the dividend. In either case, you will have the answer.

1. What is the value of 6oz. 6pwt. of gold, in Spanish dollars?

```
                              20
        pwt.                  ——
As 9 : 126 :: 8              126 pwt.                   Or,
             8                                         9)126
        ——————                 Or,                      —14
      9)1008                 9)126                     ————
        ——————                ——————                   112 Ans.
Ans.  112 Dolls.           14×8=112 Ans.               ————
        ——————
```

2. In 7oz. 13pwt. 17gr. how many dollars?

```
          oz. pwt. gr.
           7  13  17
          20
        ——————
         153 17/24
          24
        ——————
         619
        307
        ——————
        3689
```

As $\frac{9}{1} : \frac{3689}{24} :: \frac{8}{1} : \frac{29512}{216}$

```
216)29512(136 doll.
    216
    ————
     791
     648
    ————
     1432
     1296
    ————
      136
```

To find the value of this remainder.

1. In shillings, &c.

```
   136
     6
  ----
216)816(3s.
    648
   ----
    168
     12
   ----
216)2016(9d.
    1944
    ----
      72
       4
    ----
216)288(1 1/3 qr.
    216
    ---
72/216 = 1/3
```

2. In Federal Money.

Annex cyphers, as in division of decimals; the two quotient places next to dollars, will be cents; the third, mills; the others, decimals of a mill; or the remainder with the divisor, will form a fraction of a mill.

```
216)1360(62c. 9 17/27 m.
    1296
    ----
     640
     432
    ----
    2080
    1944
    ----
136/216 = 17/27
```

Prob. 3. To ascertain the weight of gold equivalent to any given sum, currency.

Rule 1. If the given sum be in pence, reverse Rule 1 Theorem 1. that is; As the numerator 8 is to the given sum in pence; so is the denominator 3 to the weight required, in grains.

What weight of gold is equal to 4s. ?

```
         d.           12
As 8 : 48 :: 3        --
          3           48
        ---
     8)144
       ---
  Ans.  18 grains.
```

Rule 2. If the given sum be in pounds, shillings and pence.

As $\frac{5\frac{1}{3}}{1}$ is equal to $\frac{16}{3}$; therefore, divide the given sum by 8, and that quotient by 2; add the two quotients together, double the last denomination, and you will have the answer.

What quantity of gold is equivalent to £45 13s. 4d.

Mark the pounds, shillings and pence, as oz. pwt. and gr.

```
      oz. pwt. gr.
    8)45   13   4
    -------------
    2)5    14   2 }
      2    17   1 } Add.
    -------------
      8    11   3+3
    -------------
  Oz. 8    11   6 Ans.
```

Prob. 4. To find the value of gold equivalent to any given sum in Federal money.

Rule. As the numerator 8 is to the number of dollars; so is the denominator 9 to the answer in pennyweights: Or, divide the dollars by the numerator 8, and add the quotient to the dividend.

Or, divide as before, and multiply the quotient by the denominator 9. In either case you will have the answer.

1. Required the weight of gold equal to 76 dollars.

As 8 : 76 :: 9 Or thus, 8)76 Or, $9\frac{1}{2}\times9=85\frac{1}{2}$pwt.

```
As 8 : 76 :: 9        Or thus, 8)76
       9                        9½
      ---                      ----
    8)684                 Ans. 85½pwt.
      ---      oz. pwt. gr.     ---
 Ans. 85½pwt.=4  5   12
```

2. Required the weight of gold equal $159 75c.

As 8 : 159·75 :: 9 : 179pwt. 17¼gr. Ans.

```
          9
      -------
    8)1437·75
      -------
      179·71875
             24
      ---------
         287500
        143750
        ------
        17·25 grains.
```

Or, $\overline{159{\cdot}75 \div 8} + 159{\cdot}75 = 179$pwt. 17¼gr. Ans.

Or, $\overline{159{\cdot}75 \div 8} \times 9 = 179$pwt. 17¼gr. as before.

RULE OF THREE DIRECT AND INVERSE.

Though Direct and Inverse Proportion, are properly only parts of the same rule, yet for the use of those who may desire it, the common distinctions will be made and the common rules given.

The Rule of Three Direct teaches, by having three numbers given, to find a *fourth*, which shall have the same ratio to the *second*, as the *third* has to the *first*.

The Rule of Three Inverse teaches, by having three numbers given, to find a *fourth*, which shall have the same ratio to the *second*, as the *first* has to the *third*. It is also called *reciprocal* or *indirect* proportion.

If *more* require *more*, or *less* require *less*, the question belongs to the Rule of Three Direct. But if *more* require *less*, or *less* require *more*, the question belongs to the Rule of Three Inverse.

The principal difficulty, which will embarrass the learner, will be to distinguish when the proportion is *direct*, and when *inverse*. This must be done by an attentive consideration of the question proposed. For *more* requires *more*, when the third term is greater than the first, and the question requires the fourth term to be greater than the second; and *less* requires *less*, when the third term is less than the first, and the fourth is required to be less than the second.

More is said to require *less*, when the third term is *greater* than the first, and the question requires the fourth to be *less* than the second; and *less* requires *more*, when the third term is *less* than the first, and the fourth is required to be *greater* than the second.

RULE OF THREE DIRECT.

Rule.

1. State the question by making that number, which asks* the question, the third term; that which is of the same name or quality as the demand, the first term; and that, which is of the same name or quality with the answer required, the second term.

2. Divide the product of the second and third terms by the first term and the quotient will be the answer.

Note. The directions under the General Rule, as well as the demonstration, apply to this rule.

Examples.

1. If 6lbs. of sugar cost 10s. what will 33lbs. cost at the same rate?

```
   lbs.   s.    lbs.
As  6  :  10 :: 33  : the answer.
                10
               ----
             6)330
               ----
               55s.= £2 15s. Ans.
```

In this example 33lbs. *asks* the question, and is made the third term; 6lbs. being of the same quality, is made the first term; and 10s. being of the quality of the answer required, is placed for the second term.

To invert the question, say,

```
    s.  lbs.  s.  lbs.
As 10 : 6 :: 55 : 33 the Ans.
```

2. If 100yds. of cloth cost $66 what will 1 yard cost?

Ans. 66c.

3. If my income be $1750 a year, and I spend 19s. 7d. a day, how much shall I have saved at the end of the year?

Ans. £167 12s. 1d.

RULE OF THREE INVERSE, OR RECIPROCAL PROPORTION.

Rule.†

State and reduce the terms as in the Rule of Three Direct; then, multiply the first and second terms together, and divide the product

* The term which *asks* or moves the question, has generally some words like these before it, viz. What will? what cost? How many? how long? how much? &c.

† The reason of this rule may be explained from the principles of Compound Multiplication and Compound Division, in the same manner as the direct rule.—

by the third; the quotient will be the answer in the same denomination as the middle term was reduced into.

If there be fractions in your question, they must be stated as before directed, and if they be vulgar, invert the third term: Then multiply the three terms continually together, and the product will be the answer.

Examples.

1. How much shalloon, that is ¾ yard wide, will line 6¾ yards of cloth which is 1¼ yard wide?

```
   yd.  yds.  qrs.
As 1¼ : 6¾ :: 3
    4    4
   --   --
    5   27
```

```
   qrs. qrs. qrs.
As 5 : 27 :: 3
             5
           ---
          3)135
           ---
          4)45
           ---
          11¼ yards, Ans.
```

The same by Vulgar Fractions.

First. $1\frac{1}{4}=\frac{5}{4}$, $6\frac{3}{4}=\frac{27}{4}$, and 3qrs.$=\frac{3}{4}$. Then,

As $\frac{5}{4} : \frac{27}{4} :: \frac{3}{4}$. And $\frac{5}{4}\times\frac{27}{4}\times\frac{4}{3}=\frac{5\times27\times4}{4\times4\times3}=\frac{540}{48}=\frac{45}{4}=11\frac{1}{4}$yds. Ans.

The same by Decimal Fractions.

$1\frac{1}{4}=1\cdot25$, $6\frac{3}{4}=6\cdot75$ and 3qrs.$=\cdot75$. Then,

```
As 1·25 : 6·75 :: ·75
          1·25
          ----
          3375
         1350
         675
        ------
·75)8·4375(11·25yds. Ans.
    75
    --
     93
     75
     --
     187
     150
     ---
      375
      375
```

2. What length of board 7½ inches wide, will make a square foot?

In.br. in.len. in.br. in.len.
As 12 : 12 :: 7½ : 19⅕ Ans.

For example, if 4 men can do a piece of work in 12 days, in what time will 8 men do it?

As 4 men : 12 days :: 8 men : $\frac{4\times12}{8}$=6 days, the Answer.

And here the product of the first and second terms, that is, 4 times 12, or 48, is evidently the time in which one man would perform the work. Therefore, 8 men will do it in one-eighth part of the time, or 6 days.

3. Suppose I lend a friend £350 for 5 months, he promising the like kindness; but, when requested, can spare but £125, how long may I keep it to balance the favour?

£ Mo. £ Mo.
As 350 : 5 :: 125 : 14 Ans.

4. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may be sufficient for those who remain 9 months?

Mo. M. Mo. M. M.
As 5 : 450 :: 9 : 250, and 450—250=200 men, Ans.

5. If a man perform a journey in 15 days, when the day is 12 hours long, in how many days will he do it, when the day is but 10 hours? Ans. 18 days.

6. If a piece of land, 40 rods in length, and 4 in breadth, make an acre, how wide must it be, when it is but 19 rods long to make an acre? Ans. 8rods 6ft. $11\frac{7}{19}$in.

7. If a piece of board be 30 inches in length, what breadth will make $1\frac{1}{2}$ square foot? Ans. 7·2 inches.

8. A wall, which was to be built 24 feet high, was raised 8 feet by 6 men, in 12 days: How many men must be employed to finish the wall in four days?

ft. m. ft. m.
As 8 : 6 :: $\overline{24—8}$: 12 to finish it in 12 days. And,
d. m. d. m.
As 12 : 12 :: 4 : 36 to finish in 4 days.

9. There is a cistern having a pipe, which will empty it in 6 hours: How many pipes of the same capacity will empty it in 20 minutes?

h. pi. mi. pi.
As 6 : 1 :: 20 : 18 Ans.

10. If a field will feed 6 cows 91 days, how long will it feed 21 cows? Ans. 26 days.

11. How much in length, that is $13\frac{7}{8}$ poles in breadth, will make a square acre? Ans. $11\frac{59}{111}$ poles.

12. If a suit of clothes can be made of $4\frac{1}{8}$ yards of cloth, $1\frac{3}{8}$ yard wide; how many yards of coating $\frac{7}{8}$ of a yard wide, will it require for the same person? Ans. 6yds. 1qr. $3\frac{5}{7}$n.

Abbreviations.

To know whether a fraction, when abbreviated, be equivalent in all respects to the original fraction.

Rule.

As the numerator of the fraction, in its lowest terms, is to its denominator; so will the numerator of the original fraction be to its own denominator.

Or, as one numerator is to the other; so will one denominator be to the other, &c.

A owes B £75 13s. 6d.; now £100 of A's money is equal to £140 of B's; what must A pay to satisfy the said debt?

$\frac{100}{140}=\frac{5}{7}$, therefore,

	£	s.	d.
	75	13	6
			5
7)	378	7	6
	£54	1	$0\frac{6}{7}$ Ans.

Now, to prove whether $\frac{5}{7}$ be equal $\frac{100}{140}$.

Num. Den. Num. Den. — Num. Num. Den. Den.

As 5 : 7 :: 100 : 140 — Or, as 5 : 100 :: 7 : 140.

All questions in the Rule of Three Direct or Inverse, may be wrought by the following

RULE.

State the questions as directed in the Rule of Three Direct; then multiply the second term by the first or third term accordingly as the answer ought to be greater or less; divide the product by the other term, and the quotient will be the answer.

COMPOUND PROPORTION,

OR DOUBLE RULE OF THREE,

TEACHES to resolve such questions as require two or more statements by Single Proportion, and hence its name. There is always an odd number of terms given, as five, seven, &c. All questions in Compound Proportion may be stated and wrought by the following

GENERAL RULE.*

1. Place that term, which is of the same kind or quality with the answer sought, for the second term.

2. Then, of the two terms in the question of the same kind, place the greater or less on the right for the third term, and the other on the left for the first term, according to the directions under the General Rule for Simple Proportion. Arrange the two remaining terms under the first and third, on the same principle.

3. Find the fourth term from the first statement, and place it for the second term in the second part of the statement, and find the fourth term from this statement, and it will be the answer required.

Note. If there be more than five terms in the question, the same mode of statement must be continued, and a third proportion formed, and so on, and the fourth term found from the last statement, will be the answer as before.

* This rule is evident from the General Rule of Three, for each statement is a particular statement under that Rule. If, then, all the separate dividends be collected into one dividend, and all the divisors into one divisor, their quotient must be the answer. Thus, in Ex. 1.

D. Int. D. Int. M. Int. M.

$$\text{As } 100 : 6 :: 400 : \frac{400\times6}{100}, \text{ and as } 12 : \frac{400\times6}{100} :: 9 : \frac{400\times6\times9}{100\times12} = 18 \text{ Int. Ans.}$$

EXAMPLES.

1. If a principal of \$100, gain \$6 interest in one year, what will \$400 gain in 9 months.

Statement and Operation.

$$\begin{array}{ccccccccccc} \text{D.} & \text{Int.} & & \text{D.} & \quad & & \text{M.} & \text{Int.} & & \text{M.} \\ \text{As } 100 & : 6 & :: & 400 & & \text{Or, } 12 & : & 6 & :: & 9 \\ \text{M.} & & & \text{M.} & & \text{D.} & & & & \\ 12 : & & :: & 9 & & 100 & : & & :: & 400. \end{array}$$

Then $100 : 6 :: 400 : \frac{400 \times 6}{100} = 24$. And $12 : \frac{400 \times 6}{100} : 9 : 18$ Int. Ans.

Or $12 : 24 : 9 : 18$ Ans.

Then $12 : 6 :: 9 : \frac{6 \times 9}{12} = 4\frac{1}{2}$ and $100 : \frac{6 \times 9}{12} : 400 : \frac{6 \times 9 \times 400}{12 \times 100} = 18$ Ans.

Or $100 : 4\frac{1}{2} : 400 : 18$ Int. Ans.

In this question, the answer sought is *interest*, and therefore \$6 must be the second term. As \$400 will gain more interest in the same time than \$100, \$400 must be placed on the right for the third term, and \$100 on the left for the first term. And as the same sum will gain more interest in 12 months than in 9 months, the 9 must be placed under the third term, and the 12 under the first term.

The operation is obvious on inspecting it.

Note. Instead of working two proportions, the whole may be reduced to one, by multiplying the *first* terms together, and also the *third* terms, and using their products for the first and third terms. This is merely changing the order of the operation, as will be seen in the preceding example.

$$\left.\begin{array}{l} \text{D. Int. D.} \\ 100 : 6 :: 400 \\ \;\;12 : \;\; :: \;\;9 \end{array}\right\} \text{This becomes, evidently,}$$

$$100 \times 12 : 6 :: 400 \times 9 : \frac{400 \times 9 \times 6}{100 \times 12} = 18, \text{ as before.}$$

The work may also frequently be contracted by dividing the *first* and *third* terms by a common divisor, or the *first* and *second* terms, and using their quotients, for the divisor will diminish the terms in the same ratio, and the proportion be still preserved. Thus, in the preceding example,

$$\begin{array}{l} 100 : 6 :: 400 \text{ becomes } 1 : 6 :: 4, \text{ by dividing by } 100. \\ \;\;12 : \;\; :: 9 \qquad\qquad\quad 4 : \;\; :: 3, \qquad\qquad\qquad\quad 3. \end{array}$$

$$\text{And } \left.\begin{array}{l} 1 : 6 :: 4 \\ 4 : \;\; :: 3 \end{array}\right\} \text{becomes } 1 : 6 : 3 : 18, \text{ Ans. as before.}$$

Ex. 2. If 950 soldiers consume 350 quarters of wheat in 7 months, how many soldiers will consume 1464 quarters in 1 month?

Ans. 27816 soldiers.

Ex. 3. If 1464 quarters of wheat be used by 27816 soldiers in a month, in what time will 950 soldiers consume 350 quarters?

Ans. 7 months.

Ex. 4. If 144 men, in 6 days of 12 hours each, dig a trench 200 feet long, 3 wide and 2 deep, how many hours long is the day,

when 30 men dig a trench 350 feet long, 6 wide and 3 deep, in 259·2 days? Ans. 7 hours.

The following Rule for the Double Rule of Three, involves the consideration of Direct and Inverse Proportion. Though the General Rule will enable the student to solve all questions with ease, this Rule is retained for the satisfaction of those who might desire to use it.

RULE.

Always place the three conditional terms in this order: That number, which is the principal cause of gain, loss or action, possesses the first place; that, which denotes the space of time, distance of place, rate, medium or mean of action, the second; and that, which is the gain, loss or action, the third: This being done, place the other two terms which move the question, under those of the same name, and if the blank place, or term sought, fall under the third place, then the question is in direct proportion: therefore,

RULE I.*

Multiply the three last terms together, for a dividend, and the two first for a divisor:—But, if the blank fall under the first or second place; then, the proportion is inverse; therefore,

RULE II.

Multiply the first, second and last terms together for a dividend, and the other two for a divisor, and the quotient will be the answer.

EXAMPLES.

1. If \$100 gain \$6 in a year; what will \$400 gain in 9 months?

D. P. Mo. D. Int.

100 : 12 :: 6 Terms in the supposition, or conditional terms.

400 : 9 Terms which move the question.

Here, the blank falling under the third place, the question is in direct proportion, and the answer must be found by the first Rule; therefore,

$400 \times 9 \times 6 = 21600$ For the dividend, and,

$100 \times 12 \quad = 1200$ For the divisor.

* 1. When the blank falls under the third term by this mode of statement, it is obvious on inspecting the statement that the proportion is *direct*, and the same terms are taken to form the dividend and divisor as in the preceding rule, or, by two statements in the Single Rule of Three direct.

2. But in Example 2nd, and when the blank falls under the first or second term, the proportion is inverse. In this Example, *more* principal and interest require *less* time, and, every statement according to the rule will make more require less. The operation by the rule is the same as from two statements by the Single Rule of Three Inverse. These statements on this Example would be thus

$$100 : 12 :: 400 : \frac{100 \times 12}{400}$$

$$18 : \frac{100 \times 12}{400} : 6 : \frac{100 \times 12 \times 18}{400 \times 6} = 9,$$ and illustrates the rule.

See the work at large.

```
     D. Pr. Mo. D. Int.
      100 : 12 :: 6
      400 :  9
        9
      ----
 100  3600
  12     6
 ---  ----
12|00)216|00(18D. Ans.
      12
      --
       96
       96
       --
```

2. If $100 will gain $6 in a year; in what time will $400 gain $18? D. Mo. D.

100 : 12 :: 6 Terms in the supposition.

400 : :: 18 Terms which move the question.

Here, the blank falling under the 2d place, the question is in reciprocal or inverse Proportion, and the answer must be sought by the second rule; therefore,

100×12×18=21600 For the dividend.

400× 6 = 2400 For the divisor.

```
 D. Pr. Mo. D. Int.
  100 : 12 ::  6
  400 :     :: 18
    6          12
 ----         ---
 2400         216
              100
            -----
       24|00)216|00(9 months, Ans.
             216
```

3. What principal, at 6 per cent. per ann. will gain $18 in 9 months?

```
Pr.  Mo.  Int.
100 : 12 ::  6
       9 :: 18
            12
          ----
       9   216
       6   100
     ---------- D.
     54)21600(400 Ans.
        216
        ---
          00
```

4. If $400 gain $18 in 9 months; what is the rate per cent. per annum?

```
 Pr.  Mo.  Int.
 400 :  9 :: 18
 100 : 12 :: $6 Ans.
```

Here, the blank falling under the first place, the proportion is inverse, and the answer found by the second rule, as in the last example.

5. If 8 men spend £32 in 13 weeks; what will 24 men spend in 52 weeks? Ans. £384.

6. If the freight of 9hhds. of sugar, each weighing 12cwt. 20 leagues, cost $50; what must be paid for the freight of 50 tierces ditto, each weighing $2\frac{1}{2}$cwt. 100 leagues? Ans. $289 35c. $1\frac{1}{9}$m.

7. There was a certain edifice completed in a year by 20 workmen; but the same being demolished, it is necessary that just such an one should be built in 5 months. I demand the number of men to be employed about it? Ans. 48 men.

8. If 6 men build a wall 20 feet long, 6 feet high and 4 feet thick, in 16 days, in what time will 24 men build one 200 feet long, 8 feet high, and 6 feet thick? Ans. 80 days.

COMPARISON OF WEIGHTS AND MEASURES.

EXAMPLES.

1. If 78 pence Massachusetts be worth 1 French crown, how many Massachusetts pence are worth 320 French crowns?

$$\begin{array}{r} \text{F.cr.}\ \ \text{d.}\ \ \text{F.cr.} \\ \text{As } 1 : 78 :: 320 \\ 78 \\ \hline 2560 \\ 2240\ \\ \hline 24960 \text{ Ans.} \end{array}$$

2. If 24 yards at Boston make 16 ells at Paris, how many ells at Paris will make 128 yards at Boston?

Bost. Par. Bost. Par.
As 24yds. : 16ells :: 128yds. : $85\frac{1}{3}$ells, Ans.

3. If 60lb at Boston make 56lb at Amsterdam, how many pounds at Boston will be equal to 350 at Amsterdam?
Ans. 375lb Boston.

4. If 95lb Flemish make 100lb American, how many American pounds are equal to 550lb Flemish? Ans. $578\frac{18}{19}$lb American.

CONJOINED PROPORTION,

IS when the coins, weights or measures of several countries are compared in the same question; or, in other words, it is joining many proportions together, and by the relation, which several antecedents have to their consequents, the proportion between the first antecedent and the last consequent is discovered, as well as the proportion between the others in their several respects.

This rule may generally be so abridged by cancelling equal quantities on both sides, and abbreviating commensurables, that the whole operation may be performed with very little trouble, and it may be proved by as many statings in the Single Rule of Three, as the nature of the question may require.

CASE I.

When it is required to find how many of the first sort of coin, weight, or measure, mentioned in the question, are equal to a given quantity of the last.

Rule.

Place the numbers alternately, that is, the antecedents at the left hand, and the consequents at the right, and let the last number stand on the left hand; then multiply the left hand column continually for a dividend, and the right hand for a divisor, and the quotient will be the answer.

Examples.

1. Suppose 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Thoulouse, and 100 canes of Thoulouse=160 ells of Geneva, and 100 ells of Geneva=200 ells of Hamburgh: How many yards of America are equal to 379 ells of Hamburgh?

Antecedents.		Consequents.	Abriged. Ant.	Con.
100 of America	=	100 of England.		
100 of England	=	50 of Thoulouse.	5	8
100 of Thoulouse	=	160 of Geneva.	379	
100 of Geneva	=	200 of Hamburgh.		
379 of Hamburgh?				

Therefore, $\frac{379\times5}{8}=236\frac{7}{8}$yds. of America=379 ells of Hamburgh.

Illustration.

The two 100s of both sides cancel each other. Let the last cyphers of the next three antecedents and consequents be cancelled, which is dividing by 10. Then divide the second antecedent and consequent by 5, and the quotients will be 2 on the side of the antecedents, and 1 on the side of the consequents; then 2 will measure the third antecedent and consequent, and the quotients will be 5 and 8. 10 will measure the 4th antecedent and consequent, and the quotients will be 1 and 2. Now, there being 2 left on each side, they cancel each other, and as there is no farther room for abridging by reason of the odd number 379, the operation is finished, and the answer found, as before.

2. If 20lb at Boston make 23lb at Antwerp, and 155 at Antwerp make 180 at Leghorn: How many at Boston are equal to 144 at Leghorn? Ans. $107\frac{19}{23}$lb.

3. If 12lb at Boston make 10lb at Amsterdam, 10lb at Amsterdam 12lb at Paris: How many pounds at Boston are equal to 80lb at Paris? Ans. 80lb.

4. If 140 braces at Venice be equal to 150 braces at Leghorn, and 7 braces at Leghorn be equal to 4 Americau yards: How many Venetian braces are equal to 32 American yards?

Ans. $52\frac{4}{13}$.

5. If 40℔ at Newburyport make 36 at Amsterdam, and 90℔ at Amsterdam make 116 at Dantzick: How many pounds at Newburyport are equal to 260℔ at Dantzick? Ans. $224\frac{4}{25}$.

CASE II.

When it is required to find how many of the last sort of coin, weight or measure, mentioned in the question, are equal to a given quantity of the first.

Rule.

Place the numbers alternately, beginning at the left hand, and let the last number stand on the right hand; then multiply the first row for a divisor, and the second for a dividend.

Examples.

1. Suppose 100 yards of America=100 yards of England, and 100 yards of England=50 canes of Thoulouse, and 100 canes of Thoulouse=160 ells of Geneva, and 100 ells of Geneva=200 ells of Hamburgh: How many ells of Hamburgh are equal to $236\frac{7}{8}$ yards of America?

Ant.		Con.			Abridged.	
					Ant.	Con.
100 Amer.	=	100	Eng.		5	8
100 Eng.	=	50	Thoul.			$236\frac{7}{8}$
100 Thoul.	=	160	Geh.			
100 Gen.	=	200	Hamb.			
		$236\frac{7}{8}$	Amer.			

$$\frac{236\frac{7}{8} \times 8}{5} = 379 \text{ Ham. Ans.}$$

This needs no further illustration. The learner will readily see, that this case being the reverse of the former, they are proofs to each other.

2. If 20℔ at Boston make 23℔ at Antwerp, and 155 at Antwerp make 180 at Leghorn: How many at Leghorn are equal to 144 at Boston? Ans. 144℔.

3. If 12℔ at Boston make 10℔ at Amsterdam, and 100℔ at Amsterdam 120℔ at Paris: How many at Paris are equal to 80℔ at Boston? Ans. 80℔.

4. If 140 braces at Venice be equal to 150 braces at Leghorn, and 7 braces at Leghorn be equal to 4 American yards: How many American yards are equal to $52\frac{4}{13}$ Venetian braces?

Ans. 32 yards.

5. If 40℔ at Newburyport make 36 at Amsterdam, and 90℔ at Amsterdam make 116 at Dantzick: How many pounds at Dantzick are equal to 244 at Newburyport? Ans $283\frac{12}{25}$℔.

ARBITRATION OF EXCHANGES.

By this term is understood how to choose, or determine the best way of remitting money from abroad with advantage; which is performed by conjoined proportion: Thus,

1. Suppose a merchant has effects at Amsterdam to the amount of \$3530, which he can remit by way of Lisbon at 840 rees per dollar, and thence to Boston, at 8s. 1d. per milree (or 1000 rees :) Or, by way of Nantz, at $5\frac{2}{3}$ livres per dollar, and thence to Boston at 6s. 8d. per crown ; It is required to arbitrate these exchanges, that is, to choose that which is most advantageous?

1 dollar at Amsterdam = 840 rees at Lisbon.
1000 rees at Lisbon = 97d. at Boston.
3530 dollars at Amsterdam.

$$\frac{840\times97\times3530}{1000\times1}=£1198\ 8s.\ 8\tfrac{4}{10}d.\ \text{by way of Lisbon.}$$

1 dollar at Amsterdam = $5\frac{2}{3}$ livres at Nantz.
6 livres at Nantz = 80 pence at Boston.
3530 dollars at Amsterdam.

$$\frac{5\frac{2}{3}\times80\times3530}{1\times6}=£1059\ \text{by way of Nantz.}$$

Here it may be observed that the difference is £139 8s. $8\frac{4}{10}$d. in favour of remitting by way of Lisbon rather than by Nantz, which depends on the *course* of exchange, at that time ; but the *course* may vary so, that, in a short time by way of Nantz may be better ; hence appears the necessity and advantage of an extensive correspondence, to acquire a thorough knowledge in the *courses* of exchange, to make this kind of remittance.

2. A merchant in England can draw directly for 1000 piastres in Leghorn at 50d. sterling per piastre ; but he chooses to remit the sum to Cadiz at 19 piastres for 7000 maravedies ; thence to Amsterdam at 189d. Flemish for 680 maravedies ; and thence to Liverpool at 9d. Flemish for 5d. sterling : what is gained by this circular remittance, and what is the value of a piastre to him?

Ans. Gain £28 14s. sterling nearly.
Valne of a piastre 56d. 3·55qr. sterling.

3. A merchant in New York orders £500 sterling, due him at London at 54d. sterling per dollar, to be sent by the following circuit ; to Hamburgh at 15 marks banco per pound sterling ; thence to Copenhagen at 100 marks banco for 33 rix dollars ; thence to Bourdeaux at one rix dollar for 6 francs ; thence to Lisbon at 125 francs for 18 milrees ; and thence to New York at $\$1\frac{1}{4}$ per milree : did he gain or lose by this circular remittance, and what was the arbitrated value of a dollar by this remittance?

Ans. He gained.
Value of a dollar was 69d. sterling nearly.

FELLOWSHIP.

THE Rules of Fellowship are those by which the accompts of several merchants or other persons, trading in partnership, are so adjusted, that each may have his share of the gain, or sustain his

share of the loss, in proportion to his share of the joint stock, together with the time of its continuance in trade.

SINGLE FELLOWSHIP

Is, when the stocks are employed for any certain equal time.

RULE.*

As the whole stock is to the whole gain or loss, so is each man's particular stock to his particular share of the gain, or loss.

PROOF. Add all the particular shares of the gain or loss together, and, if it be right, the sum will be equal to the whole gain or loss.

EXAMPLES.

1. Divide the number 360 into four parts, which shall be to each other, as 3, 4, 5 and 6.

$$\text{As } 3+4+5+6 : 360 :: \left\{\begin{array}{l} 3 : 60 \\ 4 : 80 \\ 5 : 100 \\ 6 : 120 \end{array}\right\} \text{Answer.}$$

$$\overline{360} \text{ Proof.}$$

2. A, B, C, and D companied; A put in £145; B, £219; C, £378, and D, £417, with which they gained £569: What was the share of each?

$$\begin{array}{c} \text{Whole stock.} \quad \text{Gain.} \\ \text{As } 145+219+378+417 : 569 :: \end{array} \left\{\begin{array}{rrrrll} & \text{£} & \text{s.} & \text{d.} & & \\ 145 : & 71 & 3 & 8\frac{1}{2} & \frac{1058}{1159} & \text{A's share.} \\ 219 : & 107 & 10 & 3\frac{3}{4} & \frac{375}{1159} & \text{B's ditto.} \\ 378 : & 185 & 11 & 6 & \frac{552}{1159} & \text{C's ditto.} \\ 417 : & 204 & 14 & 5\frac{1}{4} & \frac{333}{1159} & \text{D's ditto.} \end{array}\right.$$

$$\text{£}569 \text{ — Proof.}$$

3. A, B, C, and D are concerned in a joint stock of $1000; of which A's part is $150; B's $250; C's $275, and D's $325. Upon the adjustment of their accompts, they have lost $337 50c. What is the loss of each? Ans. A's loss $50 62½c. B's $84 37½c. C's $92 81¼c. and D's $109 68¾c.

4. A and B companied; A put in £45, and took ⅗ of the gain; What did B put in? 5—3=2. Then, As 3 : 45 :: 2 : 30 Ans.

5. A, B and C freighted a ship with 68900 feet of boards: A put in 16520 feet; B 28750; and C the rest; but in a storm, the captain threw overboard 26450 feet: How much must each sustain of the loss? Ans. A, 6341¾ feet. B, 11036¾ and C, 9071½ do.

6. A gentleman died, leaving three sons and a daughter, to whom he bequeathed his estate in the following manner: To the eldest son, he gave 312 moidores, to the second, 312 guineas, to the third,

* That their gain or loss, in this rule, is in proportion to their stocks is evident: For, as the times, in which the stocks are in trade, are equal, if I put in ½ of the whole stock, I ought to have ½ of the gain: If my part of the stock be ¼, my share of the gain or loss ought to be ¼ also. And generally the same ratio that the whole stock has to the whole gain or loss, must each person's particular stock have to his respective gain or loss.

312 pistoles, and to the daughter, 312 dollars; but when his debts were paid, there were but 312 half joes left: What must each have in proportion to the legacies which had been bequeathed them?

Ans. 1st son £293 0s. 3d.—2d. son £227 17s. $10\frac{3}{4}$d.—3d. son £ 179 1s. $2\frac{1}{4}$d. and the daughter £48 16s. $8\frac{1}{4}$d.

7. A ship, worth $3000, being lost at sea, of which $\frac{1}{8}$ belonged to A, $\frac{1}{2}$ to B, and the rest to C: What loss will each sustain, supposing $450 to have been insured upon her?

Ans. A's loss $312 50c.
B's 937 50
C's 625

8. A and B venturing equal sums of money, cleared by joint trade $140: By agreement, as A executed the business, he was to have 8 per cent. and B was to have 5 per cent.: What was A allowed for his trouble?

$\quad\$\quad\$\quad\$\quad\$\quad\$\qquad\qquad\$\quad\$\quad\$\quad\$\quad\$$
As 8+5 : 140 :: 8 : $86\frac{2}{13}$ And, as 8+5 : 140 :: 5 : $53\frac{11}{13}$.
Ans. $32 30c. $7\frac{9}{13}$m.

9. A bankrupt is indebted to A £120, to B £230, to C £340. and to D £450, and his whole estate amounts only to £560: How must it be divided among the creditors?

Ans. A, £58 18s. $11\frac{1}{4}$d. B, £112 19s. $7\frac{3}{4}$d. C, £167 0s. 4d. and D. £221 1s. $0\frac{1}{2}$d.

10. A, B, and C put their money into a joint stock; A put in $40; B and C together $170: They gained $126, of which B took $42; What did A and C gain, and B and C put in respectively?

Ans. $24 A's gain, $70 B's stock, $100 C's stock, $60 C's gain.

11. A, B, and C companied; A put in £40; B 60, and C a sum unknown: They gained £72; of which C took £32 for his share; What did A and B gain, and C put in?

Ans. £16 A's gain, £24 B's gain, and £80 C's stock.

12. A, B, and C put in $720, and gained $540, of which, so often as A took up $3, B took 5, and C 7: What did each put in and gain?

Instead of the above rule, you may find a common multiplier to multiply the proportions by, or multiplicand to be multiplied by the given proportions, thus, 15)720(48 multiplicand to find the stocks. And 15)540(36 multiplicand to find the gains.

$		$
48×3=144 A's stock.		36×3=108 A's gain.
48×5=240 B's ditto.	And	36×5=180 B's ditto.
48×7=336 C's ditto.		36×7=252 C's ditto. as before.

13. A, B, C, and D companied; and gained a sum of money of which A, B and C took £120. B, C and D, £180. C, D and A, £160, and D, A and B, £140: What distinct gain had each?

The sum of these 4 numbers is £600. and as each man's money is named 3 times, therefore $\frac{1}{3}$, viz. £200 is the whole gain.—Therefore £200—£ 120 A's B's and C's gain=£ 80 D's gain;—And £200—£ 180 B's, C's and D's gain=£20 A's gain.—£ 200—£160 C's, D's, and A's gain=£ 40 B's gain.—And £200—£ 140 D's, A's and B's gain=£ 60 C's gain.

14. Two merchants companied; A put in £40, and B 288 ducats. They gained £135, of which A took £60. What was the value of a ducat?

As £60, A's gain : £40, his stock :: £135 the whole gain—£60, A's gain : £50, B's stock.

	Duc.	£	Duc.	s. d.
And, as	288 :	50 ::	1 :	3 $5\frac{2}{3}$ Ans.

15. Four men spent, at a reckoning, 20 shillings, of which they agreed that A should pay $\frac{3}{4}$, B, $\frac{1}{2}$, C, $\frac{1}{4}$, and D, $\frac{1}{8}$. What must each pay in that proportion?

s.	d.		
9	$2\frac{10}{13}$	A	Answer.
6	$1\frac{11}{13}$	B	
3	$0\frac{12}{13}$	C	
1	$6\frac{6}{13}$	D	

16. A, B, and C companied; A put in £40·25; B £80·5; and C £161: they gained £120. What is each man's share?

£	£	£	£	£	£	
40·25+	80·5+	161 :	120 ::	40·25 :	17·142475	=A's
					34·28495	=B's
					68·5699	=C's
				Proof	£119·997325	

17. A, B, C, and D gain \$200 in trade, of which as often as A has \$6, B must have \$10, C \$14, and D \$20: What is the share of each? Ans. A's share \$24, B's \$40, C's \$56, and D's \$80.

18. An insolvent estate of \$633 60c. is indebted to A, \$312 75c. to B, \$297, to C, \$50 25c. to D, \$0 25c. to E, \$200, to F, \$142 50c. and to G, \$21 25c.; what proportion will each creditor receive?

		\$	c.
Ans.	A's share =	193	51·41
	B's - -	183	76·87
	C's - -	31	09·23
	D's - -	0	15·41
	E's - -	123	75·
	F's - -	88	17· 1
	G's - -	13	14·87
	Proof	\$633	59·97

19. A ship was driven on shore in a gale, and in lightening and getting her afloat again and in reloading, an expense of \$768 was incurred; the ship was valued at \$10000, freight at \$3200, molasses owned by A, at \$5200, sugar owned by B, at \$4700, and rum owned by C, at \$2500: how much is this loss on every \$100, and how much must each party pay of it?

10000+3200+5200+4700+2500=\$25600. As \$25600 : \$768 :: \$100 : \$3
Ans. on each \$100.

Then, As \$100 : \$3 :: \$1000 : \$300 to be paid by the ship,

320 :	96	- -	freight,
5200 :	156	- -	A,
4700 :	141	- -	B,
2500 :	75	- -	C, Ans.
	768 Proof.		

20. A vessel, valued at \$13000 was laden with hardware for E valued at \$3000, with cordage for F, at \$5000, with dry goods for G, at \$3200, with goods for H, at \$7900, and for I, at \$4400; the captain was obliged to prevent sinking in a storm to throw overboard three fifths of the hardware, and two fifths of the cordage, with goods of H valued at \$2700; allowing the freight to be \$3500, what will be the average of the loss on 100 dolls. and what must be paid to E, F, and H, for their property thrown overboard?

Ans. \$16 25cts. on \$100, and E, F, and H must receive together \$5443 75cts.

Note. If the property of E, F, and H, had been insured, the remainder of their loss must be paid by the insurers. See Policies of Insurance.

*DOUBLE FELLOWSHIP,**

Or, *Fellowship with Time*, is occasioned by the shares of partners being continued unequal times.

Rule.

Multiply each man's stock, or share by the time it was continued in trade. Then,

As the whole sum of the products, is to the whole gain or loss, so is each man's particular product, to his particular share of the gain or loss.

Examples.

1. A, B, and C hold a pasture in common, for which they pay £40 per annum. A put in 9 oxen for 5 weeks; B, 12 oxen for 7 weeks, and C 8 oxen for 16 weeks. What must each pay of the rent?

9×5=45. 12×7=84, and 8×16=128, then 128+84+45=257.

As 257 : 40 :: 45	As 257 : 40 :: 84	As 257 : 40 :: 128
45	84	40
200	160	£ s. d.
160	320	257)5120(19 18 $5\frac{13}{257}$
£ s. d.	£ s. d.	
257)1800(7 0 $0\frac{3}{4}\frac{149}{257}$	257)3360(13 1 $5\frac{1}{2}\frac{91}{257}$	

* When times are equal, the shares of the gain or loss are evidently as the stocks, as in Single Fellowship; and when the stocks are equal, the shares are as the times; wherefore, when neither are equal, the shares must be as their products.

	£	s.	d.
A's =	7	0	$0\frac{240}{257}$
B's =	13	1	$5\frac{191}{257}$
C's =	19	18	$5\frac{83}{257}$
Proof	40	0	0

2. Four merchants traded in company; A put in \$400 for five months, B, \$600 for 7 months, C, \$960 for 8 months, and D, \$1200 for 9 months; but by misfortunes at sea, they lost \$750. What must each man sustain of the loss.

Answer, { A, \$94 93c. $6\frac{48}{79}$m. C \$227 84c. $8\frac{8}{79}$m.
B 142 40 $5\frac{5}{79}$ D 284 81 $0\frac{18}{79}$ }

3. A, with a capital of £100 began trade January 1st, 1787, and meeting with success in his business, he took in B as a partner, on the 1st day of March following, with a capital of £150. Three months after that, they admit C as a third partner, who brought into stock £180, and after trading together until the 1st of January, 1788, they found there had been gained since A's commencing business £177 13s. How must this be divided among the partners?

Ans. A, £53 16s. 8d B, £67 5s. 10d. C, £56 10s. 6d.

4. Two merchants entered into partnership for 18 months; A, at first, put into stock \$400, and at the end of 8 months he put in \$200 more; B, at first, put in \$1100, and at 4 months' end took out \$280. Now at the expiration of the time, they found they had gained \$1052. What is each man's just share?

Ans. A, \$385 90c. B, \$666 10c.

5. A and B companied; A put in the 1st of January, £150; but B could not put in any until the 1st of May: What did he then put in, to have an equal share with A at the year's end?

Ans. £225.

6. E, F, and G companied; E put in, the first of March, £30, F, the first of May, put in 80 yards of broadcloth; and on the 1st of June, G put in \$120. On the 1st of January following, they reckoned their gains, of which E and F took £228. F and G £215 10s. and G and E £187 10s. What was the whole gain, and the gain of each? What did they value a yard of cloth at? and, what was G's dollar worth?

228l.+215l. 10s.+187l. 10s.=631l. and 631l.÷2=315l. 10s. the whole gain; then, 315l. 10s.—228=87l. 10s. G's gain. 315l. 10s. —215l. 10s.=100l. E's gain, and 315l. 10s.—187l. 10s.=128l. F's gain. To find the value of one yard of cloth, say, As 100l. E's gain : 30l. his stock :: 128l. F's gain : 38l. 8s.; then, inversely, As 10 months : 38l. 8s. :: 8 months : 48l. the value of the whole cloth.

As 80yds. : 48l. :: 1yd. : 12s. answer. Now, to find the value of a dollar. As 100l. E's gain : 30l. his stock :: 87l. 10s. G's gain : 26l. 5s.; then, inversely, As 10 months : 26l. 5s. :: 7 months : 37l. 10s.=120 dollars. Lastly : As 120 dollars : 37l. 10s. :: 1 dollar : 6s. 3d. Answer.

7. E, F and G companied; E put in \$400 for ·75 of a year; F \$300 for ·5 of a year, and G \$500 for ·25 of a year; with which they gained \$720: Required the share of each.

$400 \times \cdot 75 = 300$
$300 \times \cdot 5 = 150$
$500 \times \cdot 25 = 125$

$$575 : 720 :: 300 : \$375\tfrac{15}{23}$$
$$187\tfrac{19}{23}$$
$$156\tfrac{12}{23}$$

Proof. 720 Dollars.

8. A put in $\frac{1}{2}$ for $\frac{3}{4}$ of a year, B $\frac{2}{5}$ for $\frac{1}{2}$ a year, and C the rest for one year; their joint stock was 1, and their gain 1; what is each share?

Ans. A's is $\frac{15}{27}$
B's $\frac{8}{27}$
C's $\frac{4}{27}$

Proof. $=1$

9. A and B entered into partnership for 16 months. A put in \$1200 at first, and 9 months afterwards \$200 more; B put in at first \$1500, and at the end of 6 months took out \$500; their gain was \$772 20c.? what is the share of each?

Ans. A's share \$401 70c. B's share \$370 50c.

PRACTICE,

IS a contraction of the rule of Three Direct, when the first term happens to be a unit, or one; and has its name from its daily use among merchants and tradesmen, being an easy and concise method of working most questions which occur in trade and business.

The method of proof is by the Rule of Three, Compound Multiplication, or by varying the order of them.

A variety of rules, adapted to particular cases, is usually given under Practice. Most of the sums, however, fall under two heads, and may be wrought by two General Rules, adapted to these cases. On account of their great practical importance, these two rules should be thoroughly understood.

General Rule I.

When the price of 1 yard, 1lb, &c. is given to find the value of any number of yards, &c.

1. Suppose the price of the given quantity to be 1l. 1D. 1s. &c. then will the quantity itself be the answer, at the supposed price.

2. Divide the given price into aliquot parts, either of the supposed price or of one another, and find the quotients of the several aliquot parts; and their sum will be the true answer.

Example.

What is the value of 468 yards, at 2s. 9¼d. per yard?

	£468 s. d.	Answer at £1 s. d.
2s. 6d. is ⅛ =	58 10 0	ditto at 0 2 6
3d. is $\frac{1}{10}$ =	5 17 0	ditto at 0 0 3
¼d. is $\frac{1}{12}$ =	0 9 9	ditto at 0 0 0¼
The full price =	£64 16 9	0 2 9¼

In this example it is plain, that the quantity 468 is the answer at £1; consequently as 2s. 6d. is ⅛ of a pound, ⅛ part of that quantity, or £58 10s. is the price at 2s. 6d.; in like manner, as 3d. is the $\frac{1}{10}$ part of 2s. 6d. so $\frac{1}{10}$ part of £58 10s. or £5 17s. is the answer at 3d. and as ¼d. is $\frac{1}{12}$ of 3d. so $\frac{1}{12}$ of £5 17s. or 9s. 9d. is the answer at ¼d. Now, as the sum of all these parts is equal to the whole price (2s. 9¼d.) so the sum of the answers belonging to each price will be the answer at the full price required, and the same will be true in any example whatever.

Before the questions, hereafter given, can be wrought, the following Tables must be perfectly gotten by heart.

TABLES.

Aliquot, or even parts of Money.

Pts. of a shil. of a £.

d.	s.	£
6	= ½	= $\frac{1}{40}$
4	= ⅓	= $\frac{1}{60}$
3	= ¼	= $\frac{1}{80}$
2	= ⅙	= $\frac{1}{120}$
1½	= ⅛	= $\frac{1}{160}$
1	= $\frac{1}{12}$	= $\frac{1}{240}$
¾	= $\frac{1}{16}$	= $\frac{1}{320}$
½	= $\frac{1}{24}$	= $\frac{1}{480}$
¼	= $\frac{1}{48}$	= $\frac{1}{960}$

Parts of 2 Shill.

d.	2s.
1	= $\frac{1}{24}$
1½	= $\frac{1}{16}$
2	= $\frac{1}{12}$
3	= ⅛
4	= ⅙
6	= ¼
8	= ⅓

Parts of a Pound.

s.	d.	£
10	0	= ½
6	8	= ⅓
5	0	= ¼
4	0	= ⅕
3	4	= ⅙
2	6	= ⅛
1	8	= $\frac{1}{12}$
1	4	= $\frac{1}{15}$
1	3	= $\frac{1}{16}$
1	0	= $\frac{1}{20}$
0	10	= $\frac{1}{24}$
0	8	= $\frac{1}{30}$
0	5	= $\frac{1}{48}$
0	4	= $\frac{1}{60}$
0	3	= $\frac{1}{80}$
0	2	= $\frac{1}{120}$

Parts of a Dollar.

c.	$
50	= ½
33⅓	= ⅓
25	= ¼
20	= ⅕
16⅔	= ⅙
12½	= ⅛
8⅓	= $\frac{1}{12}$
6¼	= $\frac{1}{16}$
5	= $\frac{1}{20}$
4	= $\frac{1}{25}$
3⅓	= $\frac{1}{30}$
2	= $\frac{1}{50}$
1	= $\frac{1}{100}$

Aliquot, or even Parts of Weight.

Parts of a Cwt.			Parts of ½ Cwt.		Parts of ¼ Cwt.		Parts of a Ton.		
Qrs.	lb	Cwt.	lb	½ Cwt.	lb	¼ Cwt.	Cwt.	qr.	T.
2	0	= ½	28	= ½	14	= ½	10	0	= ½
1	0	= ¼	14	= ¼	7	= ¼	5	0	= ¼
0	16	= $\frac{1}{7}$	8	= $\frac{1}{7}$	4	= $\frac{1}{7}$	4	0	= $\frac{1}{5}$
0	14	= $\frac{1}{8}$	7	= $\frac{1}{8}$	2	= $\frac{1}{14}$	2	2	= $\frac{1}{8}$
0	8	= $\frac{1}{14}$	4	= $\frac{1}{14}$			2	0	= $\frac{1}{10}$
0	7	= $\frac{1}{16}$					1	1	= $\frac{1}{16}$
0	4	= $\frac{1}{28}$					1	0	= $\frac{1}{20}$

Another Table of aliquot Parts of Money.

Parts of a shill.

d.		s.
10	=	$\frac{5}{6}$
9	=	$\frac{3}{4}$
8	=	$\frac{2}{3}$
7½	=	$\frac{5}{8}$
4½	=	$\frac{3}{8}$

Parts of a Pound.

s.	d.		£
18	0	=	$\frac{9}{10}$
17	6	=	$\frac{7}{8}$
16	8	=	$\frac{5}{6}$
16	0	=	$\frac{8}{10}$
15	0	=	$\frac{3}{4}$
14	0	=	$\frac{7}{10}$
13	4	=	$\frac{2}{3}$
12	6	=	$\frac{5}{8}$
12	0	=	$\frac{6}{10}$
8	0	=	$\frac{4}{10}$
7	6	=	$\frac{3}{8}$
6	0	=	$\frac{3}{10}$

Parts of a Dollar.

c.		D.
93¾	=	$\frac{15}{16}$
91⅔	=	$\frac{11}{12}$
90	=	$\frac{9}{10}$
87½	=	$\frac{7}{8}$
83⅓	=	$\frac{5}{6}$
81¼	=	$\frac{13}{16}$
80	=	$\frac{4}{5}$
75	=	$\frac{3}{4}$
70	=	$\frac{7}{10}$
68¾	=	$\frac{11}{16}$
66⅔	=	$\frac{2}{3}$
60	=	$\frac{3}{5}$
58⅓	=	$\frac{7}{12}$
56¼	=	$\frac{9}{16}$
43¾	=	$\frac{7}{16}$
41⅔	=	$\frac{5}{12}$
40	=	$\frac{2}{5}$
37½	=	$\frac{3}{8}$
31¼	=	$\frac{5}{16}$
30	=	$\frac{3}{10}$
18¾	=	$\frac{3}{16}$

A TABLE OF DISCOUNT PER CENT.

£		s.	d.	
1¼	per cent =	0	3	On the Pound.
2½	——— =	0	6	
3¾	——— =	0	9	
5	——— =	1	0	
6¼	——— =	1	3	
7½	——— =	1	6	

£		s.	d.	
8¾	per cent. =	1	9	On the Pound.
10	——— =	2	0	
12½	——— =	2	6	
15	——— =	3	0	
17½	——— =	3	6	
20	——— =	4	0	

£		s.	d.	
22½	per cent. =	4	6	On the Pound.
25	——— =	5	0	
30	——— =	6	0	
35	——— =	7	0	
40	——— =	8	0	
45	——— =	9	0	
50	——— =	10	0	

EXAMPLES.

1. What will 354½ yards cost, at ¼d. per yard?

```
         s.  d.
|¼d.|1/48|354 6 value of 354½ yards, at 1s. per yard.

Ans.  £0 7 4½ value of 354½ yard, at ¼d. per yard.
```

```
 Or thus.          Or divide by 8 and 6, thus, 8)354 6
 £  s. d.   s. d.
8)17 14 6=354 6                               6) 44 3¾

6) 2   4 3¾                                    7 4½ Ans. as bef.

   7. 4½ Ans. as before.
```

2. What will 759¾ yards come to, at 3d. per yard?

```
3d.|¼|759 9 value at 1s. per yard.
                                   £  s.  d.
 2|0)18|9 11¼     Or thus, |3d.|¼|37  19   9 value at 1s. per yard.

Ans. £9 9 11¼ value at 3d.  Ans.  £9   9 11¼ value of 759¾yds. at
              per yard                         3d. per yard.
```

3. What is the cost of 227yds. at 50 cents per yard?

```
c.  $  $
50|½|227=price at $1 per yard.

   $113 50c. Ans.
```

4. What cost 927yds. at 53⅓ cents a yard?

```
 c    $     $
50  | ½  |927=price at $1 per yard.
 3⅓ |1/30|
    |    |463 50=price at 50 cents.
    |    | 30 90= do. at 3⅓ cents.

      $494 40c. Ans.
```

Questions.				Answers.			Quest.		Ans.		
yds.				£			yds.	cts.	$	c.	m.
5. 918¼	at	½d.	per yard	1	18	3⅛	13. 265	at 12½	33	12	5
6. 739½	-	1 d.	- -	3	1	7½	14. 269½	- 16⅔	44	91	7
7. 567½	-	1½d.	- -	3	10	11¼	15. 1050	- 6¼	65	62	5
8. 475¼	-	2 d.	- -	5	14	3½	16. 618	- 87½	540	75	0
9. 487½	-	5 d.	- -	10	3	1½	17. 328	- 57½	188	60	0
10. 568	-	7½d.	- -	17	15	0	18. 817	- 30	245	10	0
11. 649¼	-	10 d.	- -	27	1	0½	19. 296	- 15	44	40	0
12. 164	-	11¾d.	- -	8	0	7	20. 300½	- 17½	52	58	7½

21. 758½yds. at 1s. 9d. per yard.

```
d.   £  s. d.
6|½|37 18 6  = value at 1s. per yard.
3|¼|18 19 3  =   do.  at 6d.
 |  | 9  9 7½=   do.  at 3d.

  £66  7 4½ Ans,
```

22. 106yds. at 4s. $9\frac{1}{2}$d.

4s.	$\frac{1}{5}$	106	= value at £1 per yard.
8d.	$\frac{1}{6}$ of 4.	21 :: 4	=value at 4s.
1	$\frac{1}{8}$	3 :: 10 :: 8	- - 8d.
$\frac{1}{2}$	$\frac{1}{2}$	0 :: 8 :: 10	- - 1d.
		0 :: 4 :: 5	- - $\frac{1}{2}$d.
		£25 :: 7 :: 11 :: 2 Ans.	

		s. d.	£ s. d.
23.	$17\frac{1}{2}$lbs. at	4 :: $0\frac{1}{2}$	3 :: 10 :: $8\frac{3}{4}$ Ans.
24.	$57\frac{1}{2}$ -	1 :: $5\frac{3}{4}$	2 :: 5 :: $0\frac{5}{8}$
25.	$68\frac{1}{4}$ -	8 :: $1\frac{1}{2}$	27 :: 11 :: $8\frac{1}{4}$
26.	$67\frac{1}{2}$ -	12 :: 2	41 :: 1 :: 3
27.	614 -	16	491 :: 4 :: 0
28.	$167\frac{1}{2}$ -	19 :: 6	163 :: 6 :: 3

Note. *If there be pounds also in the price :* Multiply the quantity by the pounds, and to the product add the value of the quantity for the other parts of the price as in the preceding examples, and the sum will be the answer.

29. Find the value of 156yds. of cloth at £3 6s. 8d. per yard.

	£	
6s. 8d $=\frac{1}{3}$	156	=value at £1 a yard.
	3	
	468	=value at £3 a yd.
	52	- - 6s. 8d. a yd.
	£520 Ans.	

30. What is the cost of $224\frac{1}{2}$yds. at £5 7s. 6d. a yard ?

	£ s.	
5s. $=\frac{1}{4}$	224 :: 10	=cost at £1 a yard.
2s. 6d. $=\frac{1}{2}$	5	
	1122 :: 10	= - - £5 a yd.
	56 :: 2 :: 6d.	- - 5s. a yd.
	28 :: 1 :: 3	- - 2s. 6d. do.
	£1206 :: 13 :: 9 Ans.	

		£ s. d.	Answers. £ s. d.
31.	$345\frac{1}{2}$yds. at	6 :: 5 :: 0 per yard=	2159 :: 7 :: 6
32.	$59\frac{3}{4}$ - -	3 :: 6 :: 8 - -	199 :: 3 :: 4
33.	75 - -	5 :: 3 :: 4 - -	387 :: 10 :: 0
34.	68 -	4 :: 6 :: 0 - -	292 :: 8 :: 0

General Rule II.

When the price of one hundred weight, &c. is given of several denominations, to find the value of a quantity of several denominations also.

Multiply the price by the integers, and take parts for the rest from the price of an integer, and the sum of the product and quotients will be the answer.

Examples.

1. If 1 Cwt. cost £4 17s. 4d what will 9 Cwt. 3qrs. 14lb. cost at the same rate?

2qrs. 0℔ is $\frac{1}{2}$	4	17	4	=price of 1 Cwt.	
1qr. 0 $\frac{1}{2}$			9	=Number of Cwt.	
0 14 $\frac{1}{2}$					
	43	16	0	=cost of 9 Cwt.	
	2	8	8	=	0 2qrs.
	1	4	4	=	0 1qr.
	0	12	2	=	0 0 14℔
Ans.	£48	1	2	= Cwt. 9 3 14℔	

	Cwt.	qr.	℔		£	s.	d.		£	s.	d.
Ex. 2.	8	1	16	Sugar at	5	17	9	per Cwt. =	49	8	2$\frac{1}{4}$
3.	7	3	19	-	7	12	8	- -	60	9	0$\frac{1}{2}$
4.	12	1	24	-	3	18	10	- -	49	2	7$\frac{1}{4}$
5.	72	3	27	-	8	11	5	- -	625	11	10$\frac{3}{4}$
6.	16	2	17	Coffee	2	15	11	- -	46	11	1
7.	27℔ 10oz.				0	1	4	per ℔	1	16	10

8. 13 10 12pwt. 8grs. silver at £4 7s. 6d. a ℔ 60 14 11$\frac{1}{2}$
9. 17oz. 6pwt. 16grs. of Gold £3 16 8d. per oz. 66 8 10$\frac{2}{3}$
10. 3 Tuns 2hhd. 48gall. of wine at £5 16 9=
11. 7 Acres 3 roods 15 rods 16 feet at £7 11s. 9d. an acre=

Though the General Rules, given above, are sufficient for answering questions in Practice, yet some may perhaps be answered more easily by other rules. Several cases follow.

CASE I.

When the price is any even number of shillings under 24: Multiply the given quantity by half the price, and double the first figure of the product for shillings. The rest of the product will be pounds.*

N. B. If the price be 2s. you need only double the unit figure for shillings. The other figures will be pounds.

Examples.

1st. What will 746 yards cost at 2s. per yard?

746

Ans. £74 12 value at 2s. per yard.

* Suppose the price to be 16s.; as the quantity is the price at £1, the answer will be $\frac{16}{20}=\frac{8}{10}$ of the quantity, in pounds and decimals. The decimal is to be doubled for its value in shillings, according to a previous rule. The rule is limited to 24s. because the half of a greater number, would be an inconvenient multiplier. The reason of the rules in the ten following cases is sufficiently obvious.

Note. The above is done, by saying twice 6 (the unit figure) is 12. The other figures, viz. 74, are pounds.

2d. What will 567¾yds. at 2s. per yard come to?

Ans. £56 15s. 6d.

N. B. Before I double the unit figure, viz. 7, I consider that ¾ of a yard at 2s. per yard, will amount to 1s 6d. Then I double 7, which makes 14s. and 1s. 6d. added, makes 15s. 6d. The other figures are pounds.

Questions					Answers		
	Yds				£	s	d
3d.	129½	at	4s.	per yard	25	18	0
4th.	697	—	6s.	———	209	2	0
5th.	845	—	8s.	———	338	0	0
6th.	917¼	—	10s.	———	458	12	6

CASE II.

When the price wants an even part of 2s.: First find the value of the whole quantity at 2s. per pound, yard, &c. then divide it by *that even part* which is wanting, and subtract this quotient from the value at 2s. The remainder will be the answer.

Examples.

1. What will 95½ yards cost at 22d. per yard?

		£	s.	d.	
2d.	$\frac{1}{12}$	9	11	0	value at 2s. per yard.
		0	15	11	value at 2d. per yard.
	Ans.	£8	15	1	value at 1s. 10d. per yard.

Questions.					Answers.		
	yds.				£	s.	d.
2d.	64	at	23d.	per yd.	6	2	8
3d.	128	—	22½d.	———	12	0	0
4th.	246½	—	21d.	———	21	11	4½
5th.	375¼	—	20d.	———	31	5	5

CASE III.

When there are pence in the price which are an even part of a shilling, besides an even number of shillings under 20: First find the value of the quantity at the shillings per yard, &c. according to Case 1st.: then suppose the quantity to stand as shillings per yard; divide it by *that even part*, which the pence are of 1s. and this quotient being added to the value before found, the sum will be the answer.

Examples.

1st. What will 156½ yards come to, at 6s. 4d. per yard?

Yds.
156½
3

		s.	d.			
4d.	⅓	156	6			
				£46 19 0	value of 156½ yards at 6s. per yard.
		52s.	2d.	=2 12 2	value of ditto at 4d. per yard.
			Ans.	£49 11 2	value of ditto at 6s. 4d. per yard.

Questions.					Answers.		
	Yds.		s.	d.	£	s.	d.
2d.	17½ at		4	0½ per yd.	3	10	8¾.
3d.	59¾	—	6	0¼ ———	18	2	2¼.
4th.	68¼	—	8	1 ———	27	11	8¼.
5th	96	—	10	1½ ———	48	12	0.
6th.	67½	—	12	2 ———	41	1	3.

CASE IV.

When the price is any odd number of shillings under 20 *:* Find the value of the greatest even number contained in the price, according to Case 1st. and add thereto the value of the quantity at 1s. per yard, &c. which sum will be the answer: Or, Multiply the quantity by the price, according to the 1st or 2d Case in Simple Multiplication, and divide the product by 20, the quotient will be the answer: Or, lastly, if the price be not more than 12s. find the value of the quantity at 1s. per yard, &c. and multiply it by the number of shillings in the price of 1 yard; the product will be the answer.

Examples.

1st. What will 186 yards cost, at 3s. per yard?

£	s.	
18	12	value at 2s. per yard.
9	6	ditto at 1s. per yard.
£27	18	Ans.

Or thus.

	£	s.	
	9	6	value at 1s. per yard.
		3	
Product	£27	18	Ans.

2d. What will 647 yards cost, at 17s. per yard?
8

	£517	12	value at 16s. per yard.
	32	7	ditto at 1s. per yard.
Ans.	£549	19	ditto at 17s. per yard.

Questions.				Answers.		
	Yds.	s.		£	s.	d.
3d.	169¼ at	5	per yd.	42	6	3
4th.	248¾ —	7	———	37	1	3
5th.	139 —	9	———	62	11	0
6th.	782 —	25	———	977	10	0

CASE V.

When the price is shillings, pence and farthings; and not an even part of a pound: Multiply the given quantity by the shillings in the price of 1 yard, &c. and take parts of parts from the quantity for the pence, &c. then add them together, and their sum will be the answer, in shillings, &c. Or you may let the given quantity stand as pounds per yard, &c. then draw a line underneath, and take parts of parts therefrom; which add together, and their sum will be the answer.

N. B. I advise the learner to work the following examples both ways, by which means he will be able to discover the most concise method of performing such questions in business as may fall under this case.

EXAMPLES.

1. What will 248½ yards, at 7s. 6d. per yard, come to?

|6|½|248s. 6d. value of 248½ yards, at 1s. per yard.
7

1739 6 value of ditto at 7s. per yard.
124 3 value of ditto at 6d. per yard.

2|0)186|3 9

Ans. £93 3 9 value of ditto at 7s. 6d. per yard.

Or thus,

|6|½|12 8 6 value of 248½ yards, at 1s. per yard.
Multiply by 7

86 19 6 value of ditto at 7s. per yard.
6 4 3 value of ditto at 6d. per yard.

Ans. £93 3 9

By the latter part of this case,

		£	s.	d.	
5 0	¼	248	10	0	value of 248½ yards, at £1 per yard.
2 6	½	62	2	6	value of ditto at 5s. per yard.
		31	1	3	value of ditto at 2s. 6d. per yard.
	Ans.	£93	3	9	value of ditto at 7s. 6d. per yard.

Questions.				Answers.		
Yds.		s.	d.	£	s.	d.
2. 68½	at	4	6 per yd.	15	8	3
3. 124	—	5	8 ———	35	2	8
4. 146	—	14	9 ———	107	13	6
5. 218½	—	12	6 ———	136	11	3

CASE VI.

When the quantity is any number less than 1000, *and the price not more than* 12d. *per yard, &c.:* Find the value of the whole quan-

tity at 1d. per yard, which may be done by dividing it by 12, *mentally*, setting down the quotient only in pounds, or shillings, or both. Then multiply this sum by the pence in the price of 1 yard, and the product will be the answer.

EXAMPLES.

1. What will 759½ yards cost, at 7d. per yard?

	s.	d.	
	63	3½	value at 1d. per yard.
Or, £3	3	3½	value at 1d. per yard.
Multiply by		7	
Ans. £22	3	0½	value of 759½ yards, at 7d. per yard.

Questions.					Answers.		
	Yds.		d.		£	s.	d.
2.	975¼	at	2	per yard.	8	2	7
3.	846	—	3½	———	12	6	9

CASE VII.

When the price is at any of the rates in the second Practice Table of aliquot parts: Multiply the given quantity by the numerator, and divide that product by the denominator; if the price be pence, the quotient will be the answer in shillings; if shillings, the answer will be pounds.

EXAMPLES.

1. What will 379 yards, at 4½d. per yard come to?

```
      379
        3
     ----
   8)1137
   ------
 2|0)14|2  1½
 ---------
 Ans. £7 2 1½
```

2. What will 149 yards, at 6s. per yard come to?

```
       149
         3
      ----
   1|0)44|7
   --------
 Ans. £44 14
```

Questions.						Answers.		
	Yds.		s.	d.		£	s.	d.
3.	127	at	0	7½	per yard.	3	19	4½
4.	159	—	0	8	———	5	6	0
5.	173	—	0	9	———	6	9	9
6.	241	—	0	10	———	10	0	10
7.	249	—	7	6	———	93	7	6
8.	357	—	12	6	———	223	2	6

CASE VIII.

When the price is any even number of shillings, if it be required to know what quantity of any thing may be bought for so much money: Annex a cypher to the money, and divide it by half the price, and the quotient will be the quantity to be purchased.

EXAMPLES.

1. How many yards of cloth, at 18s. per yard, may I have for £345?

Half the price=9)3450=money with a cypher annexed.

383⅓ yards, Ans.*

Questions.	s.	£	Answers. Yds.
2. How many yds. at	2 per yd. for	427?	4270
3.	4	312	1560
4.	6	917	3056⅔
5.	8	195	487½

CASE IX.

To find the value of goods sold by particular quantities, viz. I. By the score. II. Round timber. III. By 5 score to the hundred. IV. By 112 to the hundred. V. By 6 score to the hundred. VI. By the great gross. VII. By the thousand.

I. *To find the value of goods sold by the score.*

The price of one is given, to find the price of one score.

If the given price be shillings and pence, or only pence, divide the given price, in pence, by 12. The quotient will be the answer in pounds, and the remainder will be so many times 1s. 8d. for a score is 20, and 20d. is 1s. 8d.

EXAMPLES.

1. At 9d. each: What is that per score?

12)9d.(·75=£0 15 0 Ans.

Or by inverting the question.

¼ score=20=1s. 8d.
9

15s.0

2. At 4s. 9d. each: What is that per score?

£4 15 Ans.

It may be remarked, that when the price is shillings and pence, the answer will be just so many pounds as there are shillings, and so many times 1s. 8d. as there are pence. If farthings are given, for ¼d. reckon 5d. for ½d. 10d. and for ¾d. 1s. 3d.

TABLE OF ALIQUOT PARTS. 20 THE INTEGER.

2 is $\frac{1}{10}$	6 is $\frac{3}{10}$	12 is $\frac{6}{10}$	16 is $\frac{8}{10}$
4 — $\frac{1}{5}$	8 — $\frac{4}{10}$	14 — $\frac{7}{10}$	18 — $\frac{9}{10}$
5 — $\frac{1}{4}$	10 — $\frac{1}{2}$	15 — $\frac{3}{4}$	

* At £1 a yard, the quantity would be 345yds. But as the price is 18s. you want only $\frac{18}{20}=\frac{9}{10}$, of the quantity. This explains the rule; for the course is similar for any other case.

3. What cost 7; at 2s. 9d. per score?

		s.	d.
5	$\frac{1}{4}$	2	9
2	$\frac{1}{10}$		
		0	$8\frac{1}{4}$
		0	$3\frac{1}{4}$
7	=	0	$11\frac{1}{2}$

4. What cost 17; at 19s. 10d. per score?
16s. $10\frac{1}{4}$d. Ans.

II. *Round Timber.*

Forty feet make a load or ton of round timber.
If the given price of a foot be shillings,

Rule.

Multiply the given price by 2, and the product will be the answer in pounds.

5. What cost a ton at 3s. per foot? 3s.×2=6l. Ans.*

6. What cost a ton at 9s. per foot? 9s.×2=18l. Ans.

If the given price of 1 foot be pence only, or shillings and pence, divide the given price, in pence, by 6. The quotient will be the answer in pounds, and the remainder will be so many times 3s. 4d.

7th. What cost 40 feet, at 17d. per foot?

6)17
£2 16 8 Ans.

8th. At 1s. 9d. per foot: What cost a ton?
£3 10 Ans.

If the given price of a foot be farthings only, or pence and farthings, divide the given price in farthings, by 6; then divide *that quotient* by 4, and *this last quotient* will be the answer.

9th. At $\frac{3}{4}$d. per foot: What cost a ton?

6)3
4)0 10
£0 2 6 Ans.

10th. At $13\frac{1}{4}$ per foot: What cost a ton?
£2 4 2 Ans.

* If a ton of timber had been 20 feet, then 3s. a foot would have made £3, but as a ton has twice 20 feet, the answer in pounds must be twice the number of shillings. In like manner for any number of shillings. If the price be pence a foot, as in Ex. 7, the rule is $\frac{40\times17}{12}=\frac{20\times17}{6}=\frac{17}{6}$ for pounds, with a remainder of $5\times\frac{20s.}{6}$=5 times 3s. 4d.

When the price is farthings a foot, the rule is as in Ex. 9, $\frac{40\times3}{12\times4}$ for shillings, $=\frac{20+3}{6\times4}=\frac{3}{6\times4}$ for pounds.

Or, suppose every shilling in the price to be 2l. every penny to be 3s. 4d. and every farthing to be 10d.

11th. What cost 40 feet at $\frac{3}{4}$d. per foot?

$\frac{3}{4}$d. × 10 £0 2 6 Ans.

12th. What cost 40 at 15½d. per foot?

s.	d.				
1	0 × 2=	£2	0	0
3	4 × 3=	0	10	0
0	½ × 10=	0	1	8
		£2	11	8

III.* *To find the value of goods sold by 5 score to the hundred.*

1st; If the given price be pounds and shillings, or shillings only.

RULE.

Multiply the given price in shillings, by 5, and the quotient will be the answer in pounds, for 100s. are £5.

13th. At 19s. per yard, what cost 100 yards?

19s.
5
——
£95 Ans.

14th. At 4l. 13s. per cwt. what cost 100 cwt. or 5 tons?

£465 Ans.

2d. If the given price of 1 be pence only, or shillings and pence.

RULE.

Multiply the given price in pence, by 5; then divide that product by 12. The quotient will be pounds; and the remainder so many times 1s. 8d.

15th. If 1 yard cost 9d. what cost 100 yards?

9
5
——
12)45
——
£3 15 Ans.

16th. What cost 100 bushels, at 35s. 4d. per bushel?

s. d.
35 4
12
——
424
5
——
12)2120
——
£176 13 4 Ans.

Or,

35s. 4d.
|4d.|$\frac{1}{5}$| 5
——
175
1 13 4
——
£176 13 4
——
Here 5 is divided by $\frac{1}{5}$.

* *In Federal Money.*—Remove the decimal point two places to the right for the answer.

EXAMPLES.

1. What cost 100 yards at $2 50c. per yard? $2·50×100=$250· Ans.
2. What cost 100 yards at 75c. per yard? $·75×100=$75· Ans.
3. What cost 100 yards at 5c. 6¼m. per yard? $·05625×100=$5·625 Ans.
4. What cost 100 yards at 37c. 5m. per yard? Ans. $37 50c.
5. What cost 100 yards at 68c. 7½m. per yard? Ans. $68 75c.

3. If the given price of 1 be shillings and pence; Multiply the price by 5, and the product under the place of shillings, will be the answer in pounds, and the product under the place of pence, will be so many times 1s. 8d.

17th. At 2s. 5d. per bushel: what cost 100 bushels?

s.	d.
2	5
	5
12	1

£12 1 8 Ans.

18th. At 25s 3d. per ton: what cost 100 tons?

£126 5 Ans.

4.* To find the price of *one* at so much per hundred of 5 score.

GENERAL RULE.

Multiply the given price by 12; divide the product by 5, and the quotient will be the answer in pence.

But if the price be pounds only:

RULE.

Divide the given price by 5, and the quotient will be the answer in shillings.

19th. If 100yds. cost £65, what cost 1yd?

5)65

13s. Ans.

20th. If 100yds. cost £2 18s. 4d. what is that per yard?

£	s.	d.
2	18	4
		12
5)35	0	0

7d. Answer.

21st. If 100 yards cost £11 7s. 9d. what cost 1 yard.

£	s.	d.
11	7	9
		12
5)136	13	0
12)27	6	7

2s. 3¼d. Ans.

In dividing 27 by 12 (in the 21st question) the quotient is 2s. and the remainder 3d. the 6 is $\frac{6}{20}$ of a penny = one farthing, and the 7 is of no account,

TABLE OF ALIQUOT PARTS. 100 THE INTEGER.

5 is	$\frac{1}{20}$	25 is	$\frac{1}{4}$	50 is	$\frac{1}{2}$	75 is	$\frac{3}{4}$
10 ——	$\frac{1}{10}$	30 ——	$\frac{3}{10}$	60 ——	$\frac{6}{10}$	80 ——	$\frac{8}{10}$
20 ——	$\frac{1}{5}$	40 ——	$\frac{4}{10}$	70 ——	$\frac{7}{10}$	90 ——	$\frac{9}{10}$

* *In Federal Money.*—Remove the decimal point two places to the left for the answer.

EXAMPLES.

1. If 100 yards cost \$250, what cost 1 yard? \$250÷100=\$2·50 Ans.
2. If 100 yards cost \$75· what cost 1 yard? \$75·÷100=\$·75 Ans.
3. If 100 yards cost \$5 62c. 5m. what cost 1 yard? \$5·625÷100=\$·05625=5c. 6¼m. Ans.
4. If 100 yards cost \$37 50c. what cost 1 yard? Ans. 37c. 5m.
5. If 100 yards cost \$68 75c. what cost 1 yard? Ans. 68c. 7½m.

22d.* At £3 7s. 6d. per 100: What will 23 cost?

		£	s.	d.	
20	$\frac{1}{5}$	3	7	6	
		0	13	6	Add.
2	$\frac{1}{10}$	0	1	4	
1	$\frac{1}{2}$	0	0	8	
23=		£0	15	6	Ans.

23d. At £2 1s. 10d. per 100: What cost 18?

		£	s.	d.	
20	$\frac{1}{5}$	2	1	10	
		0	8	$4\frac{2}{5}$	Sub.
2	$\frac{1}{10}$	0	0	10	
18=		£0	7	$6\frac{2}{5}$	Ans.

24th. At £5 9s. 6d. per 100: What cost 35?

		£	s.	d.	
		5	9	6	
				3	
	10)	16	8	6	
5	$\frac{1}{6}$	1	12	10	Add.
			5	$5\frac{2}{3}$	
		£1	18	$3\frac{2}{3}$	Ans.

IV. *To find the value of goods, sold by 112lb the Cwt.*

The price of 1lb is given to find the value of 1 cwt.

RULE.

For a farthing, account 2s. 4d. per cwt. For a half a penny, 4s. 8d. For three farthings, 7s. And for every penny 9s. 4d. per cwt.

25th. What cost 1cwt. at $3\frac{1}{2}$d. per lb?

	£	s.	d.	
At 1d. per lb		s.	d.	
1cwt. costs		9	4	
			3	
At 3d.	£1	8	0	Add.
At $\frac{1}{2}$d.	0	4	8	
	£1	12	8	Ans.

26th. At $8\frac{3}{4}$d. per lb: What cost 1 cwt? Ans. £4 1s. 8d.

* *To find the value of any number at a given price per 100, in federal money.*—Multiply the price per 100 by the given quantity, and point off two right hand figures, in the product more than required by multiplication of decimals. Or, point off the two right hand places in the given quantity, and multiply, and point, as in multiplication of decimals.

EXAMPLES.

1. What cost 56 yards at \$87 50c. per 100 yards?

$$\frac{\$87{\cdot}5\times56}{100}=\$49, \text{ Ans.}\quad \text{Or, } \$87{\cdot}5\times{\cdot}56=\$49, \text{ as before.}$$

V. *To find the value of goods sold by 6 score to the hundred.*

The price of 1 is given to find the price of 1 hundred.

RULE.

Suppose every penny in the price to be so many pounds, and for the farthings, such a part of a pound, as they are of a penny; then, half of that sum will be the answer.

27th. At 4½d. per yard: What cost 120 yards?

$$\begin{array}{rr} \pounds & s. \\ 2)4 & 10 \\ \hline \pounds 2 & 5 \text{ Ans.} \end{array}$$

28th. At 16s. 9¼d. per yard: What cost 120 yards.
Ans. £100 12s. 6d.

To find the price of *one*, at so much per hundred of 6 score.

RULE.

Multiply the price by 2, then call the pounds so many pence, and the shillings, such a part of a penny, as they are of a pound, and you will have the answer.

29th. If 120 yards cost £3 12s.: What cost 1 yard?

$$\begin{array}{rr} \pounds & s. \\ 3 & 12 \\ & 2 \\ \hline 7 & 4 \\ \hline \end{array}$$

Ans. 7⅕d.

30th. If 120 yards cost £5 18s. 6d.: What cost 1 yard?
Ans. 11¾d.+⅖ of farthing.

TABLE OF ALIQUOT PARTS. 120 THE INTEGER.

Also,

6 is $\frac{1}{20}$	24 is $\frac{1}{5}$	36 is $\frac{3}{10}$	72 is $\frac{3}{5}$	96 is $\frac{4}{5}$
10 — $\frac{1}{12}$	30 is $\frac{1}{4}$	45 — $\frac{3}{8}$	75 — $\frac{5}{8}$	100 — $\frac{5}{6}$
12 — $\frac{1}{10}$	40 is $\frac{1}{3}$	48 — $\frac{2}{5}$	80 — $\frac{2}{3}$	105 — $\frac{7}{8}$
15 — $\frac{1}{8}$	60 is $\frac{1}{2}$	50 — $\frac{5}{12}$	84 — $\frac{7}{10}$	108 — $\frac{9}{10}$
20 — $\frac{1}{6}$		70 — $\frac{7}{12}$	90 — $\frac{3}{4}$	110 — $\frac{11}{12}$

31st. At £3 17s. 6d. per hundred, what cost 14?

$$\begin{array}{r|r|rrr} & & \pounds & s. & d. \\ 12 & \frac{1}{10} & 3 & 17 & 6 \\ \hline & & 0 & 7 & 9 \\ 2 & \frac{1}{6} & 0 & 1 & 3\frac{1}{2} \\ \hline 14 & = & \pounds 0 & 9 & 0\frac{1}{2} \text{ Ans.} \end{array}$$

(0 7 9 and 0 1 3½: Add.)

2. What cost 45½lb. beef at $5½ per 100?

$$\frac{\$5{\cdot}5\times45{\cdot}5}{100}=\$2{\cdot}5025=\$2\ 50c.\ 2\tfrac{1}{2}m.\ \text{Ans. Or, } \$5{\cdot}5\times{\cdot}455lb.=\$2{\cdot}5025, \text{ as before.}$$

3. What cost 375 yards at $375 per 100 yards? Ans. $1404 25c.
4. What cost 54 yards at $16 per 100? Ans. $8 64c.
5. What cost 512 yards at $6 25c. per 100 yards? Ans. $32.

32. At £2 13s. 6½d. per hundred, what cost 49?

		£ s. d.
40	⅓	2 13 6½
8	⅕	0 17 10 0⅔
1	⅛	0 3 6 3⅓
		0 0 5 1

49 = £1 1 10 1 Ans.

33. At £1 19s. 3d. per hundred, what cost 75?

Ans. £1 4s. 6¼d.

VI. *To find the value of goods sold by the great gross.*

Note. 12 make 1 dozen, 12 dozen 1 small gross, 12 small gross 1 great gross.

The price of 1 dozen being given, in pence, to find the price of a great gross.

Rule.

Multiply the price of 1 dozen, in pence, by 3, then divide that product by 5, and the quotient will be the answer in pounds, &c.

For proof, do the contrary.

N. B. If the price of 1 be given, the price of 1 small gross is found after the same manner.

34. What cost 1 great gross, at 18d. per dozen?

```
  3
 ---
5)54
 ------
£10 16
```

35. At 4s. 3d. per dozen, what cost 1 great gross?

```
    4s. 3d.          Or,              Or,
    12              s. d.             £ s.
    ---              4 3           144=7 4
    51d.             12                  4
     3             -----          ---------
    ---            2 11 0    |3d.|¼|28 16 } Add.
  5)153               12               1 16 }
  -------          ------         ---------
Ans. £30 12        £30 12 0        £30 12
```

TABLE OF ALIQUOT PARTS. 144 THE INTEGER.

Also,

12 is $\frac{1}{12}$	36 is ¼	32 is $\frac{2}{9}$	84 is $\frac{7}{12}$	128 is $\frac{8}{9}$
16 — $\frac{1}{9}$	48 — ⅓	60 — $\frac{5}{12}$	96 — ⅔	132 — $\frac{11}{12}$
18 — ⅛	72 — ½	64 — $\frac{4}{9}$	108 — ¾	
24 — ⅙		80 — $\frac{5}{9}$	120 — ⅚	

36. At £2 12s. 9d. per great gross, what cost 45 dozen.

Doz.		£ s. d.	
36	$\frac{1}{4}$	2 12 9	
9	$\frac{1}{4}$	0 13 $2\frac{1}{4}$	Add.
		0 3 $3\frac{1}{2}$	
45	=	£0 16 $5\frac{3}{4}$ Ans.	

37. What cost 117 dozen, at £9 13s. 7d. per great gross?

Doz.		£ s. d.	
		9 13 7	
		3	
108	$\frac{3}{4}$	29 0 9	
		7 5 $2\frac{1}{4}$	Add.
9	$\frac{1}{12}$	12 1	
117	=	£7 17 $3\frac{1}{4}$ Ans.	

38. At £3 16s. 8d. per great gross, what cost 7 great gross and 96 dozen?

	£ s. d.
	3 16 8
	2
	3)7 13 4
Doz. 96=	2 11 $1\frac{1}{4}$
Top line ×7=	26 16 8
	£29 7 $9\frac{1}{4}$

VII.* *To find the value of goods sold by the thousand.*

The price of 1 is given to find the price of 1000.

RULE.†

Multiply the given price in pence, by 50, then divide the product by 12, and the quotient will be the answer in pounds, &c.

39. At 6d. each; what cost 1000?

6
50
12)300
£25 Ans.

Or, as 1000s. are £50, take parts, for the pence out of 50.

|6d.|$\frac{1}{2}$|50
Ans. 25

40. What cost 1000, at $2\frac{1}{4}$d. each? Ans. £9 7s. 6d.

* *In Federal Money.*—Remove the decimal point three places to the right, or left, as the case requires, for the answer.

EXAMPLES.

1. What cost 1000 yards at 5 cents per yard? ·05×1000=050·=$50 Ans.
2. What cost 1000 yards at 12 cents 5 mills per yard? Ans. $125.
3. If 1000 yards cost $37 50c. what cost 1 yard. $37·5÷1000=$·0375=3c. $7\frac{1}{2}$m. or, $3\frac{3}{4}$c. Ans.
4. If 1000 yards cost $1625, what cost 1 yard? Ans. $1 62c. 5m.

† The operation by the Rule of Three would be thus; in the first example, as, 1 : 6d. :: 1000 : 6×1000 the answer in pence; and $\frac{6\times1000}{12\times20}$=answer in pounds, $=\frac{6\times50}{12}$ pounds, and is the rule.

VIII. *To find the price of one at so much per thousand.*

RULE.

Multiply the price by 12; divide the product by 50; then take the pounds for so many pence, and the shillings for such a part of a penny as they are of a pound, which will be the answer.

41. At £5 4s. 2d. per 1000, what cost 1?

	£	s.	d.
	5	4	2
			12
50 {	5)62	10	0
	10)12	10	
	£1	5	

Ans. $1\frac{1}{4}$d.

42. At £354 3s. 4d. per 1000, what cost 1?

	£	s.	d.
	354	3	4
			12
50 {	10)4250	0	0
	5)425		
	85		

Ans. 7s. 1d.

Or,

		£	s.	d.
100	$\frac{1}{10}$	354	3	4
10	$\frac{1}{10}$	35	8	4
1	$\frac{1}{10}$	3	10	10
	Ans.	0	7	1

TABLE OF ALIQUOT PARTS. 1000 THE INTEGER.

50 is $\frac{1}{20}$	200 is $\frac{1}{5}$	300 is $\frac{3}{10}$	700 is $\frac{7}{10}$
100 — $\frac{1}{10}$	250 — $\frac{1}{4}$	375 — $\frac{3}{8}$	750 — $\frac{3}{4}$
125 — $\frac{1}{8}$	500 — $\frac{1}{2}$	400 — $\frac{2}{5}$	800 — $\frac{4}{5}$
		600 — $\frac{3}{5}$	875 — $\frac{7}{8}$
		625 — $\frac{5}{8}$	900 — $\frac{9}{10}$

43. *At £1 17s. 9d. per 1000 what cost 115?

		£	s.	d.	
100	$\frac{1}{10}$	1	17	9	
10	$\frac{1}{10}$	0	3	$9\frac{1}{4}$	Add.
5	$\frac{1}{2}$	0	0	$4\frac{1}{2}$	
		0	0	$2\frac{1}{4}$	
115=		£0	4	4	Ans.

* *To find the value of any number, at a given price per 1000 in federal money.*—Multiply the price per 1000 by the given quantity, and point off three right hand figures in the product more than required by multiplication of deci-

44th. At £2 1s. 8d. per 1000, what cost 875?

£	s.	d.
2	1	8
		7
8)14	11	8
£1	16	$5\frac{1}{2}$ Ans.

45th. What cost 33, at 24s. 8d. per 1000?

		£ s. d.	
50	$\frac{1}{20}$	1 4 8	
25	$\frac{1}{2}$	0 1 $2\frac{3}{4}$	
5	$\frac{1}{5}$	0 0 $7\frac{1}{4}$	Add.
		0 0 $1\frac{1}{4}$	
30	=	0 0 $8\frac{1}{2}$	Add.
3	$\frac{1}{10}$	0 0 $\frac{3}{4}$	
33	=	0 0 $9\frac{1}{4}$.	

CASE X.

When the price of 1 is any number of dollars and parts of a dollar: Multiply the quantity by the number of dollars; and, finding, by the general rule, the price at the parts of $1, the sum of the whole is the answer.

Examples.

1. What cost 395 yards at $3 24c. per yard?

c.		$ c.		
20	$\frac{1}{5}$$	395	= price at	$1
		3		
4	$\frac{1}{5}$ of 20c.	1185	= ditto at	3
		79	= ditto at	20c.
		15 80	= ditto at	4
	Ans.	$1279 80	= ditto at	$3 24c.

2. What cost 269 yards at $2 60c. per yard? Ans. $699 40c.
3. ——— 694 ——— 12 10 ——— 8397 40
4. ——— 318 ——— 4 $12\frac{1}{2}$ ——— 1311 75
5. ——— 175 ——— 4 44 ——— 777

CASE XI.

When the price of 1 contains the same aliquot part of a dollar any number of times exactly; or, in other words, when the price of 1 has an aliquot part, which is also an aliquot part of a dollar: First, find the value of the given quantity at the aliquot part; then multiply

mals. Or, point off the three right hand places in the given quantity: and multiply and point as in multiplication of decimals.

Examples.

1. What cost 875 at $13 per 1000?
875×13=11375; and 11375÷1000=11·375=$11 37c. 5m. Ans.
2. What cost 39175 feet of boards, at $16 per 1000? Ans. $626 80c.
3. What cost 325 nails at $1 50c. per 1000? Ans. 48c. $7\frac{1}{2}$m. or, $48\frac{3}{4}$c.

this by the number of times which the aliquot part is contained in the given sum, for the answer. Or,

Since the price in this case is always such a number, as, being divided by the aliquot part, will make the numerator of a fraction, of which the denominator is the denominator of that fraction, which the aliquot part is of a dollar; Multiply the quantity by the numerator, and divide the product by the denominator, (or, when convenient, divide the quantity by the denominator, and multiply the quotient by the numerator,) for the answer.*

EXAMPLES.

1. What cost 384 yards at 87½ cents per yard?

12½c. = ⅐ of ·875 = | $⅛ |	384·	= price at	$1
	48·	= ditto at	·12½
	×7		×7
	Ans. $336·	= ditto at	·87½

Or thus,

$\cdot875 = \$\frac{7}{8}$, and $384 \times \frac{7}{8} = \frac{384 \times 7}{8} (= \frac{384}{8} \times 7) = \$336$, Ans. as before.

2. What cost 842 yards at 66⅔c. per yard? Ans. $561 33⅓c.
3. What cost 912 yards at 55c. per yard? Ans. $501 60c.

MISCELLANEOUS QUESTIONS IN PRACTICE.

1. What cost 300 yards at 27c. per yard? Ans. $81

917	$1 12 5m.	1031 62c. 5m.
35⅕	35	12 32
862¼ ft. boards at $12 per M.?		10 34 6
32159	13 75c.	442 18 6¼

CASE XII.

I. Since 2s. is $\frac{1}{10}$ of £1, the decimal of 2s. is ·1: Wherefore any quantity being given at 2s. per lb. yard, &c. the price is found in pounds and decimal parts of a pound, by separating the unit figure of the given quantity from the rest, for a decimal.

Let it be required to find the value of 356 yards at 2s. per yard?

By pointing off the unit figure 6 for a decimal, I find the amount to be £35·6, which is known to be equal to 35l. 12s. } £35·6

II. Consequently, if the price be a multiple of 2s. (viz. any even number of shillings) the amount at 2s. being first found in pounds and decimal parts, as above, and that amount multiplied by the number which shows how often 2s. is contained in the given price, the product will be the amount required in pounds and decimal parts of a pound.

What cost 427 gallons of wine, at 8s. per gallon?

* Some of the prices which apply to this case, are to be found in the second table of parts of a dollar.

£42·7 amount at 2s. per gallon.
4

Ans. £170·8 or 170l. 16s.

The examples in Case 1st. may be worked in this manner. Likewise, if the price be pounds and even shillings.

754 yards at 1l. 8s.

75·4 amount at 2s.
14×2=28s.

3016
754

Ans. £1055·6=1055l. 12s.

Or,
754
75·4×4=301·6 } Add.

£1055·6

III. *If the price be an aliquot part of 2s.*: Find the amount at 2s. and divide it by the denominator of the part, and the quotient will be the answer.

At 8d. per lb.: What cost 976 lb.?

| 8d. | $\frac{1}{3}$ | 97·6

£32 533= £32 10 8 Ans.

IV. *If the price be not an aliquot part*: Divide it into aliquot parts.

7235 yards, at 7d.

| 4d. | $\frac{1}{6}$ | 723·5
| 3d. | $\frac{1}{8}$ | 120 583
90 437

211·02 = £211 0 4$\frac{3}{4}$ Ans.

V. *If the price be pounds and shillings, or pounds, shillings and pence*: Reduce the shillings, &c. to the decimal of a pound, and multiply the quantity thereby, or the price by the quantity.

At 15l. 12s. 6d. per Cwt: What cost 75 Cwt.?

£15 12 6 = £15·625
75

78125
109375

1171·875

£1171 17 6 Ans.

VI. *If the quantity likewise be of divers denominations*: Reduce the less denominations to the decimal of that, whereof the price is given.

9lb. 10oz. of silk, at £4 5 9 = £4·287
9lb. 10oz = 9·625

```
       21435
       8574
      25722
      38583
    ---------
    41·262375
    ---------
    £41  5  3  Ans.
```

Cases 6th. and 7th. may be wrought in this manner.
Or, you may take parts for the lower denominations.

```
|8oz.| ½ |   4·287
|2oz.| ¼ |       9
         ---------
          38·583
           2·1435
            ·535875
         ----------
          41·262375
         ----------
         £41  5  3
```

VII. When the price is any *odd* number of shillings: If it be required to know what quantity of any thing may be bought for any sum of money, in pounds: Annex *two* cyphers to the money, and divide it by half the price.

Note. As half a shilling (or 6 pence) is ·5, therefore, to halve any odd number of shillings, is only to annex ·5 to half of the greatest even number in the price.

1st. How many yds. at 7s. per yd. may I have for 435l.?

```
Half=3·5)43500(1242 10/35 yds.Ans.
         35
         --
          85
          70
          --
          150
          140
          ---
           100
            70
           ---
            30
```

2d. How many pounds of tea, at 5s. per lb. for £37?
148lb. Ans.

3d. How many yards at 9s. per yard may I have for 540l.?
Ans. 1200 yards.

BILL OF PARCELS.

Newburyport, January 1st, 1808.

Mr. Timothy Huckster

Bought of *Samuel Merchant*.

25½lb Bohea tea, at 3s. 6d. per lb.
48lb Cheese, at 9d. per lb.
15 Pair worsted hose, at 5s. 8d. per pair.
4½ Dozen women's gloves, at 36s. 6d. per dozen.
19 Dozen knives and forks, at 5s. 9d. per dozen.
9 Grindstones at 15s. 9d. per stone.
½ Cwt. Brown sugar, at 51s. per cwt.
31 lb Loaf Sugar, at 1s. 0½d. per lb.

£34 3 3½

Received payment in full,
Samuel Merchant.

TARE AND TRET.

TARE and Tret are practical rules for deducting certain allowances, which are made by merchants and tradesmen in selling their goods by weight.

Tare is an allowance, made to the buyer, for the weight of the box, barrel or bag, &c. which contains the goods bought, and is either at so much per box, &c. at so much per cwt. or at so much in the gross weight.

Tret is an allowance of 4lb in every 104lb for waste, dust, &c.

Cloff is an allowance of 2lb upon every 3cwt.

Gross weight is the whole weight of any sort of goods, together with the box, barrel, or bag, &c. which contains them.

Suttle is, when part of the allowance is deducted from the gross.

Neat weight is what remains after all allowances are made.

CASE I.*

When the tare is at so much per box, barrel or bag, &c.: Multiply the number of boxes, barrels, &c. by the tare, and subtract the product from the gross, and the remainder will be the neat weight required.

Examples.

1. In 6 hogsheads of sugar, each weighing 9cwt. 2qrs. 10lb. gross, tare 25lb. per hogshead; how much neat?

* This, as well as every other case in this rule, is only an application of the rules of Proportion and Practice.

Cwt.	qr.	lb		Cwt.	qr.	lb	
25×6=1	1	10		9	2	10	gross wt. of 1hhd.
						6	
				57	2	4	gross.
				1	1	10	tare.
			Ans.	56	0	22	neat.

2. In 5 bags of cotton, marked with the gross weight as follows, tare 23lb per bag; what neat weight?

	Cwt.	qr.	lb
A=	7	1	19
B=	3	3	27
C=	5	1	12
D=	6	0	15
E=	8	1	0

Cwt. qr. lb
Ans. 30 0 14 neat.

3. What is the neat weight of 15 hogsheads of tobacco, each 7cwt. 1qr. 13lb, tare 100lb per hogshead?

Ans. 97cwt. 0qr. 11lb.

CASE II.

When the tare is at so much per cwt.: Divide the gross weight by the aliquot parts of a cwt. subtract the quotient from the gross, and the remainder will be the neat weight.

Examples.

1. In 129cwt. 3qrs. 16lb. gross, tare 14lb. per cwt. what neat weight?

		Cwt.	qr.	lb.	
14lb.	$\frac{1}{8}$	129	3	16	gross.
		16	0	$26\frac{1}{2}$	tare.
	Ans.	113	2	$17\frac{1}{2}$	neat.

2. In 97cwt. 1qr. 7lb. gross, tare 20lb. per cwt. what neat weight.

Ans. 79Cwt. 3qrs. 21lb. neat.

3. What is the neat weight of 9 barrels of potash, each weighing 305lb. gross, tare 12lb. per cwt.?

Ans. 2450lb 14oz. $4\frac{4}{7}$dr.

4. What is the value of the neat weight of 7hhds. of tobacco, at 5l. 7s. 6d. per cwt. each weighing 8cwt. 3qrs. 10lb gross, tare 21lb per cwt.?

Ans. £270 4 $4\frac{1}{2}$ reckoning the odd ounces.

CASE III.

When tret is allowed with tare: Divide the suttle weight by 26, and the quotient will be the tret, which subtract from the suttle, and the remainder will be the neat.*

* Tret is 4lb. in 104, which is $\frac{4}{104}=\frac{1}{26}$. And Cloff is 2lb. in 3Cwt. or 336lb. which is $\frac{2}{336}=\frac{1}{168}$.

EXAMPLES.

1. In 247cwt. 2qrs. 15lb gross, tare 28lb per cwt. and tret 4lb for every 104lb what neat weight?

| 28 | $\frac{1}{4}$ | 247C.2qr.15lb gross.
61 3 17 12 tare, subtract.

| 4 | $\frac{1}{26}$ | 185 2 25 4 suttle.
7 0 16 0 tret, subtract.

Ans. 178 2 9 4 neat.

2. What is the neat weight of 4hhds of tobacco, weighing as follow: The 1st. 5cwt. 1qr. 12lb gross, tare 65lb per hhd.; the 2d. 3cwt 0qr 19lb gross, tare 75lb: the 3d. 6cwt. 3qrs. gross, tare 49lb; and the 4th, 4cwt. 2qrs. 9lb gross, tare 35lb, and allowing tret to each as usual? Ans. 17cwt. 0qr. 19lb+

CASE IV.

When tare, tret, and cloff are allowed: Deduct the tare and tret as before, and divide the suttle by 168, and the quotient will be the cloff, which subtract from the suttle, and the remainder will be the neat.

EXAMPLES.

1. What is the neat weight of 1hhd. of tobacco, weighing 16cwt. 2qrs. 20lb gross, tare 14lb per cwt. tret 4lb per 104, and cloff 2lb per 3cwt.?

14lb is $\frac{1}{8}$)16 2 20 0 gross.
2 0 9 8 tare, subtract.

4lb is $\frac{1}{26}$)14 2 10 8
0 2 6 13 tret, subtract.

2lb is $\frac{1}{168}$)14 0 3 11 suttle.
0 0 9 5 cloff, subtract.

Ans. 13 3 22 6 neat.

2. If 9hhds. of tobacco, contain 85cwt. 0qr. 2lb, tare 30lb per hhd. tret and cloff as usual, what will the neat weight come to at 6½d. per lb after deducting for duties and other charges, 51l. 11s. 8d? Ans. £187 18s. 5d.

INVOLUTION

TEACHES the method of finding the *powers* of numbers.

A *power* is the product arising from multiplying any number into itself continually a certain number of times. Thus, any number is called the *first power*, as 3; if it be multiplied by itself, the product is called the *second* power or *square*, as 3×3; if the second power be multiplied by the first power, the product is called the

third power, or *cube*, as $3 \times 3 \times 3$; if the third power be multiplied by the first power, the product is the *fourth* power, or *biquadrate*, as $3 \times 3 \times 3 \times 3$, or 81 is the fourth power of 3, and so on.

The power is often denoted by a figure placed at the right and a little above the number, which figure is called the *index* or *exponent* of that power. The index or exponent is always *one more* than the number of multiplications to produce the power, or is *equal* to the number of times the given number is used as a factor in producing the power. Thus the square of

$3, = 3 \times 3 = 3^2$; and the cube of

$3, = 3 \times 3 \times 3 = 3^3$; and the 4th power of

$3. = 3 \times 3 \times 3 \times 3 = 3^4$; and the 5th power of

$3, = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$, and so on.

In producing the square of 3, for instance, there is only *one* multiplication, or *two* factors; in producing the cube, there are *two* multiplications or *three* factors, and so on.

Hence, Involution is performed by the following

Rule.

Multiply the given number, or first power continually by itself, till the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

Note. Whence, because fractions are multiplied by taking the products of their numerators, and of their denominators, they will be involved by raising each of their terms to the power required, and if a mixed number be proposed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed by the rule.

Examples.

1. What is the 5th power of 9?

$$\begin{array}{r} 9 \\ 9 \\ \hline 81 = \text{2d. power.} \\ 9 \\ \hline 729 = \text{3d. power.} \\ 9 \\ \hline 6561 = \text{4th. power.} \\ 9 \\ \hline 59049 = \text{5th power, or answer} = 9^5. \end{array}$$

2. What is the 5th power of $\frac{3}{5}$? Ans. $\frac{243}{3125}$.

3. What is the fourth power of ·045? Ans. ·000004100625.

Here we see, that in raising a fraction to a higher power, we decrease its value.

EVOLUTION,

OR THE EXTRACTION OF ROOTS.

THE Root is a number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d. 3d. 4th. root, &c. accordingly as it is, when raised to the 2d. 3d. &c. power, equal to that power. Thus, 4 is the square root of 16, because $4\times4=16$, and 3 is the cube root of 27, because $3\times3\times3=27$, and so on.

Although there is no number of which we cannot find any power exactly, yet there are many numbers, of which precise roots can never be determined. But, by the help of decimals, we can approximate towards the root to any assigned degree of exactness.

The roots, which approximate, are called *surd roots*, and those which are perfectly accurate, are called *rational roots*.

Roots are sometimes denoted by writing the character $\surd$ before the power, with the index of the power over it; thus the 3d root of 36 is expressed $\sqrt[3]{36}$, and the 2d root of 36 is $\surd 36$, the index 2 being omitted when the square root is designed.

If the power be expressed by several numbers, with the sign $+$ or $-$ between them, a line is drawn from the top of the sign over all the parts of it. Thus the 3d root of $47+22$ is $\sqrt[3]{47+22}$, and the 2d root of $59-17$ is $\sqrt{59-17}$, &c.

Sometimes roots are designated like powers, with fractional indices. Thus, the squre root of 15 is $15^{\frac{1}{2}}$, the cube root of 21 is $21^{\frac{1}{3}}$, and 4th root of $37-20$ is $\overline{37-20}^{\frac{1}{4}}$, &c. The denominator shows the root which is to be extracted, and the numerator shows the power to which that root is to be raised. Or the number may be raised to the power indicated by the numerator, and the root, indicated by the denominator, then extracted. Thus $64^{\frac{2}{3}}=4^2=16$, $=\sqrt[3]{64^2}=\sqrt[3]{4096}=16$. Hence the square of the cube root of any quantity is the same as the cube root of the square of the same quantity.

The *index* or *exponent* of the root is *one* more than the number of multiplications, required to produce the given number or power.

A TABLE OF POWERS.

Roots, - - - -	or 1st. Pow.	1	2	3	4	5	6	7	8	9
Squares, - - -	or 2d. Pow.	1	4	9	16	25	36	49	64	81
Cubes, - - -	or 3d. Pow.	1	8	27	64	125	216	343	512	729
Biquadrates, - -	or 4th. Pow.	1	16	81	256	625	1296	2401	4096	6561
Sursolids, - -	or 5th. Pow.	1	32	243	1024	3125	7776	16807	32768	59049
Square cubes, -	or 6th. Pow.	1	64	729	4096	15625	46656	117649	262144	531441
Second Sursolids,	or 7th. Pow.	1	128	2187	16384	78125	279936	823543	2097152	4782969
Biquadrates Sqd.	or 8th. Pow.	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
Cubes Cubed, -	or 9th. Pow.	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
Sursolids Squared,	or 10th. Pow.	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
Third Sursolids,	or 11th. Pow.	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
Square Cubes Sqd.	or 12th. Pow.	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
Fourth Sursolids,	or 13th. Pow.	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
2d. Sursolids Sqd.	or 14th. Pow.	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
Sursolids Cubed,	or 15th. Pow.	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

THE EXTRACTION OF THE SQUARE ROOT.

Rule.

*1. Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points shew the number of figures the root will consist of.

2. Find the greatest square number in the first, or left hand period, place the root of it at the right hand of the given number, (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

3. Place the double of the root, already found, on the left hand of the dividend for a divisor.

4. Seek how often the divisor is contained in the dividend, (except the right hand figure) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor: Multiply the divisor with the figure last annexed by the figure last placed in the root, and subtract the product from the dividend: To the remainder join the next period for a new dividend.

5. Double the figures already found in the root, for a new divisor, (or, bring down your last divisor for a new one, doubling the right hand figure of it) and from these, find the next figure in the root as last directed, and continue the operation, in the same manner, till you have brought down all the periods.

Note 1. If when the given power is pointed off as the power requires, the left hand figure should be deficient, it must nevertheless stand as the first period.

Note 2. If there be decimals in the given number, it must be pointed both ways from the place of units: If, when there are

* In order to shew the reason of the rule, it will be proper to premise the following Lemma. The product of any two numbers can have, at most, but so many places of figures as are in both the factors, and at least but one less.

Demonstration. Take two numbers consisting of any number of places; but let them be the least possible of those places, viz. Unity with cyphers, as 100 and 10: Then their product will be 1 with so many cyphers annexed as are in both the numbers, viz. 1000; but 1000 has one place less than 100 and 10 together have: And since 100 and 10 were taken the least possible, the product of any other two numbers, of the same number of places, will be greater than 1000; consequently, the product of any two numbers can have, at least, but one place less than both the factors.

Again, take two numbers, of any number of places, which shall be the greatest possible of those places, as 99 and 9. Now, 99×9 is less than 99×10; but 99×10 ($=990$) contains only so many places of figures as are in 99 and 9; therefore, 99×9, or the product of any other two numbers, consisting of the same number of places, cannot have more places of figures, than are in both its factors.

Corollary 1. A square number cannot have more places of figures than double the places of the root, and at least but one less.

Corollary 2. A cube number cannot have more places of figures than triple the places of the root, and at least but two less.

integers, the first period in the decimals be deficient, it may be completed by annexing so many cyphers as the power requires: And the root must be made to consist of so many whole numbers and decimals as there are periods belonging to each; and when the periods belonging to the integers are exhausted, the operation may be continued at pleasure by annexing cyphers.

EXAMPLES.

1st. Required the square root of 30138696025?

```
                          . . . . . .
                         30138696025(173605 the root.
                         1
                         ---
1st. Divisor=27)201
                189
                ----
2d. Divisor=343)1238
                1029
                -----
3d. Divisor=3466)20969
                 20796
                 -------
4th. Divisor=347205)1736025
                    1736025*
                    -------
```

2d. Required the square root of 575·5?

```
       . . .
      575·50(23·98+, the root.
      4
      ---
  43)175
     129
     ---
 469)4650
     4221
     -----
4788)42900
     38304
     -----
      4596 Remainder.
```

* The Rule for the extraction of the square root, may be illustrated by attending to the process by which any number is raised to the square. The several products of the multiplication are to be kept separate, as in the proof of the rule for Simple Multiplication. Let 37 be the number to be raised to the square.

$$37\times37=1369=\!=\!=\!=37\times37$$

$$\begin{array}{l} 37 \\ \hline 49=7^2 \\ \left.\begin{array}{l}21\,.=3\times7 \\ 21\,.=3\times7\end{array}\right\}=2\times3\times7 \\ 9\,.\,.=3^2 \\ \hline \quad\ldots\ (37 \\ 2\times3)42\,.=2\times3\times7 \\ \quad 49=7^2 \end{array} \qquad \begin{array}{l} 37 \\ \hline 49=7^2 \\ \left.\begin{array}{l}210=30\times7 \\ 210=30\times7\end{array}\right\}=2\times30\times7 \\ 900=30^2 \\ \hline (30+7=37 \end{array}$$

[Carried over.

3d. What is the square root of 10342656? Ans. 3216.
4th. What is the square root of 964·5192360241? Ans. 31·05671.
5th. What is the square root of 234·09? Ans. 15·3.
6th. What is the square root of ·0000316969? Ans. ·00563.
7th What is the square root of ·045369? Ans. ·213.

Rules

For the Square Root of Vulgar Fractions and Mixed Numbers.

After reducing the fraction to its lowest terms, for this and all other roots; then,

1st. Extract the *root* of the *numerator* for a *new numerator*, and the *root* of the *denominator* for a *new denominator*, which is the best method, provided the denominator be a complete power. But if it be not,

2d. Multiply the numerator and denominator together; and the root of this product being made the numerator to the denominator of the given factor, or made the denominator to the numerator of it, will form the fractional part required.* Or,

Now, it is evident that 9, in the place of hundredths, is the greatest square in this product; put its root, 3, in the quotient, and 900 is taken from the product. The next products are $21+21=2\times3\times7$, for a dividend. Double the root already found, and it is 2×3, for a divisor, which gives 7 for the quotient, which annexed to the divisor, and the whole then multiplied by it, gives $2\times3\times7$ $(=42)$ $+7\times7$ $(=49)$ which placed in their proper places, completely exhausts the remainder of the square. The same may be shown in any other case, and the rule becomes obvious.

Perhaps the following may be considered more simple and plain. Let 37, $=30+7$, be multiplied, as in the demonstration of simple multiplication, and the products kept separate.

$$\begin{array}{l} 30+7 \\ 30+7 \\ \hline 900+30\times7 \\ \qquad 30\times7+49 \\ \hline 900+2\times30\times7+49=1369, \text{ the sum, and square.} \end{array}$$

$$\begin{array}{rl} & 900 \qquad (30+7 \\ \hline 2\times30+7\times7) & 2\times30\times7+49 \\ & 2\times30\times7+49 \\ \hline & * \;\; * \;\; * \;\; * \end{array}$$

The root of 900 is 30, and leaves the two other terms, which are exhausted by a divisor, formed and multiplied as directed in the rule.

* Let the fraction be $\sqrt{\frac{7}{2}}$, then by the rule, $\sqrt{\frac{7}{2}}=\frac{\sqrt{7\times2}}{2}=\frac{7}{\sqrt{7\times2}}=1{\cdot}87+$. The reason of which is, that the value of a fraction is not altered by multiplying both its parts by the same quantity. Thus $\sqrt{\frac{7}{2}}=\frac{\sqrt{7}\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$. But $\sqrt{7}\times\sqrt{2}=\sqrt{7\times2}$, and $\sqrt{2}\times\sqrt{2}=2$ evidently. And thus also, $\frac{\sqrt{7}\times\sqrt{7}}{\sqrt{7}\times\sqrt{2}}=\frac{7}{\sqrt{7\times2}}=\frac{\sqrt{7\times2}}{2}$ and is the rule. See Surds.

3d. Reduce the vulgar fraction to a decimal, and extract its root.

4th. Mixed numbers may either be reduced to improper fractions, and extracted by the first or second rule, or the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

Examples.

1st. What is the square root of $\frac{144}{15129}$?

By Rule 1.

$\frac{144}{15129}=\frac{16}{1681}$

16(4 root of the numerator.
16

1681(41 root of the denominator.
16

81)81 Therefore, $\frac{4}{41}$=the root of the given fraction.
81

By Rule 2.

$16\times1681=26896$, and $\sqrt{26896}=164$. Then,

$\frac{164}{1681}=\frac{16}{164}=\frac{4}{41}=\cdot09756+$

By Rule 3.

$1681)16(\cdot0095181439+$. And $\sqrt{.0095181439}=\cdot09756+$.

2d. What is the square root of $\frac{2723}{8208}$? Ans. $\frac{7}{12}$.

3d. What is the square root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.

Note. In extracting the square or cube root of any surd number, there is always a remainder or fraction left, when the root is found. To find the value of which, the common method is, to annex *pairs* of cyphers to the resolvend, for the square, and *ternaries* of cyphers to that of the cube, which makes it tedious to discover the value of the remainder, especially in the cube, whereas this trouble might be saved if the true denominator could be discovered.

As in division the divisor is always the denominator to its own fraction, so likewise it is in the square and cube, each of their divisors being the denominators to their own particular fractions or numerators.

In the square the quotient is always doubled for a new divisor; therefore, when the work is completed, the root doubled is the true divisor or denominator to its own fraction; as, if the root be 12, the denominator will be 24, to be placed under the remainder, which vulgar fraction, or its equivalent decimal, must be annexed to the quotient or root, to complete it.*

If to the remainder, either of the square or cube, cyphers be annexed, and divided by their respective denominators, the quotient will produce the decimals belonging to the root.

* Although these denominators give a small matter too much in the square root, and too little in the cube, yet they will be sufficient in common use, and are much more expeditious than the operation with cyphers.

APPLICATION AND USE OF THE SQUARE ROOT.

PROB. I. *To find a mean proportional between two numbers.*

RULE. Multiply the given numbers together, and extract the square root of the product; which root will be the mean proportional sought.

Note. When the first is to the second as the second is to the third, the second is called a mean proportional between the other two. Thus, 4 is a mean proportional between 2 and 8, for 2 : 4 :: 4 :: $\frac{4\times4}{2}=8$, or 4 is as much greater than 2, as 8 is greater than 4. By Theorem I. of Geometrical Proportion $2\times8=4\times4=4^2$. To find a mean proportional between 2 and 8, take the square root of their product. The same must be true in every case, and is the rule.

EXAMPLE.

What is the mean proportional between 24 and 96?

$\sqrt{96\times24}=48$. Answer.

PROB. II. *To find the side of a* square *equal* in area *to any given* superficies *whatever.*

RULE. Find the *area*, and the square root is the side of the square sought.*

EXAMPLES.

1st. If the area of a circle be 184·125, What is the side of a square equal in area thereto?

$\sqrt{184\cdot125}=13\cdot569+$ Answer.

2d. If the area of a triangle be 160, what is the side of a square equal in area thereto? $\sqrt{160}=12\cdot649+$ Answer.

PROB. III. A certain general has an army of 5625 men: pray How many must he place in rank and file, to form them into a square? $\sqrt{5625}=75$ Answer.†

PROB. IV. Let 10952 men be so formed, as that the number in rank may be double the file. 74 in file, and 148 in rank.

PROB V. If it be required to place 2016 men so as that there may be 56 in rank and 36 in file, and to stand 4 feet distance in rank and as much in file, How much ground do they stand on?

To answer this, or any of the kind, use the following proportion: As unity : the distance :: so is the number in rank less by one : to a fourth number; next, do the same by the file, and mul-

* A square is a figure of four equal sides, each pair meeting perpendicularly, or, a figure whose length and breadth are equal. As the area, or number of square feet, inches, &c. in a square, is equal to the product of two sides which are equal, the second power is called the square. Hence the rule of Prob. II. is evident.

† If you would have the number of men be double, triple, or quadruple, &c as many in rank as in file, extract the square root of ½, ⅓, ¼, &c. of the given number of men, and that will be the number of men in file, which double, triple quadruple, &c. and the product will be the number in rank.

tiply the two numbers together, found by the above proportion, and the product will be the answer.*

As 1 : 4 :: 56—1 : 220. And, as 1 : 4 :: 36—1 : 140. Then, 220×140=30800 square feet, the Answer.

PROB. VI. Suppose I would set out an orchard of 600 trees, so that the length shall be to the breadth as 3 to 2, and the distance of each tree, one from the other, 7 yards: How many trees must it be in length, and how many in breadth? and, How many square yards of ground do they stand on?

To resolve any question of this nature, say, as the ratio in length : is to the ratio in breadth :: so is the number of trees : to a fourth number, whose square root is the number in breadth. And as the ratio in breadth : is to the ratio in length :: so is the number of trees : to a fourth, whose root is the number in length.

As 3 : 2 :: 600 : 400. And $\sqrt{400}$=20=number in breadth.

As 2 : 3 :: 600 : 900. And $\sqrt{900}$=30=number in length.

As 1 : 7 :: 30—1 : 203. And as 1 : 7 :: 20—1 : to 133. And 203×133=26999 square yards, the Answer.

PROB. VII. Admit a leaden pipe ¾ inch diameter will fill a cistern in 3 hours; I demand the diameter of another pipe which will fill the same cistern in 1 hour.

RULE. As the given time is to the square of the given diameter, so is the required time to the square of the required diameter †

¾=·75 : and ·75×·75=·5625. Then, as 3h. : ·5625 :: 1h. : 1·6875 inversely, and $\sqrt{1\cdot6875}$=1·3 inches nearly, Ans.

PROB VIII. If a pipe whose diameter is 1·5 inches, fill a cistern in 5 hours, in what time will a pipe whose diameter is 3·5 inches fill the same?

1·5×1·5=2·25; and 3·5×3·5=12·25. Then, as 2·25 : 5 :: 12·25 : ·918+hour, inversely,=55 min. 5 sec. Answer.

PROB. IX. If a pipe 6 inches bore, will be 4 hours in running off a certain quantity of water, In what time will 3 pipes, each four inches bore, be in discharging double the quantity?

6×6=36. 4×4=16, and 16×3=48. Then, as 36 : 4h. :: 48 : 3h. inversely, and as 1w. : 3h. :: 2w. : 6h. Answer.

PROB. X. Given the diameter of a circle, to make another circle which shall be 2, 3, 4, &c. times greater or less than the given circle.

RULE. Square the given diameter, and if the required circle be greater, multiply the square of the diameter by the given proportion, and the square root of the product will be the required diameter. But if the required circle be less, divide the square of the diameter by the given proportion, and the root of the quotient will be the diameter required.

* The above rule will be found useful in planting trees, having the distance of ground between each given.

† For more water will run as the area of the pipe is greater, and the areas of circular pipes vary as the square of their diameters.

There is a circle whose diameter is 4 inches; I demand the diameter of a circle 3 times as large?

$4\times4=16$; and $16\times3=48$; and $\sqrt{48}=6\cdot928+$ inches, Answer.

PROB. XI. To find the diameter of a circle equal in area, to an ellipsis, (or oval) whose transverse and conjugate diameters are given.*

RULE. Multiply the two diameters of the ellipsis together, and the square root of that product will be the diameter of a circle equal to the ellipsis.

Let the transverse diameter of an ellipsis be 48, and the conjugate 36: What is the diameter of an equal circle?

$48\times36=1728$, and $\sqrt{1728}=41.569+$ the Answer.

Note. The square of the hypothenuse, or the longest side of a right angled triangle, is equal to the sum of the squares of the other two sides; and consequently the difference of the squares of the hypothenuse and either of the other sides is the square of the remaining side.

PROB. XII. A line 36 yards long will exactly reach from the top of a fort to the opposite bank of a river, known to be 24 yards broad. The height of the wall is required?

$36\times36=1296$; and $24\times24=576$. Then, $1296-576=720$, and $\sqrt{720}=26\cdot83+$ yards, the Answer.

PROB. XIII. The height of a tree growing in the centre of a circular island 44 feet in diameter, is 75 feet, and a line stretched from the top of it over to the hither edge of the water, is 256 feet. What is the breadth of the stream, provided the land on each side of the water be level?

$256\times256=65536$; and $75=75=5625$: Then, $65536-5625=59911$ and $\sqrt{59911}=244\cdot76+$ and $244\cdot76-\frac{44}{2}=222\cdot76$ feet, Ans.

PROB. XIV. Suppose a ladder 60 feet long be so planted as to reach a window 37 feet from the ground, on one side of the street, and without moving it at the foot, will reach a window 23 feet high on the other side; I demand the breadth of the street?

102.64 feet the Answer.

PROB. XV. Two ships sail from the same port; one goes due north 45 leagues, and the other due west 76 leagues: How far are they asunder?† 88·32 leagues, Answer.

PROB. XVI. Given the sum of two numbers, and the difference of their squares, to find those numbers.

RULE. Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference. The two num-

* The transverse and conjugate are the longest and shortest diameters of an ellipsis; they pass through the centre, and cross each other at right angles, and the diameter of the equal circle is the square root of the product of the diameters of the ellipsis.

† The square root may in the same manner be applied to navigation; and, when deprived of other means of solving problems of that nature, the following proportion will serve to find the course.

As the sum of the hypothenuse (or distance) and half the greater leg (whether difference of latitude or departure) is to the less leg; so is 86, to the angle opposite the less leg.

bers may then be found, from their sum and difference, by Prob. 4, page 57.

Ex. The sum of two numbers is 32, and the difference of their squares is 256, what are the numbers?

Ans. The greater is 20. The less 12.

Prob. XVII. Given the difference of two numbers, and the difference of their squares, to find the numbers.

Rule. Divide the difference of the squares by the difference of the numbers, and the quotient will be their sum. Then proceed by Prob. 4, p. 57.

Ex. The difference of two numbers is 20. and the difference of their squares is 2000; what are the numbers?

Ans. 60 the greater. 40 the less.

Examples for the two preceding problems.

1. A and B played at marbles, having 14 apiece at first; B having lost some, would play no longer, and the difference of the squares of the numbers which each then had, was 336; pray how many did B lose? Ans. B lost 6.

2. Said Harry to Charles, my father gave me 12 apples more than he gave brother Jack, and the difference of the squares of our separate parcels was 288; Now, tell me how many he gave us, and you shall have half of mine.

Ans. Harry's share 12.
Jack's share 6.

EXTRACTION OF THE CUBE ROOT.

A cube is any number multiplied by its *square*. To extract the cube root, is to find a number which, being multiplied into its square, shall produce the given number.

FIRST METHOD.

Rule.

*1. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. Find the greatest cube in the left hand period, and put its root in the quotient.

3. Subtract the cube, thus found, from the said period, and to the remainder bring down the next period, and call this the *dividend*.

4. Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the *divisor*.

5. Seek how often the divisor may be had in the dividend, and place the result in the quotient.

* The reason of pointing the given number, as directed in the rule, is obvious from *Coroll.* 2, to the *Lemma* made use of in demonstrating the square root.

The process for extracting the Cube Root may be illustrated in the same manner as that for the Square Root. Take the same number 37, and multiply

6. Multiply the triple square by the last quotient figure, and write the product under the dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure and call their sum the *subtrahend*.

as before, collecting the twice 21 into one sum, as they belong to the same place and the operation will be simplified, $37^3=50653$.

$$37^2=\begin{cases}49=7^2\\ 42\cdot=2\times3\times7\\ 9\cdot\cdot=3^2\end{cases}\qquad\begin{array}{l}49=7^2\\ 420=2\times30\times7\\ 900=30^2\end{array}$$

37 the multiplier. 37

$$37^3=\begin{cases}343=7^3\\ 294\cdot=2\times3\times7^2\\ 63\cdot\cdot=3^2\times7\\ 147\cdot=3\times7^2\\ 126\cdot\cdot=2\times3^2\times7\\ 27\cdot\cdot\cdot=3^3\end{cases}\qquad\begin{array}{l}343=7^3\\ 2940=2\times30\times7\\ 6300=30^2\times7\\ 1470=30\times7^2\\ 12600=2\times30^2\times7\\ 27000=30^3\end{array}$$

$27\cdots(37$ $\qquad 27000\ (30\times7$

$$\left.\begin{array}{r}3\times3^2\\ 3\times3\end{array}\right)\begin{array}{l}189\cdot\cdot=3\times3^2\times7\\ 441\cdot=3\times3\times7^2\end{array}\quad\left.\begin{array}{r}3\times30^2\\ 3\times30\end{array}\right)\begin{array}{l}18900=3\times30^2\times7\\ 4410=3\times30\times7\end{array}$$

As 27 or 27000 is the greatest cube, its root is 3 or 30, and that part of the cube is exhausted by this extraction. Collect those terms which belong to the same places, and we have $3^2\times7=63$, and $2\times3^2\times7=126$, and $63+126=3\times3^2\times7=189$; and $2\times3\times7^2=294$ and $3\times7^2=147$, and $294+147=441=3\times3\times7^2$, for a dividend, which divided by the divisor, formed according to the rule, the quotient is 7, for the next figure in the root. And it is evident on inspecting the work, that that part of the cube not exhausted is composed of the several products which form the *subtrahend*, according to the rule. The same may be shown in any other case, and the universality of the rule hence inferred.

The other method of illustration, employed on the square root, is equally applicable in this case.

$$37=30+7,\text{ and }\overline{30+7}^2=30^2+2\times30\times7+7^2$$
$$30+7\text{ the multiplier.}$$

$$30^3+2\times30^2\times7+30\times7^2$$
$$30^2\times7+2\times30\times7^2+7^3$$

$$37^3=50653=30^3+3\times30^2\times7+3\times30\times7^2+7^3\,(30+7=37$$
$$30^3$$

$$\text{Divisor }3\times30^2+3\times30\quad)3\times30^2\times7+3\times30\times7^2+7^3\text{ dividend.}$$

$$3\times30^2\times7+3\times30\times7^2+7^3\text{ subtrahend.}$$

It is evident that 30^3 is the greatest cube. When its root is extracted, the next three terms constitute the dividend; and, the several products formed by means of the quotient or second figure in the root, are precisely equal to the remaining parts of the power, whose root was to be found.

The arithmetical demonstrations of the Rules for extracting either the square or cube root, are not only more consistent with the plan of an Arithmetick than demonstrations on the figure, called a square, and the solid, called a cube, but they are much more readily understood by those unaccustomed to the mathematical consideration of solid bodies.

7. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on till the whole be finished.

Note. The same rule must be observed for continuing the operation, and pointing for decimals, as in the square root.

EXAMPLES.

1st. Required the cube root of 436036824287?

```
            436036824287(7583 root.7×7×300=        14700=1st.Trip.sq.
            343                    7×  30  =          210=1st.do.quo.
            ———                                      ————
1st.Divis.=14910)93036=1st. Dividend.              14910=1st. divisor.
            ———                                      ————
            73500              14700×5=            73500
             5250              5×5×210=             5250
              125              5×5×5   =             125
            ———                                      ————
            78875=1st. Subtrahend.                 78875=1st. Subtra.
            ————           ——————————————
2.Div.=1689750)14161824=2d. Divid. 75×75×300=  1687500=2d. Trip.sq.
            ————              75×30      =         2250=2d. do. quo.
            13500000                             ————
              144000                             1689750=2d. Divisor.
                 512                             ————
            ————              1687500×8=  13500000
            13644512=2d. Subtra. 2250×8×8=    144000
            ————              8×8×8     =        512
3.Div.=172391940)517312287=3d. Divid.            ————
                ————                      13644512=2d. Subtra.
                517107600       ——————————————
                   204660       758×758×300=172369200=3d. Trip.sq.
                       27       758×30              22740=3d. do. quo.
                ————                               ————
                517312287=3d. Subtra.           172391940=3d. Divisor.
                ————                               ————
                .........       172369200×3=517107600
                                22740×3×3 =   204660
                                3×3×3     =       27
                                                ————
                                             517312287=3d. Subtra.
```

2d. What is the cube root of 34965783? Ans. 327.

3d. What is the cube root of 84·604519? Ans. 4·39.

4th. What is the cube root of ·008649? Ans. ·2052+.

5th. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.

To find the true denominator, to be placed under the remainder, after the operation is finished.

In the extraction of the cube root, the quotient is said to be squared and tripled for a new divisor; but is not really so, till the triple number of the quotient be added to it; therefore when the operation is finished, it is but squaring the quotient, or root, then multiplying it by 3, and to that number adding the triple number of the root, when it will become the divisor, or true denominator to its own fraction, which fraction must be annexed to the quotient, to complete the root.

Suppose the root to be 12, when squared it will be 144, and multiplied by 3, it makes 432, to which add 36, the triple number of the root, and it produces 468 for a denominator.*

SECOND METHOD.

Rule.

1. Having pointed the given number into periods of three figures each, find the greatest cube in the left hand period, subtracting it therefrom and placing its root in the quotient; to the remainder bring down the next period and call it the *dividend.*

2. Under this dividend write the triple square of the root, so that units in the latter may stand under the place of hundreds in the former; and under the said triple square, write the triple root, removed one place to the right hand, and call the sum of these the *divisor.*

3. Seek how often the divisor may be had in the dividend, exclusive of the place of units, and write the result in the quotient.

4. Under the divisor write the product of the triple square of the root by the last quotient figure, setting down the unit's place of this line, under the place of tens in the divisor; under this line, write the product of the triple root by the square of the last quotient figure, so as to be removed one place beyond the right hand figure of the former; and, under this line, removed one place forward to the right hand, write down the cube of the last quotient figure, and call their sum the *subtrahend.*

5. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on until the whole be finished.

Example.

Required the Cube Root of 16194277?

* It may not be amiss to remark here, that the denominators, both of the square and cube, shew how many numbers they are denominators to, that is, what numbers are contained between any square or cube number and the next succeeding square or cube number, exclusive of both numbers, for a complete number, of either, leaves no fraction, when the root is extracted, and consequently has no use for a denominator, but all the numbers contained between them have occasion for it: Suppose the square root to be 12, then its square is 144, and the denominator 24, which will be a denominator to all the succeeding numbers, until we come to the next square number, *viz.* 169, whose root is 13, with which it has nothing to do, for between the square numbers 144 and 169 are contained 24 numbers excluding both the square numbers. It is the same in the cube; for, suppose the root to be 6, the cube number is 216, and its denominator 126 will be a denominator to all the succeeding numbers, until we come to the next cube number, *viz.* 343, whose root is 7, with which it has nothing to do, as ceasing then to be a denominator; for between the cube 343 and 216 are 126 numbers, excluding both cubes. And so it is with all other denominators, either in the square or cube.

16194277(253=Root.
8

8194	= First dividend.
12	= Triple square of 2.
6	= Triple of 2.
126	= First divisor.
60	= Triple square of 2, multiplied by 5.
150	= Triple of 2 multiplied by the square of 5.
125	= Cube of 5.
7625	= First Subtrahend.
569277	= Second dividend.
1875	= Triple square of 25.
75	= Triple of 25.
18825	= Second divisor.
5625	= Triple square of 25 multiplied by 3.
675	= Triple of 25 multiplied by the square of 3.
27	= Cube of 3.
569277	= Second subtrahend.

FIRST METHOD BY APPROXIMATION.

RULE.

1. Find, by trial, a cube near to the given number, and call it the *supposed* cube.

2. Then as twice the supposed cube, added to the given number, is to twice the given number, added to the supposed cube, so is the root of the supposed cube, to the true root, or an approximation to it.

3. By taking the cube of the root, thus found, for the supposed cube, and repeating the operation, the root will be had to a greater degree of exactness.

EXAMPLE.

It is required to find the cube root of 54854153?

Let 64000000=supposed cube, whose root is 400:

Then,	64000000		54854153	
	2		2	
	128000000		109708306	
	54854153		64000000	
As	182854153	:	173708306	:: 400
			400	

182854153)69483322400(379=root nearly.

Again, let 54439939 = supposed cube, whose root is 379.

```
Then, 54439939     54854153
             2            2
      ---------   ---------
      108879878   109708306
       54854153    54439939
      ---------   ---------
As    163734031 : 164148245 :: 379
                        379
                 ----------
                 1477334205
                1149037715
                492444735
                -----------
163734031)62212184855(379·958793+=root corrected.
```

SECOND METHOD BY APPROXIMATION.

Rule.

1. Divide the resolvend by three times the assumed root, and reserve the quotient.

2. Subtract one twelfth part of the square of the assumed root from the quotient.

3. Extract the square root of the remainder.

4. To this root add one half of the assumed root, and the sum will be the true root, or an approximation to it; take this approximation as the assumed root, and, by repeating the process, a root farther approximated will be found, which operation may be farther repeated, as often as necessary, and the root discovered to any assigned exactness.

Note. In order to find the value of the first assumed root, in this or any other power, divide the resolvend into periods by beginning at the place of units, and including in each period, so many figures as there are units in the exponent of the root; viz. 3 figures in the cube root; 4 for the biquadrate, and so on; then, by a table of powers, or otherwise, find a figure, which (being involved to the power whose exponent is the same with that of the required root) is the nearest to the value of the first period of the resolvend at the left hand, and to that figure annex so many cyphers as there are periods remaining in the integral part of the resolvend; this figure, with the cyphers annexed, will be the assumed root. and equal to r in the theorem; and it is of no importance whether the figure thus chosen be, when involved, greater or less than the left hand period, as the theorem is the same in both cases.

1st. What is the cube root of $43\dot{6}03\dot{6}82\dot{4}28\dot{7}$?

7000=assumed root.
3

21|000)436036824287(20763658·2994
Subtract 7000×7000÷12=4083333·3333

$\sqrt{16680324\cdot9661}=4084\cdot15$
Add ½ the assumed root=3500

And it gives the approximated root=7584 15

For the second operation, use the approximated root as the assumed one, and proceed as above.

THIRD METHOD BY APPROXIMATION.

1. Assume the root in the usual way, then multiply the square of the assumed root, by 3, and divide the resolvend by this product; to this quotient add ⅔ of the assumed root, and the sum will be the true root, or an approximation to it.

2. For each succeeding operation let the last approximated root be the assumed root, and proceeding in this manner, the root may be extracted to any assigned exactness.

1st. What is the cube root of 7?

Let the assumed root be 2. Then. $\overline{2\times2}\times3=12$ the divisor.

12)7 0(·583 to this add ⅔ of 2=1·333, &c. that is, ·583+1·333=1 916 approximated root.

Now assume 1·916 for the root. Then, by the second process, the root is $\frac{7}{3\times1\cdot916^2}+\overline{\frac{2}{3}\times1\cdot916}=1\cdot9126$, &c.

2d. What is the cube root of 9? Let 2 be the assumed root as before. Then, $\frac{9}{12}+\overline{\frac{2}{3}\times2}=2\cdot08$ the approximated root. Now assume 2·08. Then, $\frac{9}{3\times2\cdot08^2}+\overline{\frac{2}{3}\times2\cdot08}=2\cdot08008$, &c.

3d. What is the cube root of 282? Let 6 be the assumed root.—Then, 6×6×3=108)282(2 611, &c. and 2·611+⅔ of 6=6·611 approximated root. Now assume 6·611, and it will be 6·611×6·611×3=131·116)282(2·1507, &c. and 2 1507+⅔ of 6·611=6·558 a farther approximated root.

4th. What is the cube root of 1728?—Here the assumed root is 10. Then, 10×10×3=300)1728(5·76, and 5·76+⅔ of 10=12 426.—Now assume 12 426, then 12 426×12 426×3=463·216428)1728(3·732, and 3·732+⅔ of 12·426=12·014 a farther approximated root, and so on.

APPLICATION AND USE OF THE CUBE ROOT.

1. To find *two* mean proportionals between any two given numbers.

RULE.

1. Divide the *greater* by the *less*, and extract the cube root of the quotient.

2. Multiply the root, so found, by the least of the given numbers, and the product will be the *least*.

3. Multiply this product by the same root, and it will give the greatest.*

EXAMPLES.

1st. What are the two mean proportionals between 6 and 750?

$750 \div 6 = 125$, and $\sqrt[3]{125} = 5$. Then, $5 \times 6 = 30 =$ least, and $30 \times 5 = 150 =$ greatest. Answer 30 and 150.

Proof. As 6 : 30 :: 150 : 750.

2d. What are the two mean proportionals between 56 and 12096?

Answer 336 and 2016.

Note. The solid contents of similar figures are in proportion to each other, as the cubes of their similar sides or diameters.

3d. If a bullet 6 inches diameter weigh 32lb; What will a bullet of the same metal weigh, whose diameter is 3 inches?

$6 \times 6 \times 6 = 216$. $3 \times 3 \times 3 = 27$. As 216 : 32lb :: 27 : 4lb. Ans.

4th. If a globe of silver of 3 inches diameter, be worth £45, What is the value of another globe, of a foot diameter?

Ans. £2880.

The side of a cube being given, to find the side of that cube which shall be double, triple, &c. in quantity to the given cube.†

RULE.

Cube your given side, and multiply it by the given proportion between the given and required cube, and the cube root of the product will be the side sought.

5th. If a cube of silver, whose side is 4 inches, be worth £50, I demand the side of a cube of the like silver, whose value shall be 4 times as much?

$4 \times 4 \times 4 = 64$, and $64 \times 4 = 256$. $\sqrt[3]{256} = 6{\cdot}349 +$ inches, Ans.

6th. There is a cubical vessel, whose side is 2 feet; I demand the side of a vessel, which shall contain three times as much?

Ans. 2ft. $10\frac{5}{8}$ inches.

7th.‡ The diameter of a bushel measure being $18\frac{1}{2}$ inches, and the height 8 inches, I demand the side of a cubic box, which shall contain that quantity. Ans. $12{\cdot}907 +$ inches.

* As two mean proportionals are required to two given numbers, there will be four terms in the proportion, in which the first is to the second, as the second to the third, and the third to the fourth. The numbers therefore belong to a geometrical progression of four terms. The first part of the rule is explained in Prob. VIII. of Geometrical Progression, and the second and third parts of the rule are evident from the proof of Prob. I. of Geometrical Progression.

† The solid, called a cube, has its length and breadth and height all equal. As the number of solid feet, inches, &c. in a cube are found by multiplying the height and length and breadth together, that is, by multiplying one side into itself twice, the third power of a number is called the *cube* of that number.

‡ Multiply the square of the diameter by ·7854, and the product by the height: the cube root of the last product is the answer. See *Mensuration of Superficies and Solids, Art.* 30.

8. Suppose a ship of 500 tons has 89 feet keel, 36 feet beam, and is 16 feet deep in the hold: What are the dimensions of a ship of 200 tons, of the same mould and shape?

$89\times89\times89=704969=$cubed keel.

As 500 : 200 :: 704969 : 281987·6 cube of the required keel.

$\sqrt[3]{281987\cdot6}=65\cdot57$ feet the required keel.

As 89 : 65·57 :: 36 : 26·522=26½ feet, beam, nearly.

As 89 : 65·57 :: 16 : 11·7 feet, depth of the hold, nearly.

9. From the proof of any cable to find the strength of any other.

Rule.—The strength of cables, and consequently the weights of their anchors, are as the cubes of their peripheries

If a cable, 12 inches about, require an anchor of 18cwt.: Of what weight must an anchor be, for a 15 inch cable?

Cwt. Cwt.

As $12\times12\times12$: 18 :: $15\times15\times15$: 35·15625 Ans.

10. If a 15 inch cable require an anchor 35·15625cwt.: What must the circumference of a cable be, for an anchor of 18cwt?

12 inches, Answer.

EXTRACTION OF THE BIQUADRATE ROOT.

Rule.

Extract the square root of the resolvend, and then the square root of that root, and you will have the biquadrate root.

What is the biquadrate root of 20736?

```
20736(144          144(12 root required.
1                  1
---                ---
24)107           22)44
    96                44
   ----               ---
284)1136              ..
    1136
```

TWO METHODS OF EXTRACTING THE BIQUADRATE ROOT BY APPROXIMATION.

Rule I.

1. Divide the resolvend by *six* times the square of the assumed root, and from the quotient subtract $\frac{1}{12}$ part of the square of the assumed root.

2. Extract the square root of the remainder.

3. Add $\frac{2}{3}$ of the assumed root to the square root, and the sum will be the true root, or an approximation to it.

4. For every succeeding operation, either in this or the following method, proceed in the same manner, as in the first, each time using the last approximated root for the assumed root.

The biquadrate root of 20736 is required.

Here 10 is the assumed root.

$10\times10\times6=600)20736(34\cdot56$
Subtract $10\times10\div18 = 5.5555$

$\sqrt{29\cdot0044}=5\cdot38$
Add $\frac{2}{3}$ of $10 \quad =6\ 66$

Approximated root 12·04, to be made the assumed root for the next operation.

Rule II.

Divide the resolvend by *four* times the cube of the assumed root: to the quotient add *three fourths* of the assumed root, and the sum will be the true root, or an approximation to it.

Let the biquadrate of 20736 be required, as before?
The assumed root is 10
$10\times10\times10\times4=4000)20736(5.184$
Add $\frac{3}{4}$ of $10=7\cdot5$

Approximated root 12·684, to be made the assumed root for the next operation.

EXTRACTION OF THE SURSOLID ROOT BY APPROXIMATION.

A Particular Rule.*

1. Divide the resolvend by *five* times the assumed root, and to the quotient add *one twentieth* part of the *fourth* power of the same root.
2. From the square root of this sum subtract *one fourth* part of the square of the assumed root.
3. To the square root of the remainder add *one half* of the assumed root, and the sum is the root required, or an approximation to it.

Note. This rule will give the root true to *five* places, at the least, (and generally to eight or nine places) at the first process.

Required the sursolid root of 281950621875?
200=assumed root.
5
$1000)\ 281950621\cdot875$ quotient.
Add $200\times200\times200\times200\div20 = 80000000$

$\sqrt{361950621\cdot875}=19025$ nearly,
Subtract $200\times200\div4=10000$
$\sqrt{9025}=95$
Add half the assumed root $= 100$
Required root 195

$$* \ r\pm e=\sqrt{\sqrt{\frac{G}{5r}+\frac{r^4}{20}}-\frac{rr}{4}}+\frac{r}{2}$$

A General Rule for Extracting the Roots of all Powers.

1.* Prepare the given number, for extraction, by pointing off from the unit's place, as the required root directs.

2. Find the first figure of the root by trial, or by inspection into the table of powers, and subtract its power from the left hand period.

3. To the remainder bring down the first figure in the next period, and call it the *dividend*.

4. Involve the root to the next inferiour power to that which is given, and multiply it by the number denoting the given power, for a *divisor*.

5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.

6. Involve the whole root to the given power, and subtract it from the *given number*, as before.

7. Bring down the first figure of the next period to the remainder for a new dividend, to which find a new divisor, as before, and, in like manner, proceed till the whole be finished.

Examples.

1st. What is the cube root of 20346417?

20346417(273 2×2×2=8 root of the 1st. period, or 1st. Subtrahend.

2^3 = 8 = 1st. Subtrah. 2×2=4(=next inferiour power,) and, 4×3=(the index of the given pow.)= 12 1st. Divisor.

2^2×3=12)123=Dividend 27×27×27=19683=2d. Subtrahend.

27×27=729 (next inferior power) and,

27^3 = 19683=2d. Subt. 729×3 (=index of the given pow.)= 2187=2d. Divisor.

37^2×3=2187)6634=2d.Di. 273×273×273=27346417=3d. Subtra.

273^3 = 20346417=3d. Subtrahend.

. . . .

* The extracting of roots of very high powers will, by this rule, be a tedious operation: The following method, when practicable, will be much more convenient.

When the index of the power, whose root is to be extracted, is a composite number, take any two or more *indices*, whose product is equal to the given *index*, and extract out of the given number a root answering to another of the indices, and so on to the last.

Thus, the fourth root=square root of the square root; the sixth root=square root of the cube root; the eighth root=square root of the fourth root; the ninth root=the cube root of the cube root; the tenth root=square root of the fifth root; the twelfth root=the cube root of the fourth, &c.

The general rule for extracting the roots of all powers, may be illustrated in the same way, as those for the square and cube roots. Any student may at once see the truth of the rule, in exhausting the several products of the case illustrating the rule for the cube root. And the same will be evident by raising the number to any higher power.

2d. What is the biquadrate root of 34827998976 ? Ans. 431·9+.

3d. Extract the sursolid, or fifth root of 281950621875 ?

Ans. 195.

4th. Extract the square cubed, or sixth root of 1178420166015625?

Ans. 325.

A General* Rule for Extracting Roots by Approximation.

1. Subtract *one* from the exponent of the root required, and multiply half of the remainder by that exponent, and this product by that power of the assumed root, whose exponent is *two* less than that of the root required.

* The general theorem for the extraction of all roots, by approximation, from whence the rule was taken, and the Theorems deducible from it, as high as the twelfth power. Let G=resolvend whose root is to be extracted. ‡$\overline{r \pm e}$= root required; r being assumed as near the true root, and m=exponent of the power—then the equation will stand thus.

$$\overline{r \pm e}\sqrt{\frac{G}{\frac{m-1}{2}m\,r^{m-2}} - \frac{\overline{m-2}\,r^2}{m\,\overline{m-1}^2}} + \frac{m-2}{m-1}r. \text{ Hence,}$$

Theorem for the cube root $\overline{r \pm e} = \sqrt{\frac{G}{3r} - \frac{rr}{12}} + \frac{r}{2}$

For the Biquadrate - - - - $\sqrt{\frac{G}{6rr} - \frac{rr}{18}} + \frac{2r}{3}$

For the Sursolid - - - - $\sqrt{\frac{G}{10r^3} - \frac{3rr}{80}} + \frac{3r}{4}$

For the squared cube root $\sqrt{\frac{G}{15r^4} - \frac{2rr}{75}} + \frac{3r}{5}$

For the second sursolid - - $\sqrt{\frac{G}{21r^5} - \frac{5rr}{252}} + \frac{5r}{6}$

For the squared Biquadrate $\sqrt{\frac{G}{28r^6} - \frac{3rr}{196}} + \frac{6r}{7}$

For the cubed cube - - - $\sqrt{\frac{G}{36r^7} - \frac{7rr}{576}} + \frac{7r}{8}$

For the squared sursolid - $\sqrt{\frac{G}{45r^8} - \frac{4rr}{405}} + \frac{8r}{9}$

For the third sursolid - - $\sqrt{\frac{G}{55r^9} - \frac{9rr}{1100}} + \frac{9r}{10}$

For the squared square cube $\sqrt{\frac{G}{66r^{10}} - \frac{5rr}{726}} + \frac{10r}{11}$ &c.

‡ By this Theorem the fraction is obtained in numbers to the lowest terms in all the odd powers; and in the even powers only by having the numerator and denominator found by this equation.

2. Divide the given number by the last product; and from the quotient subtract a fraction, whose numerator is obtained by subtracting *two* from the exponent, and multiplying the remainder by the square of the assumed root; and whose denominator is found by subtracting *one* from the exponent and multiplying the square of the remainder by the exponent.

3. After this subtraction is made, extract the square root of the remainder.

4. From the exponent subtract *two*, and place the remainder as a numerator; then subtract one from the exponent, and place the remainder under the numerator for a denominator.

5. Multiply this fraction by the assumed root; add the product to the square root, before found, and the sum will be the root required, or an approximation to it.

Example.

What is the square cubed root of 1178420166015625?
Let the assumed root = 300

Exponent of the required root is 6. Then, $\frac{\overline{6-1}}{2}\times 6=15$.

$300^4=8100000000$ and this multiplied by $15=121500000000$.
$1178420166015625\div 121500000000=9698\cdot 9314$, from this

Subtract $\frac{\overline{6-2}\times 300^2}{6\times\overline{6-1}^2}$ $=2400$

And $\sqrt{7298\cdot 9314}=85\cdot 43$

To which add $\frac{6-2}{6-1}\times 300=$ 240

And the sum is the approximated root= 325·43

For the 2d operation, let 325·43 = assumed root.

ANOTHER METHOD BY APPROXIMATION.*

Rule.

1. Having assumed the root in the usual way, involve it to that power denoted by the exponent less 1.

* A rational formula for extracting the root of any pure power by approximation.

Let the resolvend be called G, and let $r\pm e$ be the required root, r being assumed in the usual way.

Let $G^{\frac{1}{m}}$ be required; then $r\pm e=\frac{G}{mr^{m-1}}+\frac{m-1}{m}r$ the general Theorem.

Hence, For the cube root $r\pm e=\frac{G}{3r^2}+\frac{2}{3}r$.

For the biquadrate - - $\frac{G}{4r^3}+\frac{3}{4}r$.

2. Multiply this power by the exponent.

3. Divide the resolvend by this product, and reserve the quotient.

4. Divide the exponent of the given power, less 1, by the exponent, and multiply the assumed root by the quotient.

5. Add this product to the reserved quotient, and the sum will be the true root, or an approximation.

6. For every succeeding operation, let the root last found, be the assumed root.

EXAMPLE.

What is the square cubed root of 117842016601562 5?

The exponent is 6. Let the assumed root be 300.

Then $300^5 \times 6 = 14580000000000$

14580000000000)117842016601562 5(80·824.

Add $\frac{5}{6} \times 300 = 250$

330·824=approximated root.

For the next operation, let 330·824 be the assumed root.

SURDS.

I. SURDS are quantities, whose roots cannot be obtained exactly, but may be approximated to any definite extent by continuing the extraction of the roots. Surds are expressed by fractional indices or exponents, or by the radical sign $\sqrt{}$. Thus, $3^{\frac{1}{2}}$, or $\sqrt{3}$,

For the sursolid - - - $\frac{G}{5r^4} + \frac{4}{5}r$.

For the square cubed - $\frac{G}{6r^5} + \frac{5}{6}r$.

For the seventh root - - $\frac{G}{7r^6} + \frac{6}{7}r$.

For the eighth - - - - $\frac{G}{8r^7} + \frac{7}{8}r$.

For the ninth - - - - $\frac{G}{9r^8} + \frac{8}{9}r$.

For the tenth - - - - $\frac{G}{10r^9} + \frac{9}{10}r$.

For the eleventh - - $\frac{G}{11r^{10}} + \frac{10}{11}r$,

For the twelfth - - - $\frac{G}{12r^{11}} + \frac{11}{12}r$. &c.

denotes the square root of 3. The value of $2^{\frac{1}{2}}$ or $\sqrt{2}$, to the hundredth place of decimals, is 1·41, and to the millionth place is 1·414213. The value of $\sqrt{2}$ may be obtained more nearly by continuing the extraction, but can never be obtained with perfect accuracy, as is easily proved in the following section.

Surds are often called *irrational* quantities, because their value cannot be expressed by figures. They are thus distinguished from assignable quantities, which are called *rational* quantities. Thus, 2 is a *rational*, and $\sqrt{2}$, an *irrational* quantity.

A surd is always connected with a *rational* quantity expressed or understood. Thus, as the square root of 2, or $\sqrt{2}$, is that quantity taken *once*, unity is understood, and the surd is expressed either $\sqrt{2}$, $1\sqrt{2}$, or $1\times\sqrt{2}$. If the surd is to be taken more than once, the number of times is always expressed; thus $3\sqrt{3}$, or $3\times\sqrt{3}$, means *thrice* the square root of 3, or the surd taken *three* times.

Hence it is that an expression of this form, $1\sqrt{2}$, or $3\sqrt[3]{5}$, is considered as consisting of 2 parts, a *rational*, and an *irrational* part, the rational part always expressing the number of times the surd is taken.

From the notation of powers, and surds, these expressions are equivalent; viz. $3^{\frac{5}{4}}=\sqrt[4]{3^5}$; and $2^{\frac{2}{3}}=\sqrt[3]{2^2}$. Also, $5^{\frac{3}{2}}=\sqrt{5^3}$, that is, the square root of the cube of 5, or the cube of the square root of 5.

Note. Though surds are expressed by means of fractional indices or the radical sign, yet it is common to apply the same indices or radical sign, to *complete* powers, whose roots are to be extracted. The student will observe, therefore, that quantities expressed in the form of surds are not necessarily *surd quantities*. One number also may be a complete power of one kind, but not of another. Thus $4^{\frac{1}{2}}$ is 2, but $4^{\frac{1}{3}}$ is a surd; and $64^{\frac{1}{2}}$ is 8, and $64^{\frac{1}{3}}$ is 4, but $64^{\frac{1}{4}}$ and $64^{\frac{1}{5}}$ or $\sqrt[4]{64}$ and $\sqrt[5]{64}$ are surds.

II. As few numbers are complete powers, surds must very often occur in arithmetical operations. *If the root of a whole number is not a whole number, neither is it a whole number and a decimal,* which can be assigned. For, supposing the entire root to be obtained, when it was raised to the power, it would produce a whole number and a decimal; while the supposition requires that only a whole number should be produced. Thus, supposing the square root of 2, or $\sqrt{2}$, to be exactly 1·41, or 1·414213, this root raised to the square should produce 2; but it is obvious that the square would be a whole number and a decimal, and not the number 2.

It is equally evident, that the root of a vulgar fraction cannot be assigned, unless both parts of the fraction when reduced to its lowest terms, are complete powers of the roots required. Thus $\sqrt{\frac{4}{16}}=\sqrt{\frac{1}{4}}=\frac{1}{2}$; but $\sqrt{\frac{4}{8}}=\sqrt{\frac{1}{2}}$ is a surd, and the entire value of the square root of the fraction cannot be obtained.

III. Though the value of a surd cannot be assigned, its power is assignable. From the definition of a root, it is evident that $2^{\frac{1}{2}}$ or $\sqrt{2}$ is such a number as multiplied by itself, the product or square will be 2. Thus $\sqrt{2}\times\sqrt{2}$ or $2^{\frac{1}{2}}\times2^{\frac{1}{2}}=2$. And $3^{\frac{1}{3}}\times3^{\frac{1}{3}}\times3^{\frac{1}{3}}=3$, and thus for other surds.

IV. Arithmetical calculations are often simplified by certain operations on surds, or quantities in the form of surds. Rules for several operations follow.

1. Any number may be reduced to the form of a surd, by raising it to the power denoted by the index of the surd, and then placing the power under the radical sign. Thus to reduce 2 to the form of the square root; because $2\times2=2^2=4$, $2=\sqrt{2^2}=\sqrt{4}$.

Reduce 2 to the form of the fifth root. Ans. $\sqrt[5]{32}$.

Reduce 5 to the form of the third root. Ans. $\sqrt[3]{125}$.

Reduce 7 to the form of the fourth root. Ans.

2. Surds are reduced to their *most simple terms*, by resolving the quantity under the radical sign into two factors, one of which shall be a complete power of the given root; and then placing the root of this power before the other factor under the radical sign. Thus $\sqrt{27}=\sqrt{\overline{9\times3}}=\sqrt{9}\times\sqrt{3}=3\times\sqrt{3}$ or $3\sqrt{3}$. Also, $\sqrt[4]{32}=\sqrt[4]{\overline{16\times2}}=\sqrt[4]{16}\times\sqrt[4]{2}=2\sqrt[4]{2}$.

Reduce $\sqrt{125}$ to its most simple terms. Ans. $5\sqrt{5}$.

Reduce $\sqrt[3]{3584}$ to its most simple terms. Ans. $8\sqrt[3]{7}$.

Reduce $\sqrt[4]{\frac{96}{4375}}$ to its simplest terms. Ans. $\frac{2}{5}\sqrt{\frac{6}{7}}$.

Reduce $\sqrt{481}$, $\sqrt[3]{351}$, and $\sqrt[7]{896}$ to their most simple terms.

Reduce $5\sqrt{20}$ to its simplest terms. Ans. $10\sqrt{5}$.

Hence, it is obvious, that if a factor be multiplied into a surd, the whole may be reduced to the form of a surd, by raising the factor to the power denoted by the surd, multiplying the power into the surd, and placing the product under the radical sign. Thus $3\sqrt{3}=\sqrt{3^2}\times\sqrt{3}=\sqrt{\overline{9\times3}}=\sqrt{27}$; and $8\sqrt[3]{7}=\sqrt[3]{8^3}\times\sqrt[3]{7}=\sqrt[3]{\overline{512\times7}}=\sqrt[3]{3584}$.

3. Surds of the same radical sign may be added together, when the quantities under the radical sign are the same, by prefixing the *sum* of the rational parts to the surd quantity. Thus $1\sqrt{2}+1\sqrt{2}$, or $\sqrt{2}+\sqrt{2}=2\sqrt{2}$, or twice $\sqrt{2}$; and $3\sqrt[3]{5}+4\sqrt[3]{5}=7\sqrt[3]{5}$.

If the surds are not already in their most simple terms, they may often be added after the reduction is made. Thus $\sqrt{20}+\sqrt{80}=2\sqrt{5}+4\sqrt{5}=6\sqrt{5}$; and, $\sqrt[4]{162}+\sqrt[4]{1350}=3\sqrt[4]{2}+5\sqrt[4]{2}=8\sqrt[4]{2}$.

What is the sum of $\sqrt[3]{56}$ and $\sqrt[3]{3584}$? Ans. $10\sqrt[3]{7}$.

4. Surds of the same radical sign may be subtracted, if the surd part be the same, by placing the difference of the rational parts before the surd. If the quantities are not already in their simplest

terms, they should be reduced to this form. Thus $\sqrt[3]{4}-\sqrt[3]{4}=0$; and $3\sqrt{3}-2\sqrt{3}=1\sqrt{3}$ or $\sqrt{3}$. Also $7\sqrt[5]{5}-4\sqrt[5]{5}=3\sqrt[5]{5}$.

What is the difference between $\sqrt[4]{1350}$ and $\sqrt[4]{162}$? Ans. $2\sqrt[4]{2}$.

Note 1. Surds, apparently incapable of addition or subtraction except by their signs, may sometimes be reduced to a common surd, by the following process, and their sum and difference readily found. Thus let the surds be $\sqrt{\frac{2}{3}}$ and $\sqrt{\frac{3}{8}}$. As $\sqrt{\frac{2}{3}}=\sqrt{\frac{2}{3}\times\frac{2}{2}}=\sqrt{\frac{4}{6}}=2\sqrt{\frac{1}{6}}$, and as $\sqrt{\frac{3}{8}}=\sqrt{\frac{9}{24}}=\frac{3}{2}\sqrt{\frac{1}{6}}$, then $\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{8}}=2\sqrt{\frac{1}{6}}+\frac{3}{2}\sqrt{\frac{1}{6}}=\frac{7}{2}\sqrt{\frac{1}{6}}=\frac{7}{2}\sqrt{\frac{6}{36}}=\frac{7}{2}\times\frac{1}{6}\sqrt{6}=\frac{7}{12}\sqrt{6}$, their sum: And $2\sqrt{\frac{1}{6}}-\frac{3}{2}\sqrt{\frac{1}{6}}=\frac{1}{2}\sqrt{\frac{1}{6}}=\frac{1}{12}\sqrt{6}$, their difference.

What is the sum and difference of $\sqrt{\frac{2}{3}}$ and $\frac{3}{5}\sqrt{\frac{3}{2}}$?

Ans. Their sum is $\frac{19}{30}\sqrt{6}$. Their diff. is $\frac{1}{30}\sqrt{6}$.

What is the sum of $\frac{4}{45}\sqrt{15}$ and $\sqrt{\frac{5}{27}}$? Ans. $\sqrt{\frac{3}{5}}$.

What is the difference of $\sqrt{\frac{5}{3}}$ and $\sqrt{\frac{5}{12}}$? Ans. $\frac{1}{6}\sqrt{15}$.

Note 2. If the same quantity is under different radical signs, or if the same radical sign has different quantities under it when the surds are in their simplest terms, the surds can be added or subtracted only by the signs of addition or subtraction. Thus it is evident, that $\sqrt{2}+\sqrt[3]{2}$, is neither twice the square root of 2 nor twice the cube root of 2; and that $3\sqrt{3}-2\sqrt[3]{3}$, is neither the square root of 3 nor the cube root of 3. It is equally obvious, that $2\sqrt{3}+2\sqrt{2}$, is neither four times the square of 3 nor of 2; and that $4\sqrt{2}-2\sqrt{3}$, is neither twice the square of 2 nor of 3.

5. Surds of the same radical sign are multiplied like other numbers, but the product must be placed under the same radical sign. Thus $\sqrt[3]{27}\times\sqrt[3]{64}=\sqrt[3]{27\times64}=\sqrt[3]{1728}=12$, for $\sqrt[3]{27}=3$, and $\sqrt[3]{64}=4$, and $\sqrt[3]{27}\times\sqrt[3]{64}=3\times4=12$. And $\sqrt{2}\times\sqrt{3}=\sqrt{2\times3}=\sqrt{6}$. Also $3\sqrt{3}\times4\sqrt{5}=12\sqrt{15}$ or $\sqrt{27}\times\sqrt{80}$, and $\sqrt{27}\times\sqrt{80}=\sqrt{27\times80}=12\sqrt{15}=\sqrt{2160}$.

Sometimes the product of the surds becomes a complete power of that root, and the root should then be extracted, as in the first of the preceding examples. Also in this example; $\sqrt{2}\times\sqrt{200}=\sqrt{400}=20$.

It is evident from the first example in this section, that, when quantities are under the same radical sign, the *root of the product of quantities is equal to the product of their roots.*

If a surd be raised to a power denoted by the index of the root, the power will be rational. Thus, $\sqrt{3}\times\sqrt{3}$, or $3^{\frac{1}{2}}\times3^{\frac{1}{2}}=3$. In this example 2 is the index of the root, and the surd is raised to the second power or square. Also $\sqrt[3]{4}\times\sqrt[3]{4}\times\sqrt[3]{4}=4$. If fractional indices be used, in order to multiply surds of the *same* root, you have only to add the indices. Thus $4^{\frac{1}{3}}\times4^{\frac{1}{3}}\times4^{\frac{1}{3}}=4^{\frac{3}{3}}=4^{1}$ or 4, unity being the implied index of 4, or of the first power of any number. In all cases when *the sum of the numerators contains the com-*

mon denominator a certain number of times exactly, the product will be rational. Thus $3^{\frac{2}{3}}\times3^{\frac{2}{3}}\times3^{\frac{2}{3}}=3^{\frac{2+2+2}{3}}=3^{\frac{6}{3}}=3^2=9$, and $7^{\frac{5}{2}}\times7^{\frac{5}{2}}=7^{\frac{5+5}{2}}=7^{\frac{10}{2}}=7^5$.

As 5^4 may be expressed according to the notation of powers, thus, $5^{\frac{4}{1}}$, and 5^3 by $5^{\frac{3}{1}}$, hence $5^{\frac{4}{1}}\times5^{\frac{3}{1}}=5^{\frac{4+3}{1}}=5^{\frac{7}{1}}=5^7$. Therefore, to multiply *different* powers of the same root, you have only to add the indices of the given root, and place the sum for the index of the power which is produced. Thus $3^2\times3^2=3^4$, or the square of a number multiplied by itself produces the fourth power; the cube by the cube, the sixth power, and so on. Thus also $2^{\frac{5}{4}}\times2^{\frac{3}{4}}=2^{\frac{5}{4}+\frac{3}{4}}=2^{\frac{8}{4}}=2^{\frac{2}{1}}=2^2$.

6. Surds of the same radical sign are divided like whole numbers, but the quotient must be placed under the same radical sign. Thus $\sqrt[3]{1728}\div\sqrt[3]{64}=\sqrt[3]{1728\div64}=\sqrt[3]{27}=3$; and $\surd6\div\surd3=\sqrt{6\div3}=\surd2$. Sometimes the quotient becomes a complete power, as in the first example, in which case the root should be extracted. So also in the following; $\surd400\div\surd100=\sqrt{400\div100}=\surd4=2$.

As $\sqrt[3]{1728}=12$, and $\sqrt[3]{64}=4$, then $\sqrt[3]{1728}\div\sqrt[3]{64}=12\div4=3=\sqrt[3]{27}$. Hence, *the quotient of the roots of quantities is the same as the root of their quotient*, if the quantities are under the same radical sign.

Divide $\surd108$ by $\surd6$. Now $\surd108\div\surd6=\sqrt{108\div6}=\surd18=\sqrt{9\times2}=3\surd2$.

Divide $9\surd100$ by $3\surd2$. Now $9\surd100=\surd8100$, and $3\surd2=\surd18$, and $\surd8100\div\surd18=\sqrt{8100\div18}=\surd450=15\surd2$. Or $9\surd100\div3\surd2=9\div3\times\surd100\div\surd2=9\div3\times\sqrt{100\div2}=9\div3\times\surd50=3\surd50=3\times5\surd2=15\surd2$.

Divide $\frac{1}{3}\sqrt[3]{\frac{18}{125}}$ by $\frac{1}{2}\sqrt[3]{\frac{1}{3}}$. Now $\frac{1}{3}\div\frac{1}{2}=\frac{2}{3}$; and $\sqrt[3]{\frac{18}{125}}\div\sqrt[3]{\frac{1}{3}}=\sqrt[3]{\frac{54}{125}}=\frac{3}{5}\sqrt[3]{\frac{2}{1}}=\frac{3}{5}\surd2$; and $\frac{2}{3}\times\frac{3}{5}\surd2=\frac{2}{5}\surd2$.

Divide $\sqrt[4]{48}$ by $\sqrt[4]{\frac{1}{3}}$; $5\surd60$ by $3\surd15$; and $\frac{1}{4}\surd\frac{1}{3}$ by $\frac{1}{8}\surd\frac{2}{3}$.

If the quantities under the radical sign be the same, the quotient will be found by dividing the rational parts only. Thus $\surd2\div\surd2=\surd1=1$. or $\surd2$ is contained in $\surd2$ once. Also $\frac{1}{4}\sqrt[3]{3}\div\frac{1}{8}\sqrt[3]{3}=2$, and $2\surd5\div5\surd5=\frac{2}{5}$.

To divide one power by another of the same root, *place the difference of the indices for the index of the given root.* This is merely reversing a process given in the preceding section. The reason of the process may also be seen in the following manner. Thus $2^4\div2^2=\frac{2^4}{2^2}=\frac{2^2\times2^2}{2^2}=2^2=2^{4-2}$. Also, $2^2\div2^{\frac{3}{4}}=2^{\frac{8}{4}}\div2^{\frac{3}{4}}$, by reduc-

ing the indices to a common denominator; and $2^{\frac{8}{4}}\div 2^{\frac{3}{4}}=\frac{\sqrt[4]{2^8}}{\sqrt[4]{2^3}}=$ $\sqrt[4]{\frac{2^8}{2^3}}=\sqrt[4]{\frac{2^3\times 2^5}{2^3}}=\sqrt[4]{2^5}=\sqrt[4]{2^{8-3}}=2^{\frac{5}{4}}$.

If the index of the divisor exceed that of the dividend, the index of the quotient will be their difference with the sign of subtraction before it. Thus, $5^2\div 5^5=5^{2-5}=5^{-3}$. Now, as $5^2\div 5^5=\frac{5^2}{5^5}=\frac{5^2}{5^2\times 5^3}=\frac{1}{5^3}$, $5^{-3}=\frac{1}{5^3}$. Hence a power whose index has the sign of subtraction before it, is the same power of the *reciprocal* of that quantity. Hence, there is an obvious method of transferring powers from the numerator to the denominator of a fraction. Thus, $\frac{1}{2^2}=2^{-2}$, and $\frac{3^{-2}}{4}=\frac{1}{4\times 3^2}$, and $\frac{2}{3^{-2}}=2\times 3^2$. There is also an obvious method of finding the value of a quantity whose index has the sign of subtraction before it. Thus $2^{-3}=\frac{1}{2^3}=\frac{1}{8}$, and $\overline{\tfrac{1}{4}|}^{-2}=\overline{\tfrac{4}{1}|}^{2}=16$, or, $\cdot 25^{-2}=\frac{1}{\cdot 25^2}=\frac{1}{\cdot 0625}=16$. And $2^{-\frac{1}{2}}=\frac{1}{2^{\frac{1}{2}}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$.

7. To raise surds to any power, multiply the index of the surd by the index of the required power. Thus $2^{\frac{1}{2}}$ raised to the square is $2^{\frac{1}{2}+\frac{1}{2}}=2^{\frac{2}{2}}=2^{\frac{1}{2}\times 2}=2$; and the cube of $3^{\frac{1}{4}}=3^{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}=3^{\frac{1}{4}\times 3}$ $=3^{\frac{3}{4}}$.

If there be rational parts with the surd, they must be raised to the given power, and prefixed to the required power of the surd. Thus, $3\sqrt[3]{3}$, or $3\times 3^{\frac{1}{3}}$ raised to the square, is $3^2\times 3^{\frac{1}{3}+\frac{1}{3}}=3^2\times 3^{\frac{2}{3}}$ $=3^2\times 3^{\frac{1}{3}\times 2}=9\sqrt[3]{3^2}$, or $9\sqrt[3]{9}$. And the cube of $2^{\frac{1}{2}}=2^{\frac{1}{2}+\frac{1}{2}+\frac{1}{2}}=$ $2^{\frac{1}{2}\times 3}=2^{\frac{3}{2}}=\sqrt{2^3}=\sqrt{8}=2\sqrt{2}$, when reduced to its simplest terms. Also, the fourth power of $\frac{1}{4}\sqrt{2}$ is $\frac{1}{256}\times 2^2=\frac{1}{64}$.

Required the fifth power of $\frac{1}{3}\sqrt[3]{\frac{1}{3}}$.

8. To extract any root of a surd quantity, divide the index of the quantity by the index of the required root. Thus, the square root of $2^{\frac{1}{3}}$ is $2^{\frac{1}{3}\div 2}=2^{\frac{1}{6}}$ or $\sqrt[6]{2}$, and the cube root of $3^{\frac{2}{4}}$ is $3^{\frac{2}{4}\div 3}$ $=3^{\frac{2}{3\times 4}}=3^{\frac{1}{6}}$ or $\sqrt[6]{3}$.

If there be rational parts with the surd, their root must be prefixed to the required root of the irrational part. Thus, the square root of $9\sqrt[3]{9}$, or $9\times 9^{\frac{1}{3}}=9^{\frac{1}{2}}\sqrt[3]{3^2}=3\sqrt[3]{3}$. The process must evidently be the reverse of that in the preceding section, and the reason of it is obvious.

What is the cube root of $\frac{8}{27}\sqrt[3]{8}$? Ans. $\frac{4}{3}$.

What is the square root of 10^5? Ans. $10^{\frac{5}{2}}$ or $100\sqrt{10}$.

Examples.

1. Multiply $6^{\frac{1}{4}}$ by $6^{\frac{1}{5}}$, and the product is $6^{\frac{9}{20}}$, or $\sqrt[20]{6^9}$.

2. Divide $6^{\frac{1}{4}}$ by $6^{\frac{1}{5}}$, and the quotient is $6^{\frac{1}{20}}$ or $\sqrt[20]{6}$.

3. Add $\sqrt[3]{32}$ and $\sqrt[3]{108}$, and multiply the same by $\sqrt[3]{\frac{4}{125}}$.

Ans. $16^{\frac{1}{3}}$ or $2\sqrt[3]{2}$.

4. Add $\sqrt[3]{32}$ and $\sqrt[3]{108}$, and divide the sum by $\sqrt[3]{\frac{4}{125}}$. Ans. 5^2.

5. Find the shortest method of dividing 3 by $\sqrt{2}$, to any given place of decimals.

$$\text{Now } \frac{3}{\sqrt{2}}=\frac{3\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=\frac{3\sqrt{2}}{2}=\frac{\sqrt{18}}{2}=\frac{4\cdot242640\ \&c.}{2}=2\cdot121320$$
&c.

6. Find the sum of $\sqrt[3]{\frac{1}{4}}$ and $\sqrt[3]{\frac{1}{32}}$, and also their difference.

Ans. Their sum is $\frac{1}{4}\sqrt[3]{54}$, or $\frac{3}{4}\sqrt[3]{2}$. Their diff. is $\frac{1}{2}\sqrt[3]{2}$, or $\sqrt[3]{\frac{1}{2}}$.

7. What is the sum and difference of $\sqrt[3]{\frac{2}{3}}$ and $\sqrt[3]{\frac{9}{32}}$.

Ans. Their sum is $\frac{7}{12}\sqrt[3]{18}$. Their diff. $\frac{1}{12}\sqrt[3]{18}$.

8. There are four spheres each 4 inches in diameter, lying so as to touch each other in the form of a square, and on the middle of this square is put a fifth ball of the same diameter; what is the perpendicular distance between the two horizontal planes which pass through the centres of the balls?

$$\text{Ans. } \frac{4}{\sqrt{2}}=\frac{4\sqrt{2}}{2}=2\sqrt{2}=\sqrt{8}=2\cdot8284+\text{ inches.}$$

Note. It may be seen from this example that the diameter of the ball divided by $\sqrt{2}$, will give the distance between the planes, whatever be the diameter of the ball, or, which is the same, half the diameter of the ball multiplied by the square root of 2.

9. There are two balls, each four inches in diameter, which touch each other, and another, of the same diameter is so placed between them that their centres are in the same vertical plane; what is the distance between the horizontal planes which pass through their centres?

$$\text{Ans. } \sqrt{\frac{4^2\times3}{4}}=\frac{4}{2}\sqrt{3}=2\sqrt{3}\text{ inches.}$$

Note. It is evident from this example, that in all similar cases, half the diameter of the ball multiplied by the square root of 3, gives the distance between the planes.

10. There is a quantity to whose square $\frac{1}{6}$ is to be added; of the sum the square root is to be taken and raised to the cube; to this power $\frac{4}{9}$ are to be added, and the sum will be $\frac{2}{9}\sqrt{15}$; what is that quantity? Ans. $\sqrt{\frac{1}{2}}$.

OF PROPORTION IN GENERAL.

NUMBERS are compared together to discover the relations they have to each other.

There must be two numbers to form a comparison: the number, which is compared, being written first, is called the *antecedent;* and that, to which it is compared, the *consequent.*

Numbers are compared with each other two different ways: The one comparison considers the *difference* of the two numbers, and is called arithmetical relation, the difference being sometimes named the arithmetical ratio; and the other considers their *quotient,* which is termed geometrical relation, and the quotient, the geometrical ratio. Thus, of the numbers 12 and 4, the difference or arithmetical ratio is $12-4=8$; and the geometrical ratio is $\frac{12}{4}=3$, and of 2 to 3 is $\frac{2}{3}$.

If two, or more, couplets of numbers have equal ratios, or differences, the equality is termed proportion; and their terms, similarly posited, that is, either all the greater, or all the less taken as antecedents, and the rest as consequents, are called proportionals. So the two couplets 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8, 6, are arithmetical proportionals; and the two couplets, 2, 4, and 8, 16, taken thus 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals.*

* To denote numbers as being geometrically proportional, the couplets are separated by a double colon, and a colon is written between the terms of each couplet; we may, also, denote arithmetical proportionals by separating the couplets by a double colon, and writing a colon turned horizontally between the terms of each couplet. So the above arithmeticals may be written thus, 2 .. 4 :: 6 .. 8, and 4 .. 2 :: 8 .. 6; where the first antecedent is less or greater than its consequent by just so much as the second antecedent is less or greater than its consequent: And the geometricals thus, 2 : 4 :: 8 : 16, and 4 : 2 :: 16 : 8; where the first antecedent is contained in, or contains its consequent, just so often, as the second is contained in, or contains its consequent.

Four numbers are said to be *reciprocally* or *inversely* proportional, when the fourth is less than the second, by as many times, as the third is greater than the first, or when the first is to the third, as the fourth to the second, and vice versa. Thus 2, 9, 6 and 3, are reciprocal proportionals.

Note. It is common to read the geometricals 2 : 4 :: 8 : 16, thus, 2 is to 4 as 8 to 16, or, As 2 to 4 so is 8 to 16.

Harmonical proportion is that, which is between those numbers which assign the lengths of musical intervals, or the lengths of strings sounding musical notes; and of *three numbers* it is, when *the first is to the third, as the difference between the first and second is to the difference between the second and third,* as the numbers 3, 4, 6. Thus, if the lengths of strings be as these numbers, they will sound an octave 3 to 6, a fifth 2 to 3, and a fourth 3 to 4.

Again, between 4 numbers, when *the first is to the fourth, as the difference between the first and second is to the difference between the third and fourth,* as in the numbers 5, 6, 8, 10; for strings of such lengths will sound an octave 5 to 10; a sixth greater, 6 to 10; a third greater 8 to 10; a third less 5 to 6; a sixth less 5 to 8; and a fourth 6 to 8.

Let 10, 12, and 15, be *three* numbers in harmonical proportion, then by the preceding definition, 10 : 15 :: 12—10 : 15—12, and by Theorem I. of Geometri-

Proportion is distinguished into continued and discontinued. If, of several couplets of proportionals, written down in a series, the difference or ratio of each consequent, and the antecedent of the next following couplet, be the same as the common difference or ratio of the couplets, the proportion is said to be continued, and the numbers themselves, a series of continued arithmetical or ge-

cal Proportion, $10\times\overline{15-12}=15\times\overline{12-10}$, or $10\times15-10\times12=15\times12-15\times10$, whence if any two of the three terms be given, the other may be found in the following manner.

Case 1. Given the 1st and 2d terms to find the 3d.

As $10\times15-10\times12=15\times12-15\times10$, then $10\times15-15\times12+15\times10=10\times12$, or $2\times10\times15-12\times15=10\times12$, or, $\overline{2\times10-12}\times15=10\times12$, and $15=\frac{10\times12}{2\times10-12}$, that is, 15, *the third is equal to the product of the first and second terms, divided by the difference of twice the first term and the second term.*

2. Given the 1st and 3d to find the second term.

From the same equivalent expression, we get $2\times10\times15=15\times12+10\times12=\overline{15+10}\times12$, and $\frac{2\times10\times15}{10+15}$, $=12$, that is, the *second term is equal to twice the product of the first and third terms, divided by the sum of the first and second terms.*

3. Given the second and third to find the first term.

From the same expression, we get $2\times10\times15-10\times12=15\times12$, or $\overline{2\times15-12}\times10=15\times12$, and $10=\frac{15\times12}{2\times15-12}$, that is, the *first term is equal to the product of the second and third terms, divided by the difference of twice the third term and the second term.*

Ex. Find from third term, or *monochord*, 50, and the first term, or *octave*, 25, the second term.

By Case 2, $\frac{2\times25\times50}{25+50}=\frac{2500}{75}=33{\cdot}33$, the second term, and is the length of that chord, which is called a *fifth*.

If there be *four* harmonical proportionals, as, 5, 6, 8 and 10; then, according to the definition, $5:10::6-5:10-8$, and as before, $5\times\overline{10-8}=10\times\overline{6-5}$, or $5\times10-5\times8=10\times6-10\times5$. From this expression, we may find any one of four harmonical proportionals from the other three. Thus, the first three being given to find the fourth; $2\times10\times5-10\times6=5\times8$, and $10=\frac{5\times8}{2\times5-6}$, that is, the *fourth term is equal to the product of the first and third divided by the difference of twice the first term and the second term.*

In the same manner, it may be shown, that the *third* term of four harmonical proportionals is equal to the *difference of twice the product of the first and fourth terms and the product of the second and fourth terms, divided by the first term.* If the terms be 5, 6, 8, and 10, then $8=\frac{2\times5\times10-6\times10}{5}$.

Also, The *second* term is *equal to the difference of twice the fourth and the third term, multiplied by the quotient of the first divided by the fourth term.* If the terms be as before, $6=\overline{2\times10-8}\times\frac{5}{10}$.

Also, The *first* term is *equal to the product of the second and fourth terms, divided by the difference of twice the fourth and the third term.* Thus $5=\frac{6\times10}{2\times10-8}$.

ometrical proportionals. So 2, 4, 6, 8, form an arithmetical progression; for 4—2=6—4=8—6=2; and 2, 4, 8, 16, a geometrical progression; for $\frac{4}{2}=\frac{8}{4}=\frac{16}{8}=2$.

But, if the difference or ratio of the consequent of one couplet, and the antecedent of the next couplet be not the same as the common difference or ratio of the couplets, the proportion is said to be discontinued. So 4, 2, 8, 6, are in discontinued arithmetical proportion; for 4—2=8—6=2=common difference of the couplets, 8—2=6=difference of the consequent of one couplet and the antecedent of the next; also, 4, 2, 16, 8, are in discontinued geometrical proportion; for $\frac{4}{2}=\frac{16}{8}=2$=common ratio of the couplets, and $\frac{16}{2}=8$=ratio of the consequent of one couplet and the antecedent of the next.

ARITHMETICAL PROPORTION.

Theorem I.

IF any four quantities 2, 4, 6, 8 be in arithmetical proportion,* the sum of the two means is equal to the sum of the two extremes.†

And if any three quantities, 2, 4, 6, be in arithmetical proportion, the double of the mean is equal to the sum of the extremes.

Theorem II.

In any continued Arithmetical Proportion (1, 3, 5, 7, 9, 11) the sum of the two extremes, and that of every other two terms, equally distant from them, are equal. Thus, 1+11=3+9=5+7.‡

When the number of terms is odd, as in the proportion 3. 8. 13. 18. 23, then, the sum of the two extremes being double to the mean or middle term, the sum of any other two terms, equally remote from the extremes, must likewise be double to the mean.

* Although in the comparison of quantities according to their differences, the term *proportion* is used: yet the word *progression*, is frequently substituted in its room, and is indeed more proper; the former form being, in the common acceptation of it, synonymous with ratio, which is only used in the other kind of comparison.

† For since 4—2=8—6, therefore 4+6=2+8.

‡ Since, by the nature of progressionals, the second term exceeds the first by just so much as its corresponding term, the last but one, wants of the last, it is evident that when these corresponding terms are added, the excess of the one will make good the defect of the other, and so their sum be exactly the same with that of the two extremes, and in the same manner it will appear that the sum of any two other corresponding terms must be equal to that of the two extremes.

THEOREM III.

In any continued Arithmetical Proportion, as 4, 4+2, 4+4, 4+6, 4+8, &c. the last or greatest term is equal to the sum of the first or least term and the common difference of the terms, multiplied by the number of the terms less one.*

THEOREM IV.

The sum of any rank, or series of quantities in continued Arithmetical Proportion (1. 3. 5. 7. 9. 11.) is equal to the sum of the two extremes multiplied into half the number of terms.†

ARITHMETICAL PROGRESSION.

ANY rank of numbers, more than two, increasing by a common excess, or decreasing by a common difference, is said to be in Arithmetical Progression.

If the succeeding terms of a progression exceed each other, it is called an ascending series or progression; if the contrary, a descending series.

So { 0. 2. 4. 6. 8. 10, &c. is an ascending arithmetical series.
1. 2. 4. 8. 16. 32, &c. is an ascending geometrical series.

And { 10. 8. 6. 4. 2. 0, &c. is a descending arithmetical series.
32. 16. 8. 4. 2. 1, &c. is a descending geometrical series.

* For since each term, after the first, exceeds that preceding it by the common difference, it is plain that the last must exceed the first by so many times the common difference as there are terms after the first; and therefore must be equal to the first, and the common difference repeated that number of times.

† For, because (by the second Theorem) the sum of the two extremes, and that of every other two terms, equally remote from them are equal, the whole series, consisting of half so many such equal sums as there are terms, will therefore be equal to the sum of the two extremes, repeated half as many times as there are terms.

The same thing also holds, when the number of terms is odd, as in the series 4, 8, 12, 16, 20; for then, the mean, or middle term, being equal to half the sum of any two terms, equally distant from it on contrary sides, it is obvious that the value of the whole series is the same as if every term thereof were equal to the mean, and therefore is equal to the mean (or half the sum of the two extremes) multiplied by the whole number of terms; or to the sum of the extremes multiplied by half the number of terms.

The sum of any *number* of terms of the arithmetical series of odd numbers 1, 3, 5, 7, 9, &c. is equal to the square of that *number*.

For, 0+1 or the sum of 1 term $= 1^2$ or 1
1+3 or the sum of 2 terms $= 2^2$ or 4
4+5 or the sum of 3 terms $= 3^2$ or 9
9+7 or the sum of 4 terms $= 4^2$ or 16
16+9 or the sum of 5 terms $= 5^2$ or 25, &c.

By continuing the addition, the rule would be true for any number of terms.

EXAMPLE.

The first term, the ratio, and number of terms given, to find the sum of the series.

A gentleman travelled 29 days, the first day he went but 1 mile, and increased every day's travel 2 miles; How far did he travel? $29\times 29=841$ miles, Ans.

The numbers which form the series, are called the *terms* of the progression.

Note. The first and last terms of a progression are called the extremes, and the other terms the means.

Any three of the five following things being given, the other two may be easily found.

1. The *first* term.
2. The *last* term.
3. The *number* of terms.
4. The *common difference.*
5. The *sum* of all the terms.

Problem I.

The first term, the last term, and the number of terms being given, to find the common difference.

Rule.*

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference sought.

Examples.

1st. The extremes are 3 and 39, and the number of terms is 19: What is the common difference?

$$\left.\begin{array}{r}39\\ -\ 3\end{array}\right\}\text{ Extremes.}$$

Divide by the number of terms less $1=19-1=18)36(2$ Ans.
$$\begin{array}{r}36\\ \hline\end{array}$$

Or, $\dfrac{39-3}{19-1}=2.$

2d. A man had 10 sons, whose several ages differed alike; the youngest was 3 years old, and the eldest 48; What was the common difference of their ages? Ans. 5.

3d. A man is to travel from Boston to a certain place in 9 days, and to go but 5 miles the first day, increasing every day by an equal excess, so that the last day's journey may be 37 miles: Required the daily increase? Ans. 4.

Problem II.

The first term, the last term, and the number of terms being given, to find the sum of all the terms.

Rule.† Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

* The difference of the first and last terms evidently shews the increase of the first term by all the subsequent additions, till it becomes equal to the last; and as the number of those additions was one less than the number of terms, and the increase, by every addition, equal, it is plain that the total increase, divided by the number of additions, must give the difference of every one separately; whence the rule is manifest.

† Suppose another series of the same kind with the given one be placed under it in an inverse order; then will the sum of any two corresponding terms be the

Examples.

1st. The extremes of an arithmetical series are 3 and 39, and the number of terms 19: Required the sum of the series?

$$\left.\begin{array}{r} 39 \\ +\ 3 \end{array}\right\} \text{Extremes.}$$

$$\begin{array}{rr} \text{Sum}= & 42 \\ \text{Number of terms}= & \times\ 19 \\ \hline & 378 \\ & 42 \\ \hline & 2)798 \\ \hline & 399 \text{ Ans.} \end{array}$$

Or, $\frac{\overline{39+3}\times 19}{2}=399.$

2d. It is required to find how many strokes the hammer of a clock would strike in a week, or 168 hours, provided it increased 1 at each hour? Ans. 14196.

3d. Suppose a number of stones were laid a yard distant from each other for the space of a mile, and the first a yard from a basket: What length of ground will that man travel over, who gathers them up singly, returning with them one by one to the basket? Ans. 1761 miles.

N. B. In this question, there being 1760 yards in a mile, and the man returning with each stone to the basket, his travel will be doubled; therefore the first term will be 2, and the last 1760×2, and the number of terms 1760.

4th. A man bought 25 yards of linen in Arithmetical Progression; for the 4th yard he gave 12 cents, and for the last yard 75 cents: What did the whole amount to, and what did it average per yard?

$\frac{75-12}{22-1}=3$ the common difference by which the first term is found [to be 3.

Then $\frac{\overline{75+3}\times 25}{2}=$ \$9 75c. and the average price is 39cts. per yard.

5th. Required the sum of the first 1000 numbers in their natural order? Ans. 500500.

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same as that of the first and last; consequently, any one of those sums, multiplied by the number of terms, must give the whole sum of the two series.

Let 1, 2, 3, 4, 5, 6, 7, 8, be the given series.

And 8, 7, 6, 5, 4, 3, 2, 1, the same inverted.

Then, $9+9+9+9+9+9+9+9=9\times 8=72$, and

$1+2+3+4+5+6+7+8=\frac{72}{2}=36.$

PROBLEM III.

Given the extremes and the common difference, to find the number of terms.

RULE.* Divide the difference of the extremes by the common difference, and the quotient increased by 1 will be the number of terms required.

EXAMPLES.

1st. The extremes are 3 and 39, and the common difference 2: What is the number of terms?

$$\left.\begin{array}{r} 39 \\ -\ 3 \end{array}\right\} \text{Extremes.}$$

$$\begin{array}{rr} \text{Common difference}=2) & 36 \\ \hline \text{Quotient}= & 18 \\ \text{Add} & 1 \\ \hline & 19 \text{ Ans.} \end{array}$$

Or, $\frac{39-3}{2}+1=19.$

2d. A man going a journey, travelled the first day 7 miles, the last day 51 miles, and each day increased his journey by 4 miles: How many days did he travel, and how far?

Ans. 12 days, and 348 miles.

PROBLEM IV.

The extremes and common difference given, to find the sum of all the series.

RULE. Multiply the sum of the extremes by their difference increased by the common difference, and the product divided by twice the common difference will give the sum.†

EXAMPLES.

1st. If the extremes are 3 and 39, and the common difference 2: What is the sum of the series?

* By the first Problem, the difference of the extremes, divided by the number of terms less 1, gave the common difference; consequently the same divided by the common difference, must give the number of terms less 1; hence, this quotient, augmented by 1, must be the answer to the question.

† By the 3d Problem find the number of terms, and then, with the number of terms and the extremes, find, by Prob. 2, the sum of the series. This is the rule, which is contracted in the text. Thus in the 1st Example, by Problem 3, $\frac{39-3}{2}+1$= the number of terms, and by Prob. 2, $\overline{39+3}\times\overline{\frac{39-3}{2}+1}$= twice the sum of the series. But $\frac{39-3}{2}+1$ is also $\frac{39-3+2}{2}$. Therefore, $\frac{\overline{39+3}\times\overline{39-3+2}}{2\times2}$= the sum of the series, and is the rule.

39+3=42 sum of the extremes.
39—3=36=difference of extremes.
36+2=38=difference of extremes increased by the common difference.

$$\begin{array}{r} 42 \\ \times\ 38 \\ \hline 336 \\ 126\ \ \\ \hline \end{array}$$

$$\text{Twice the common difference}=4)\begin{array}{r} 1596 \\ \hline 399 \end{array}$$

$$\text{Or, } \frac{\overline{39+3}\times\overline{39-3+2}}{2\times2}=399.$$

2d. A owes B a certain sum, to be discharged in a year, by paying 6d. the first week, 18d. the second, and thus to increase every weekly payment by a shilling, till the last payment be 2l. 11s. 6d.: What is the debt? Ans. £67 12s.

Problem V.

The extremes and sum of the series given, to find the number of terms.

Rule.

Twice the sum of the series, divided by the sum of the extremes, will give the number of terms.*

Examples.

1st. Let the extremes be 3 and 39, and the sum of the series 399: What is the number of terms?

$$\begin{array}{r} \text{Sum of the series}=399 \\ \times\ \ 2 \\ \hline \end{array}$$

$$\text{Sum of the extremes}=39+3=42)798(19 \text{ Ans.}$$

$$\begin{array}{r} 42 \\ \hline 378 \\ 378 \end{array}$$

$$\text{Or, } \frac{399\times2}{39+3}=19.$$

2d. A owes B 67l. 12s. to be paid weekly in Arithmetical Progression, the first payment to be 6d. and the last to be 51s. 6d.: How many payments will there be, and how long will he be in discharging the debt?

Ans. 52 payments, and as many weeks.

* This Problem is the reverse of Prob. II. and the reason of the rule is obvious from the demonstration of the Rule, Prob. II.

Problem VI.

The extremes and the sum of the series given, to find the common difference.

Rule.

Divide the product of the sum and difference of the extremes, by the difference of twice the sum of the series, and the sum of the extremes, and the quotient will be the common difference.*

Examples.

1st. Let the extremes be 3 and 39, and the sum 399: What is the common difference?

$$\begin{array}{rr}
\text{Sum of the extremes} = 39+3 = & 42 \\
\text{Diff. of the extremes} = 39-3 = & \times 36 \\
\hline
& 252 \\
& 126 \\
\hline
399\times2-42=756) & 1512(2 \text{ Ans.} \\
& 1512
\end{array}$$

$$\text{Or, } \frac{\overline{39+3}\times\overline{39-3}}{\overline{399\times2}-\overline{39+3}}=2.$$

2d. A owes B £67 12s. to be discharged in a year, by weekly payments; the first payment to be 6d. and the last, £2 11s 6d.: What is the common difference of the payments, and what will each payment be?

$$\frac{\overline{51\cdot5+\cdot5}\times\overline{51\cdot5-\cdot5}}{\overline{1352\times2}-\overline{51\cdot5+\cdot5}}=1\text{s. and } 6\text{d.}+1\text{s.}=1\text{s. }6\text{d.}=2\text{d payment,}$$

1s. 6d.+1s.=2s. 6d.=3d payment, &c.

Problem VII.

The first term, the common difference, and the number of terms given, to find the last term.

Rule.

The number of terms less 1, multiplied by the common difference, and the first term added to the product, will give the last term.†

Examples.

1st. If the first term be 3, the common difference 2, and the number of terms 19: What is the last term?

* This rule is only a contraction of the following process. By Prob. V. find the *number* of terms, and, then, from the extremes and number of terms, find by Prob. I. the common difference.

† By Prob. I. the difference of the last and first terms divided by the number of terms less 1, gives the common difference, whence the common difference multiplied by the number of terms less 1, and the product increased by the first term, must give the last term.

Number of terms = 19
— 1

Number of terms less 1 = 18
Common difference = × 2

36
First term = + 3

39 the Ans.

Or, $\overline{19-1}\times2+3=39$.

2d. A owes B a certain sum to be paid in Arithmetical Progression; the first payment is 6d. the number of payments 52, and the common difference of the payments is 12d.: What is the last payment? Ans. £2 11s. 6d.

Problem VIII.

The first term, common difference, and number of terms given, to find the sum of the series.

Rule.

To the first term add the product of the number of terms less 1 by half the common difference, and their sum, multiplied by the number of terms, will give the sum of the progression.*

Examples.

1st. If the first term be 3, the common difference 2, and number of terms 19: What is the sum of the series?

First term = 3

Add the product of the number of terms less 1 by ½ common difference } $=\overline{19-1}\times1=18$

Their sum 21
Multiply by the number of terms = 19

189
21

Or, $19\times\overline{3+\overline{19-1}\times1}=399$ Ans. = 399

2d. Sixteen persons gave charity to a poor man; the first gave 7c. and the second 12c. and so on in arithmetical progression; I demand what sum the last person gave, and how much the poor man received in all?

Ans. 82c. the last gave, and $7 12c. the whole sum.

* Find by Prob. VII. the last term, and then by Prob. II. the sum of the progression. The rule is merely a contraction of this process.

PROBLEM IX.

Given the first term, number of terms, and the sum of the series, to find the last term.

RULE.

Divide twice the sum by the number of terms; from the quotient take the first term, and the remainder will be the last.*

EXAMPLES.

1st. If the first term be 3, the number of terms 19, and the sum 399; What is the last term?

Sum of the terms = 399
Multiply by 2

Divide by the number of terms = 19)798

Quotient = 42
Subtract the first term = 3

Answer = 39

Or, $\frac{399\times2}{19}-3=39.$

2d. A merchant being indebted to 12 creditors $2460, ordered his clerk to pay the first $40, and the rest increasing in arithmetical progression: I demand the difference of the payments, and the last payment?

Ans. $30=diff. and $370 last payment.

PROBLEM X.

Given the last term, the number of terms, and the sum of the terms, to find the first term.

RULE.

Divide twice the sum by the number of terms; from the quotient subtract the last term, and the remainder will be the first.†

EXAMPLES.

1. If the last term be 39, the number of terms 19, and the sum of the series 399; what is the first term?

Sum of the series = 399
Multiply by 2

Divide by the number of terms = 19)798

Quotient = 42
From the quotient take the last term = 39

Or, $\frac{399\times2}{19}-39=3.$ Remainder = 3 Ans.

* By Prob. II. the product of the sum of the series and the number of terms, divided by 2, gives the sum of the series; whence twice the sum of the series divided by the number of terms, and the quotient diminished by the first term, will give the last term.

† By Prob. II. half the product of the sum of the extremes and the number of terms, gives the sum of the terms; whence, twice the sum of the terms divided by the number of terms, and the quotient diminished by the last term, must give the first term.

2. A man had 10 sons, whose several ages differed alike; the eldest was 48 years old, and the sum of all their ages was 255: What was the age of the youngest? Ans. 3 years.

PROBLEM XI.

The common difference, number of terms, and the last term given, to find the first term.

RULE.

From the last term subtract the product of the terms less 1 by the common difference, and the remainder will be the first term.*

EXAMPLES.

1. If the common difference be 2, the number of terms 19, and the last term 39; what is the first? Last term = 39

Subtract the number of terms less 1 multiplied by the common difference } $=\overline{19-1}\times 2 = 36$

Remains 3 Ans.

Or, $39-\overline{19-1\times 2} = 3$.

2. A man travelled 6 days, each day going 4 miles farther than on the preceding day, till the last day's journey was 40 miles; how far did he ride the first day? Ans. 20 miles.

PROBLEM XII.

The common difference, the number of terms, and last term given, to find the sum of the series.

RULE.

From the last term take the number of terms minus 1, multiplied by half the common difference, and the remainder, multiplied by the number of terms, will give the sum.†

EXAMPLES.

1. If the common difference be 2, number of terms 19, and the last term 39; what is the sum of the series? Last term=39

Subtract the number of terms less 1 multiplied by $\frac{1}{2}$ the common difference } $=\overline{19-1}\times 1 = 18$

Remainder = 21

Multiply by the number of terms = 19

189
21

Answer, 399

Or, $19\times 39-\overline{19-1\times 1}=399$

* By Prob. I. the difference of the extremes divided by the number of terms less 1, gives the common difference, whence the last term diminished by the product of the common difference and the number of terms less 1, must give the first term.

† By Prob. XI. find the first term, and then by Prob. VIII. find the sum of the progression. The rule is only a contraction of this process, as may be seen in working an example, and keeping the several terms separate in the operation.

2. A man performed a journey in 6 days, and, each day, travelled 4 miles farther than on the preceding day, till his last day's travel was 40 miles; how far did he travel in the whole?

Ans. 180 miles.

Problem XIII.

The sum of the terms, the number of terms, and the common difference given, to find the first term.

Rule.

Divide the sum by the number of terms; from the quotient take half the product of the number of terms, minus unity, by the common difference, and the remainder will be the first term.*

Examples.

1. If the sum of the series be 399, the number of terms 19, and the common difference 2; what is the first term?

Number of terms = 19)399 = sum.

Quotient = 21

Subtract ½ the product of the number of terms, less 1, by the common difference $\left.\right\} = \dfrac{\overline{19-1}\times 2}{2} = 18$

Ans. 3

$$\text{Or, } \frac{399}{19} - \frac{2\times\overline{19-1}}{2} = 3.$$

2. A man travelled 180 miles in 6 days; he increased his journey, each day by 4 miles: how far did he travel the first day?

Ans. 20 miles.

Problem XIV.

The sum of the terms, number of terms, and the common difference given, to find the last term.

Rule.

Divide the sum of the series by the number of terms; to the quotient add half the product of the number of terms minus unity by the common difference, and the sum will be the last term.†

Examples.

1. If the sum of the series be 399, the number of terms 19, and the common difference 2; what is the last term?

Divide by the number of terms = 19)399 sum.

Quotient = 21

Add ½ the product of the number of terms, less 1, by the common difference $\left.\right\} = \dfrac{\overline{19-1}\times 2}{2} = 18$

Ans. = 39

$$\text{Or, } \frac{399}{19} + \frac{2\times\overline{19-1}}{2} = 39.$$

* By Prob. VIII. the product of the number of terms less 1, and of half the common difference, added to the first term, and the sum multiplied by the number of terms, gives the sum of the progression, whence divide the sum of the series by the number of terms, and diminish the quotient by the product of the number of terms less 1 and half the common difference, or by half the product of the number of terms less 1 and the common difference, and you have the first term.

† This rule is obtained from the rule of Prob. XII. in a similar manner as the preceding rule from Prob. VIII.

2. A person bought a farm for £510 to be paid monthly in arithmetical progression, and to be completed in a year, each payment to exceed that preceding by £5: What were the first and last payments?

Ans. £15 the first payment, and £70 the last payment.

The following Table contains a summary of the whole doctrine of Arithmetical Progression.

Note. The table contains several cases, whose rules are not given in the text, because they are not very easily demonstrated without the aid of Algebra. Each of these cases however is illustrated by examples, which follow the table, so that the expression for the process in the table, may be more intelligible to the learner.

It should be observed that where two letters or a figure with a letter or letters, occur in the rules in the table, without a sign between them, the product of the quantities is intended. Thus, $d5$ means $d\times5$, and $8ds$ signifies $8\times d\times s$.

CASES OF ARITHMETICAL PROGRESSION.

Case	Given	Required	Solution
1.	$a, l, n,$	d	$\frac{l-a}{n-1}$ Prob. I.
		s	$\frac{\overline{a+l}\times n}{2}$ Prob. II.
2.	$a, l, \&\ d$	n	$\frac{l-a}{d}+1$ Prob. III.
		s	$\frac{\overline{l+a}\times\overline{l-a+d}}{2d}$ Prob. IV.
3.	$a, l, s,$	n	$\frac{2s}{a+l}$ Prob. V.
		d	$\frac{\overline{l+a}\times\overline{l-a}}{2s-\overline{l+a}}$ Prob. VI.
4.	$a, d, s,$	n	$\frac{\sqrt{\overline{2a-d}\vert^2+8ds}-\overline{2a-d}}{2d}$
		l	$\frac{\sqrt{\overline{2a-d}\vert^2+8ds}-d}{2}$

Case	Given	Required	Solution
5.	$a, d, n,$	l	$\overline{n-1}\times d+a$ Prob. VII.
		s	$n\times\overline{a+\overline{n-1}\times\frac{d}{2}}$ Prob. VIII.
6.	$a, n, s,$	d	$\frac{2\times\overline{s-an}}{\overline{n-1}\times n}$
		l	$\frac{2s}{n}-a$ Prob. IX.
7.	$l, d, s,$	a	$\frac{d\pm\sqrt{\overline{2l+d}\vert^2-8ds}}{2}$
		n	$\frac{2l+d\pm\sqrt{\overline{2l+d}\vert^2-8ds}}{2d}$
8.	$l, n, s,$	a	$\frac{2s}{n}-l$ Prob. X.
		d	$\frac{2\times\overline{nl-s}}{\overline{n-1}\times n}$.
9.	$l, n, d,$	a	$l-\overline{n-1}\times d$ Prob. XI.
		s	$n\times\overline{l-\overline{n-1}\times\frac{d}{2}}$ Prob. XII.
10.	$d, n, s,$	a	$\frac{s}{n}-\frac{d\times\overline{n-1}}{2}$ Prob. XIII.
		l	$\frac{s}{n}+\frac{d\times\overline{n-1}}{2}$ Prob. XIV.

Here
- a = first term, or least term.
- l = last term.
- n = number of terms.
- d = common difference.
- s = sum of all the terms.

Examples in Arithmetical Progression.

1. Given the first term 3, the common difference 2, and the sum of the series 399, to find the number of terms and the last term.

By the 4th Case in the table we have,

$$\frac{\sqrt{\overline{3\times2-2}|^2+399\times2\times8}-3\times2-2}{2\times2}=19, \text{ the number of terms.}$$

And, $$\frac{\sqrt{\overline{3\times2-2}|^2+399\times2\times8}-2}{2}=39, \text{ the last term.}$$

2. Given the first term 3, the number of terms 19, and the sum of the terms 399, to find the common difference.

By the first rule of Case 6th, we have,

$$\frac{2\times\overline{399-3\times19}}{\overline{19-1}\times19}=2, \text{ the common difference.}$$

3. Given the common difference 2, the last term 39, and the sum of the series 399, to find the first term and the number of terms.

By Case 7, we have,

$$\frac{2\mp\sqrt{\overline{39\times2+2}|^2-399\times2\times8}}{2}=3, \text{ the common difference.}$$

And, $$\frac{39\times2+2\pm\sqrt{\overline{39\times2+2}|^3-399\times2\times3}}{2\times2}=19 \text{ the number of}$$
terms.

4. A merchant owed to several persons \$1080; to the greatest creditor he paid \$142, to the greatest but one \$132, and so on, in Arithmetical Progression; What was the number of creditors, and what did the least creditor receive?

Ans. The number of creditors was 15, and the least creditor received \$2.

5. Given the last term 39, the number of terms 19, and the sum of the series 399, to find the common difference.

By the 2d rule in Case 8, we have,

$$\frac{2\times\overline{19\times39-399}}{\overline{19-1}\times19}=2, \text{ the common difference.}$$

6. Sixteen persons gave in charity to a poor man in such a manner as to form an arithmetical series; the last gave 65 cents, and the whole sum was \$5 60c.; What did each give less than the other, from the last down to the first.

Ans. 4 cents.

GEOMETRICAL PROPORTION.

Theorem I.

IF four quantities, 2. 6. 4. 12, be in Geometrical Proportion, the product of the two means, 6×4 will be equal to that of the two extremes, 2×12, whether they are continued, or discontinued,* and, if three quantities, 2. 4. 8, the square of the mean is equal to the product of the two extremes.

Theorem 2.

If four quantities, 2. 6. 4. 12, are such, that the product of two of them, 2×12 is equal to the product of the other two, 6×4, then are those quantities proportional.†

* It was stated under Proportion in General, that the *geometrical* ratio of two quantities is expressed by the *quotient*, arising from dividing the antecedent by the consequent; thus, the geometrical ratio of 6 to 2 is $3,=\frac{6}{2}$, and of 2 to 6, is $\frac{2}{6}$ or $\frac{1}{3}$, and of 3 to 8 is $\frac{3}{8}$: and that in a proportion there must be two, or more, couplets which have *equal ratios*. Hence, four numbers will be *geometrical proportionals*, when the ratios, obtained in this manner, are equal. Thus 2, 4, 8, 16, are geometrically proportional, because $\frac{2}{4}=\frac{8}{16}=$ each to $\frac{1}{2}$; and thus also, 9, 3, 12, 4, because $\frac{9}{3}=\frac{12}{4}=$each to 3. From these principles, it is easy to prove in a given example, the theorems in Geometrical Proportion. The factors should be kept separate by the sign of multiplication.

Let 2, 4, 3, 6, be the geometrical proportionals; then $\frac{2}{4}=\frac{3}{6}$. Multiply both fractions by the product of the second and fourth terms, and the fractions will obviously still be equal, and we have $\frac{2\times4\times6}{4}=\frac{3\times4\times6}{6}$. Then cancel the equal terms in the fractions, and $2\times6=3\times4$, that is, the product of the extremes, 2×6, is equal to the product of the means, 4×3. The same may be shown in any other case, and, hence the general rule be inferred.

Again; Three numbers are geometrical proportionals, when the ratio of the first and second terms is equal to the ratio of the second and third. Thus, 2, 4, 8, are three geometrical proportionals, for $\frac{2}{4}=\frac{4}{8}=$each to $\frac{1}{2}$. Proceed as before, and we have $\frac{2\times4\times8}{4}=\frac{4\times4\times8}{8}$, and $2\times8=4\times4$, or 4^2, that is, the product of the extremes, 2×8, is equal to the square of the mean, 4×4 or 4^2.

† Let the four quantities be 2, 6, 4, and 12, so that $2\times12=6\times4$. Divide these equal products by the quantity 6×12, and the quotients will obviously be equal, or $\frac{2\times12}{6\times12}=\frac{6\times4}{6\times12}$. Cancel the equal terms in these two fractions, and we have $\frac{2}{6}=\frac{4}{12}$; whence $2:6::4:12$, by the definition of geometrical proportionals.

In the same way it may be shown, that if $2\times8=4\times4$ or 4^2, then $2:4::4:8$, and 2, 4, and 8, are three geometrical proportionals.

Theorem 3.

If four quantities, 2. 6. 4. 12, are proportional, the product of the means, divided by either extreme, will give the other extreme.*

Theorem 4.

The products of the corresponding terms of two Geometrical Proportions are also proportional.

That is, if $2 : 6 :: 4 : 12$, and $2 : 4 :: 5 : 10$, then will $2\times2 : 6\times4 :: 4\times5 : 12\times10$.†

Theorem 5.

If four quantities, 2, 6, 4, 12, are directly proportional,

Then,	1. Directly,	$2 : 6 :: 4 : 12$
	2. Inversely,	$6 : 2 :: 12 : 4$
	3. Alternately,	$2 : 4 :: 6 : 12$
	4. Compoundedly,	$2 : 8 :: 4 : 16$
	5. Dividedly,	$2 : 4 :: 4 : 8$
	6. Mixtly,	$8 : 4 :: 16 : 8$
	7. By Multiplication,	$2\times5 : 6\times5 :: 4 : 12$
	8. By Division,	$\frac{2}{5} : \frac{6}{5} :: 4 : 12$

Because the product of the means, in each case, is equal to that of the extremes, and therefore the quantities are proportional by Theorem 1.

Theorem 6.

If three numbers, 2, 4, 8, be in continued proportion, the square of the first will be to *that* of the second, as the first number to the third; that is, $2\times2 : 4\times4 :: 2 : 8$.‡

* Let the four proportionals be 2, 4, 5, and 10; then $2\times10=4\times5$, by Theorem 1. Divide both expressions by 2, and $\frac{2\times10}{2}=\frac{4\times5}{2}$; or $10=\frac{4\times5}{2}$; or, divide, as before, by 10, and $\frac{2\times10}{10}=\frac{4\times5}{10}$; or $2=\frac{4\times5}{10}$, that is, the product of the means divided by one extreme, gives the other extreme. Hence, if the two means and one extreme be given, the other extreme, or geometrical proportional may be found.

† Let there be given, $2 : 6 :: 4 : 12$, whence $\frac{2}{6}=\frac{4}{12}$ by Theorem 1: and also, $3 : 5 :: 6 : 10$, whence $\frac{3}{5}=\frac{6}{10}$. Multiply the corresponding parts of these equal fractions together, and we have $\frac{2\times3}{6\times5}$ and $\frac{6\times4}{10\times12}$ and these products are obviously equal, or $\frac{2\times3}{6\times5}=\frac{6\times4}{10\times12}$. Hence by the definition of geometrical proportionals, $2\times3 : 6\times5 :: 6\times4 : 10\times12$, and is the theorem.

Hence, if four quantities are proportional, their squares, cubes, &c. will likewise be proportional. Thus, let the terms be $2 : 6 :: 4 : 12$, then $2\times2 : 6\times6 :: 4\times4 : 12\times12$, or $2^2 : 6^2 :: 4^2 : 12^2$, and hence also, $2^3 : 6^3 :: 4^3 : 12^3$ and $2^5 : 6^5 :: 4^5 :: 12^5$.

‡ For since $2 : 4 :: 4 : 8$, thence will $2\times8=4\times4$, by Theorem 1; and therefore $2\times2\times8=2\times4\times4$, by equal multiplication; consequently, $2\times2 : 4\times4 :: 2 : 8$, by Theorem 2.

In like manner it may be proved that, of four quantities continually proportional the *cube* of the first is to *that* of the second, as the first quantity to the fourth.

THEOREM 7.

In any continued Geometrical Proportion, 1, 3, 9, 27, 81, &c. the product of the two extremes, and *that* of every other two terms equally distant from them are equal.*

THEOREM 8.

The sum of any number of quantities, in continued Geometrical Proportion, is equal to the difference of the product of the second and last terms, and the square of the first, divided by the difference of the first and second terms.†

GEOMETRICAL PROGRESSION.

A GEOMETRICAL Progression is, when a *rank* or *series* of numbers increases, or decreases, by the continual multiplication, or division, of some equal number, which is called the *ratio*.

* For, the ratio of the first term to the second being the same as *that* of the last but one to the last, these four terms are in proportion; and therefore by Theorem 1, the product of the extremes is equal to *that* of their two adjacent terms; and after the same manner, it will appear that the product of the third and last but two is equal to *that* of their two adjacent terms, the second and last but one, and so of the rest; whence the truth of the proposition is manifest.

† Take any series of continued geometrical proportionals, as 2, 6, 18, 54, 162, 486, and its sum is, $2+6+18+54+162+486=728$, by addition. Multiply the whole, by that number by which any term of the series is multiplied or divided to form the succeeding term, which is in this example, 3, and you have,

- - $6+18+54+162+486+1458=2184$. Subtract the first series,

$2+6+18+54+162+486$ - - - $=728$. As all the terms in the upper series are cancelled by those in the lower, except the last in the former and the first in the latter, those two terms become $-2+1458$, or, $1458-2=2184-728$. Now, the upper series exceeds the lower three times, and, of course, from *thrice* the series there has been taken *once* the series, and the remainder must be *twice*, or $\overline{3-1}$ times, the series and equal to *twice* or $\overline{3-1}$ times, the sum of the series, that is, $\frac{1458-2}{3-1}=728$, the sum of the series. As $\frac{1458-2}{3-1}$ is also $\frac{3\times486-2}{3-1}$, multiply both parts of the fraction by 2, which will not alter its value, and you have $\frac{2\times3\times486-2\times2}{2\times\overline{3-1}}$ or $\frac{6\times486-4}{6-2}$, which must also be equal to the sum of the series, and is the rule. For 6 is the second term of the given series; 486, the greatest term; 4, the square of the second term; and the divisor, $\overline{6-2}$, is the difference of the first and second terms.

In this demonstration it is shown that $\frac{3\times486-2}{3-1}=$ the sum of the series; that is, that the greatest term multiplied by the number by which the series increases, and the product diminished by the least term, and this divided by a number one less than that by which the series increases, the quotient is the sum of the series. Let the series be 1, 4, 16, 64, 256, 1024, whose multiplier is 4. Then $\frac{4\times1024-1}{4-1}=1365$, the sum of the series.

PROBLEM I.

Given one of the extremes, the ratio, and the number of the terms of a geometrical series, to find the other extreme.

RULE.

Multiply or divide, (as the case may require) the given extreme by such power of the ratio, whose exponent* is equal to the *number* of terms less 1, and the product or quotient, will be the other extreme.†

* As the *last term* or any term near the last, is very tedious to be found, by continual multiplication, it will often be necessary in order to ascertain them, to have a series of numbers in Arithmetical Proportion, called *indices*, or *exponents*, beginning with a cypher, or a unit whose common difference is *one*.

When the *first* term of the series and the *ratio* are *equal*, the *indices* must begin with a unit, and, in this case, the product of any two terms is equal to that term signified by the *sum* of their *indices*. This is obvious on inspection of this example.

Thus, { 1. 2. 3. 4. 5. 6, &c. indices, or arithmetical series.
2. 4. 8. 16. 32. 64, &c. geometrical series (leading terms.)

Now, $6+6=12$=the index of the twelfth term, and

$64\times64=4096$=the twelfth term.

But, when the *first* term of the series and the *ratio* are *different*, the *indices* must begin with a cypher, and the sum of the *indices*, made choice of, must be *one less* than the *number of terms*, given in the question; because 1 in the *indices* stands over the *second term*, and 2 in the *indices*, over the *third* term, &c. And, in this case, the *product* of any *two* terms divided by the *first*, is equal to that term *beyond* the first, signified by the *sum* of their *indices*, as is obvious from this example.

Thus, { 0. 1. 2. 3. 4. 5. 6, &c. indices.
1. 3. 9. 27. 81. 243. 729, &c. geometrical series.

Here, $6+5=$ 11 the index of the 12th term.

$729\times243=177147$ the 12th term, because the first term of the series and the ratio are different, by which means a cypher stands over the first term.

Thus, by the help of these indices, and a few of the first terms in any geometrical series, any term, whose distance from the first term is assigned, though it were ever so remote, may be obtained without producing all the terms.

† The rule is evident from the manner in which a geometrical progression is formed. Let 2, 4, 8, 16, 32, &c. be the series, whose ratio is 2. The *second* term is formed by multiplying the first term by the ratio; the *third* term, by multiplying the second by the ratio, and so on. The series may therefore be written thus, $2, 2\times2, 2\times2\times2, 2\times2\times2\times2, 2\times2\times2\times2\times2$, &c. or thus $2, 2\times2^1, 2\times2^2, 2\times2^3, 2\times2^4$, and so on. Any term after the first is evidently that power of the ratio whose index is one less than the number of the term multiplied by the first term. Thus, the 3d term is 2×2^2; the 4th term is 2×2^3, and the 8th term would be 2×2^7, and so on. In an *ascending* series, therefore, multiply the first term by that power of the ratio whose index is one less than the number of the term sought, and the product is the term sought.

In a *descending* series, as 243, 81, 27, 9, 3, 1, whose ratio is 3, and which is also $1\times3^5, 1\times3^4, 1\times3^3, 1\times3^2, 1\times3^1, 1$, the last term, $1, =\frac{1\times3^5}{3^5}$. Hence the rule is evident, whatever be the first term or the ratio.

Note 1. If the ratio of any geometrical series be 2, the *difference* of the *greatest* and *least* terms is *equal* to the *sum* of all the terms except the greatest; if the ratio be 3, the *difference* is *double* the sum of all the terms except the greatest; if the ratio be 4, the *difference* is *triple* the sum, &c. Let the series be 1, 2, 4, 8, 16, 32, whose ratio is 2, and whose sum is 63, and the sum of whose first five terms is 31. Now $32-1$, the difference of the extremes, is *equal* to the sum of

EXAMPLES.

1. If the first term be 4, the ratio 4, and the number of terms 9: What is the last term?

1. 2. 3. 4+ 4= 8

4. 16. 64. 256.×256=65536=power of the ratio, whose exponent is less by 1, than the number of terms.

65536=8th power of the ratio
Multiply by 4=first term.

262144=last term.

Or, $4\times4^8=262144$=the Answer.

2. If the last term be 262144, the ratio 4, and the number of terms 9, what is the first term? 4=the first term.

the first five terms. For, by the principle proved under Theorem 8, of Geometrical Proportion, the sum of the first five terms is $\frac{2\times16-1}{2-1}$, or $\frac{32-1}{2-1}=32-1$, the difference of the greatest and least terms.

Let the series be, 3, 9, 27, 81, 243, whose ratio is 3; then 243—3=twice the sum of all the terms except the greatest. Now by the same principle $\frac{3\times81-3}{3-1}$ =sum of all the terms except the last, $=\frac{243-3}{3-1}=\frac{243-3}{2}$, that is, the sum of all the terms except the greatest is half the difference of the greatest and least terms, or this difference is twice the sum of all the terms except the greatest.

Let the series be 1, 4, 16, 64, 256, whose ratio is 4, as before $\frac{4\times64-1}{4-1}$= the sum of all the terms except the greatest, $=\frac{256-1}{4-1},=\frac{256-1}{3}$, or the difference of the greatest and least terms is equal to *thrice* the sum of the series except the last term.

Note 2. In any geometrical progression decreasing to infinity, or beyond any assignable limit, *the square of the first term divided by the difference between the first and second terms, will be the sum of the series.*

Let the series be $\frac12$, $\frac14$, $\frac18$, $\frac1{16}$, &c. to infinity. By Theorem 8 of Geometrical Proportion, the sum of any number of terms, as the first four terms, is $\overline{\frac14-\frac14\times\frac1{16}}\div\frac14$, or from the square of the first term the product of the second and fourth terms is to be taken and the remainder divided by the difference of the first and second terms. But if the series be infinite, the last term is infinitely small and must be considered 0; and then the product to be taken from the square of the first term is 0. For when $\frac1{16}$ is supposed infinitely small or 0, then $\frac14\times\frac1{16}$ becomes $\frac14\times0=0$; and the expression becomes $\frac14\div\frac14$, or $\frac14\times\frac41=1$, the sum of the above series continued without end. Hence the rule is manifest.

The above expression $\overline{\frac14-\frac14\times\frac1{16}}\div\frac14$ is also $\overline{\frac12-\frac12\times\frac1{16}}\div\frac12$, that is, *multiply the last term by the ratio, and divide the difference between this product and the first term by the difference between* 1 *and the ratio*, and the quotient will be the sum of the series. Let the series be 1, $\frac13$, $\frac19$, $\frac1{27}$, whose sum is, by addition, $\frac{40}{27}$. Then by the rule, $\overline{1-\frac13\times\frac1{27}}\div\overline{1-\frac13}=\overline{1-\frac13\times\frac1{27}}\div\frac23,=\frac{80}{81}\times\frac32=\frac{40}{27}$, as before. If the series were continued infinitely, then as before the product to be subtracted from the first term would be 0, and $1\div\frac23=1\times\frac32=1\frac12$ would be the sum. That is, if the series descend to infinity, *divide the first term by the difference between unity and the ratio*, and the quotient will be the sum of the

Again, given the first term, and the ratio, to find any other term assigned.

RULE I.*

When the indices begin with a unit.

1. Write down a few of the leading terms of the series, and place their indices over them.

2. Add together such indices, whose sum shall make up the entire index to the term required.

3. Multiply the terms of the geometrical series, belonging to those indices, together, and the product will be the term sought.

EXAMPLES.

1. If the first term be 2, and the ratio 2, what is the 13th term?

1. 2. 3. 4. $5+5\times3=$ 13

2. 4. 8. 16. $32\times32\times8=8192$ Ans. Or, $2\times2^{12}=8192$.

2. A merchant wanting to purchase a cargo of horses for the West-Indies, a jockey told him he would take all the trouble and expense upon himself, of collecting and purchasing 30 horses for the voyage, if he would give him what the last horse would come to by doubling the whole number by a half penny, that is, two farthings for the first, four for the second, eight for the third, &c. to which, the merchant, thinking he had made a very good bargain, readily agreed: Pray what did the last horse come to, and what did the horses, one with another, cost the merchant?

1. 2. 3. 4. 5. $6+6=12$th. $12+12+6=$last term.

2. 4. 8. 16. 32. $64\times64=4096$, and $4096\times4096\times64=$ 1073741824qrs. = £1118481 1s. 4d. and their average price was £37282 14s. 0½d. a piece.

RULE II.*

When the indices begin with a cypher.

1. Write down a few of the leading terms of the series, as before and place their indices over them.

infinite series. Thus, the sum of $\frac{1}{1\cdot05}, \frac{1}{1\cdot05^2}, \frac{1}{1\cdot05^3}, \frac{1}{1\cdot05^4}$, &c. to infinity, is $\frac{1}{1\cdot05} \div \overline{1-\frac{1}{1\cdot05}} = \frac{1}{1\cdot05} \times \frac{1\cdot05}{\cdot05} = \frac{1}{\cdot05} = \frac{100}{5} = 20$, Ans.

And the sum of four terms of the series, $\frac{1}{1\cdot05}, \frac{1}{1\cdot05^2}, \frac{1}{1\cdot05^3}, \frac{1}{1\cdot05^4}$, is $\overline{\frac{1}{1\cdot05} - \frac{1}{1\cdot05^5}} \div \overline{1-\frac{1}{1\cdot05}}$, which is also $\overline{\frac{1}{1\cdot05} - \frac{1}{1\cdot05^5}} \div \frac{1\cdot05-1}{1\cdot05}$ which is $\overline{\frac{1}{1\cdot05} - \frac{1}{1\cdot05^5}} \times \frac{1\cdot05}{1\cdot05-1}$, which is $\overline{1-\frac{1}{1\cdot05^4}} \times \frac{1}{1\cdot05-1}$, which is $1-\frac{1}{1\cdot05^4} \times \frac{1}{\cdot05} = \frac{1}{\cdot05} - \frac{1}{\cdot05} \times \frac{1}{1\cdot05^4}$.

* These rules are only particular cases of the preceding general rule, and the reason of them is obvious from the demonstration of that rule.

2. Add together the most convenient indices to make an index, less by 1, than the number expressing the place of the term sought.

3. Multiply the terms of the geometrical series together, belonging to those indices, and make the product a dividend.

4. Raise the first term to a power, whose index is one less than the *number* of terms multiplied, and make the result a divisor, by which divide the dividend, and the quotient will be that term *beyond the first*, signified by the sum of those indices, or the term sought.

5. If the first term be 5, and the ratio 3; what is the 7th term?

0. 1. 2. $3.+2+1=$ $6=$ind. to 6th term beyond the 1st or 7th

5. 15. 45. $135.\times45\times15=91125=$dividend.

The number of terms, multiplied, is 3 (viz. $135\times45\times15$,) and $3-1=2$ is the power to which the term 5 is to be raised; but the 2d. power of 5 is $5\times5=25$, and therefore $91125\div25=3645$ the 7th term required.

PROBLEM II.

Given the first term, the ratio, and number of terms to find the sum of the series.

RULE.

Raise the ratio to a power, whose index shall be equal to the number of terms, from which subtract 1; divide the remainder by the ratio, less 1, and the quotient, multiplied by the first term, will give the sum of the series.*

EXAMPLES.

1. If the first term be 5, the ratio 3, and the number of terms 7; what is the sum of the series?

Ratio$=3\times3\times3\times3\times3\times3\times3=2187=$7th power of the ratio.

Subtract 1

Divide by the ratio less $1=\overline{3-1}=2)2186$

Quotient$=1093$

Multiply by the first term $=$ 5

Sum of the series$=5465$

$$\text{Or, } \frac{3^7-1}{3-1}\times5=5465 \text{ Ans.}$$

* This rule is a contraction of the following process. Find by Problem I. the last term of the progression, and then find by Theorem 8, of Geometrical Proportion, the sum of the series. Thus in Ex. 1. where 5 is the first term and 3 the ratio, and the sum for 7 terms is required. By Prob. I. the 7th term is 5×3^6. Then by Theorem 8, $\frac{5\times3\times5\times3^6-5^2}{5\times3-5}=$the sum of the terms. The expression $5\times3\times5\times3^6$ may be written $5\times5\times3\times3^6=5^2\times3^7$, for 3×3^6 raises 3^6 to the next higher power, or makes it 3^7. Also $5^2\times3^7-5^2$ is $\overline{3^7-1}\times5^2$, and $5\times3-5$ is also $\overline{3-1}\times5$, as is seen by multiplying the terms. Therefore $\frac{5\times3\times5\times3^6-5^2}{5\times3-5}$ becomes $\frac{\overline{3^7-1}\times5^2}{\overline{3-1}\times5}$ or $\frac{\overline{3^7-1}\times5\times5}{\overline{3-1}\times5}$ and, cancelling the equal terms in this fraction, it becomes $\frac{\overline{3^7-1}\times5}{3-1}$ or $\frac{3^7-1}{3-1}\times5$, and is the rule. As the same may be shown in any other example, the general rule is obvious.

2. A shop keeper sold 13 yards of cloth on the following terms, viz. 2d. for the first yard, 4d. for the second, 8d. for the third, &c. I demand the price of the cloth?

$$\frac{2^{13}-1}{2-1}\times 2 = 16382\text{d.} = £68\ 5\text{s. } 2\text{d. Ans.}$$

3. A gentleman, whose daughter was married on a new year's day, gave her a guinea, promising to triple it on the first day of each month in the year; pray what did her portion amount to?

Ans. 265720 guineas.

4. What debt can be discharged in a year, by paying 1 cent the first month, 10c. the second, and so on, each month in a tenfold proportion? Ans. 111111111111c.=$1111111111 11c.

5. A man threshed wheat 9 days for a farmer, and agreed to receive but eight wheat corns for the first day's work, 64 for the second, and so on, in an eightfold proportion; I demand what his 9 day's labour amounted to, rating the wheat at 5s. per bushel?*

Ans. 153391688 corns. Amount= £78 0s. $5\frac{1}{4}$d.

6. An ignorant fop wanting to purchase an elegant house, a facetious gentleman told him he had one which he would sell him on these moderate terms, viz. that he should give him a cent for the first door, 2 cents for the second, 4 cents for the third, and so on, doubling at every door, which were 36 in all: It is a bargain, cried the simpleton, and here is a guinea to bind it: Pray what did the house cost him?

$$\frac{2^{36}-1}{2-1}\times 1 = 68719476735\text{c.} = \$687194767\ 35\text{c. Ans.}$$

7. A young fellow, well skilled in numbers, agreed with a rich farmer to serve him 10 years, without any other reward, but the produce of one wheat corn for the first year, and that produce to be sowed from year to year, till the end of the time, allowing the increase but in a tenfold proportion; what is the sum of the whole produce, and what will it amount to at $1 25c. per bushel?

Amount=$22605 61.c. 3m.+

8. Suppose one farthing had been put out at 6 per cent. per annum, Compound Interest,† at the commencement of the Christian era; what would it have amounted to in 1784 years; and suppose the amount to be in standard gold, allowing a cubic inch to be worth £53l. 2s. 8d. how large would the mass have been?

Ans. $\frac{2^{150}-1}{2-1}\times 1 =$ £1486716346568748209435714551509890767065361 11 3¾

=27980859722121230415979571232933594210766 cubick inches of gold.

As 355 : 113 :: $\overline{360\times 69{\cdot}5}$: 7964 earth's diameter. $360\times 69{\cdot}5\times 7964\times 1327{\cdot}33$ =264482820122 cubick miles in the globe,

=67273337308854741368832000 cubick inches in the globe. Then,

27980859722121230415979571232933594210766

* *Note.* 7680 wheat or barley corns are supposed to make a pint.

† Any sum at 6 per cent. per annum, compound interest, will double in eleven years and three hundred and twenty five days, or 11·889 years, or 11·89 is near enough; then, if you divide 1784 by 11·89, it will give the number of terms in this case equal to 150: the ratio will be 2, and the first term 1.

9·67273337308854741368832000=415930899840288·8, which, however incredible it may appear to some, is more than four hundred and fifteen millions of millions, nine hundred and thirty thousand, eight hundred and ninety-nine millions, eight hundred and forty thousand, two hundred and eighty-eight times larger than the globe we inhabit.*

For the solution of the four following questions, see last part of note under Problem I.

9. A frigate pursues a ship at 8 leagues distance, and sails twice as fast as the ship; how far must the frigate sail, before she comes up with her?

First, 8. 4. 2. 1. $\frac{1}{2}$. $\frac{1}{4}$. &c. $8\times8=64$, and $64\div\overline{8-4}=16$ leagues, Ans.

10. Suppose a ball to be put in motion by a force which impels it 10 rods the first minute, 8 the second, and so on decreasing by a ratio of 1·25 each minute to infinity; what space would it move through? Ans. 50 rods.

11. Required the value of ·999, to infinity, or ·9† ?

The first 9 or ·9, $=\frac{9}{10}$, the second, or ·09$=\frac{9}{100}$; therefore, $\overline{\cdot9\times\cdot9}\div\overline{\cdot9-\cdot09}=1$ Ans.

12. Required the sum of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, &c. to infinity? Ans. 1.

13. What is the sum of $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, &c. to infinity? Ans. $\frac{1}{2}$.

14. What is the sum of $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, &c. to infinity? Ans. $\frac{1}{3}$.

15. What is the sum of ·1, ·01, ·001, &c. to infinity? Ans. $\frac{1}{9}$.

16. What is the sum of 1, $\frac{1}{5}$, $\frac{1}{25}$, $\frac{1}{75}$, &c. to infinity? Ans. $1\frac{1}{4}$.

17. What is the sum of $\frac{4}{3}$, $\frac{4}{9}$, $\frac{4}{27}$, $\frac{4}{81}$, for 7 terms? Ans. $\frac{4372}{2187}$.

18. What is the sum of $\frac{1}{1\cdot06}$, $\frac{1}{1\cdot06^2}$, $\frac{1}{1\cdot06^3}$ &c. to infinity?

Ans. $16\frac{2}{3}$.

Problem III.

The first term, the last term (or the extremes) and the ratio given, to find the sum of the series.‡

Rule I.

Divide the difference of the extremes by the ratio less by 1; add the greater extreme to the quotient, and the result will be the sum of all the terms.

Rule II.

Or, Multiply the greatest term by the ratio, from the product subtract the least term; then divide the remainder by the ratio, less by 1, and the quotient will be the sum of all the terms.

* To find the solid content of a globe. See Art. 34th. of Mensuration of Solids. Note, that ·523698 is two thirds of ·785398 the area of a circle, whose diameter is 1.

† It will be seen, when we come to *circulating decimals*, that ·9 is the manner of expressing ·999, &c, to infinity.

‡ Rule 1. In Note 1, under Prob. I. it is shown that the difference of the greatest and least terms divided by the ratio less 1, gives the sum of the series except the last term. To this quotient add the last term, and the sum will be the sum of the series.

Rule 2 and 3, are demonstrated under Theorem 8 of Geometrical Proportion.

Rule III.

Or, When all the terms are given, then, from the product of the *second* and *last* terms, subtract the *square* of the *first* term; this remainder being divided by the *second* term *less* the *first*, will give the sum of the series.

Examples.

1. If the series be 2. 6. 18. 54. 162. 486. 1458. 4374. what is its sum total?

First Method.

From the greatest term = 4374
Subtract the least = 2

Divide by the ratio, less 1=3—1=2)4372 diff. of extremes.

Quotient= 2186
Add the greater extreme= 4374

6560

$$\text{Or, } \frac{4374-2}{3-1}+4374=6560 \text{ Ans.}$$

Second Method.

Greatest term=4374
Multiply by the ratio= 3

Product= 13122
Subtract the least term= 2

Divide by the ratio, less by 1=3—1=2)13120

6560 Ans.

$$\text{Or, } \frac{\overline{4374\times3-2}}{3-1}=6560$$

Third Method.

Greatest term=4374
Multiply by the second term= 6

Product=26244
Subtract the square of the first term=2×2= 4

Divide the remainder by the 2d. term less the first=6—2=4)26240

Ans. 6560

$$\text{Or, } \frac{\overline{4374\times6-4}}{6-2}=6560.$$

2. A man travelled 6 days, the first day he went 4 miles, and each day doubling his day's travel, his last day's ride was 128 miles; how far did he go in the whole? Ans. 252 miles.

3. A gentleman, dying, left 5 sons, to whom he bequeathed his estate as follows, viz. to his youngest son £1000; to the eldest £5062 10s and ordered that each son should exceed the next younger by the equal ratio of $1\frac{1}{2}$; what did the several legacies amount to? Ans. £13187 10s.

Problem IV.

Given the extremes and ratio, to find the number of terms.

Rule.

Divide the greatest term by the least; find what power of the ratio is equal to the quotient, then, add one to the index of that power, and the sum will be the number of terms.*

Examples.

1. If the least term be 2, the greatest term 4374, and the ratio 3; what is the number of terms?

Divide by the least term=2)4374=greatest term.

$3\times3\times3\times3\times3\times3\times3$=quotient, 2187=7th pow. then $7+1=8$, Ans.

2. A gentleman travelled 252 miles; the first day he rode 4 miles; the last 128, and each day's journey was double to the preceding one: How many days was he in performing the journey? Ans. 6 days.

Problem V.

Given the least term, the ratio, and the sum of the series, to find the last term.

Rule. Multiply the sum of the series by the ratio less 1, to that product add the first term, and the result, divided by the ratio, will give the last term.†

Examples.

1. If the first term be 2, the ratio 3, and the sum of the series 6560: What is the last term?

* Let the series be 1, 2, 4, 8, 16, or, 1, 1×2, 1×2^2, 1×2^3, 1×2^4. Divide the greatest term by the least, and $\frac{1\times2^4}{1}=2^4$. But the exponent is always 1 less than the number of terms, whence $4+1$, or 1 added to the index of the last term will give the number of terms. Though the index of the last is not commonly given, yet as the quotient arising from dividing the last term by the first term, is always some power of the ratio, it is readily found by multiplication, as in the 1st Example.

† This rule follows directly from Rule 2, Prob. III. For, as the product of the last term and ratio, diminished by the first term, and the remainder divided by the ratio diminished by 1, gives the sum of the series; multiply the sum of the series by the ratio diminished by 1, to the product add the first term, and divide the sum by the ratio, and you will have the last term.

Sum of the series=6560
Multiply by the ratio less 1= 2

Product=13120
Add the least term= 2

Divide their sum by the ratio=3)13122

4374 Ans.

Or, $\frac{\overline{3-1}\times 6560+2}{3}=4374$ Ans.

2. A gentleman performed a journey of 252 miles; the first day he rode 4 miles, and each day after the first, twice so far as the day before: How far did he ride the last day?

Ans. 128 miles.

Problem VI.

Given the least term, the ratio, and the sum of the series, to find the number of terms.

Rule.

To the product of the sum of the series, and the ratio minus 1, add the first term; which sum, divided by the first term, will give that powor of the ratio signified by the number of terms.*

Example.

If the first term be 2, the ratio 3, and the sum of the series 80; What is the number of terms?

Sum=80
Multiply by the ratio less 1=3—1= 2

160
Add the first term= 2

Divide by the first term=2)162

81 which, found in the Table of Powers, is the fourth power of the ratio, therefore the number of terms is 4.

Problem VII.

Given the extremes, and the sum of the series, to find the ratio.

Rule. From the sum of the series subtract the least term; divide the remainder by the sum of the series minus the greatest term, and the quotient will be the ratio.†

* By Prob. II. the difference between 1 and that power of the ratio indicated by the number of terms, divided by the ratio less 1, and the quotient multiplied by the first term, gives the sum of the series. Whence, if the sum of the series be multiplied by the ratio less 1, and the product be added to the least term, you will have the product of the first term and that power of the ratio signified by the number of terms. Divide then the former product by the least term, and the quotient will be that power of the ratio signified by the number of terms.

† This rule is deduced from Rule 2, Prob. III. in the easiest manner.

EXAMPLES.

1. If the least term be 2, the greatest term 4374, and the sum of the series 6560: What is the ratio?

Sum of the series=6560
Subtract the least term= 2

Divide the re. by the sum of the series, minus greatest term } =6560—4374=2186)6558(3 Ans.
6558

2. A debt of $252 was paid in Geometrical Progression, the first payment was $4 and the last $128: In what ratio did the payments exceed each other? Ans. 2, viz. a double ratio.

PROBLEM VIII.

The first term, the number of terms, and the last term given, to find the ratio.

RULE.

Divide the greater extreme by the less, and extract such root of the quotient, whose index is equal to the number of terms, less 1. Or, find the quotient in the Table of Powers, the root of which is the answer.*

EXAMPLES.

1. Given the extremes 2 and 4374, and the number of terms 8: What is the ratio?

Divide by the least term=2)4374=greatest term.

$\sqrt[7]{2187}=3$

$$\text{Or, } \left.\frac{4374}{2}\right|^{\frac{1}{8-1}}=3, \text{ Ans.}$$

PROBLEM IX.

The extremes and number of terms given, to find the sum of the series.

RULE.

1. Subtract the least term from the greatest, and make the difference a dividend.

2. Divide the greatest term by the least, and extract such root of the quotient, whose index is equal to the number of terms less 1; take 1 from the said root, and make the remainder a divisor. (Or find the quotient in the table of powers, which will shew the root, from which subtract 1.)

* Let the series be 1, 1×3, 1×3^2, 1×3^3, 1×3^4. Divide the last term by the least term, and $\frac{1\times3^4}{1}=3^4$, and the quotient is that power of the ratio, whose index is 1 less than the number of terms. Extract that root of the quotient, whose index is 1 less than the number of terms, and that root will be the ratio.

3. Divide the dividend by the divisor, and the greatest term, added to the quotient, will give the sum of the series.*

EXAMPLES.

Given the extremes 2 and 4374, and the number of terms 8: What is the sum of the series?

From the greatest term=4374
Take the least= 2

Make this remainder a dividend 4372

Divide the greatest term by the least 2)4374

And extract the 7th root of the quotient, $\sqrt[7]{2187}=3$: Then,

$3-1=2)4372$

Quotient=2186
Add the greatest term=4374

6560 Ans.

$$\text{Or, } 4374+\frac{4374-2}{\left|\frac{4374}{2}\right|^{8-1}-1}=6560$$

PROBLEM X.

Given the ratio, the number of terms, and the greatest term, to find the least term.

RULE.

Divide the greatest term by such power of the ratio, whose index is equal to the number of terms less 1, and the quotient will be the least term.†

EXAMPLE.

If the ratio be 2, the number of terms 6, and the greatest term, 128: What is the least?

Divide the last term by $2\times2\times2\times2\times2$=5th power of the ratio $\}=32)128(4$ Ans.
128

$$\text{Or, } \frac{128}{2^{6-1}}=4$$

* This rule is a combination of the following steps. Find by Prob. VIII. the ratio, and then find by Note 1, under Prob. I. the sum of the series except the last term, by dividing the difference of the extremes by the ratio less 1. To this quotient add the last term, and the sum will evidently be the sum of the series.

† Let the geometrical series be, 2, 2×3, 2×3^2, 2×3^3, 2×3^4. And the least term, 2, is evidently equal to the last term, $\frac{2\times3^4}{3^4}=2$. As the index of the last term is one less than the number of terms; the reason of the rule is evident.

PROBLEM XI.

Given the ratio, the number of terms, and the greatest term, to find the sum of the series.

RULE.

1. Divide the greatest term by such power of the ratio, whose index is equal to the number of terms less 1: take the quotient from the last term, and make the remainder a dividend.

2. Divide the dividend by the ratio less 1, and the quotient, added to the greatest term, will give the sum of the series.*

EXAMPLE.

If the ratio be 4, the number of terms 6, and the greatest term 3072: What is the sum of the series?

Divide the last term by the 5th power of the ratio $=4\times4\times4\times4\times4=1024)3072(3$
3072

From the last term $=3072$
Take the quotient $=\ \ 3$

Divide by the ratio less $1=4-1=3)3069$

Quotient $=1023$
Add the greatest term $=3072$

Ans. $=4095$

$$\text{Or, } 3072+\frac{3072-\dfrac{3072}{4^{6-1}}}{4-1}=4095.$$

PROBLEM XII.

Given the ratio, the number of terms, and the sum of the series, to find the least term.

RULE.

Divide the ratio, less 1, by such power, less 1, of the ratio, whose index is equal to the number of terms, and the quotient, multiplied by the sum of the series, will give the least term.†

EXAMPLE.

If the ratio be 4, the number of terms 6, and the sum of the series 4095: What is the least term.

* Find by Prob. X. the least term, and then find by Rule 1, Prob. III. the sum of the series. This process corresponds to the rule in the text.

† By Prob. II. the ratio is raised to a power whose index is the number of terms, and then diminished 1, and the remainder is divided by the ratio less 1. This quotient *multiplied* by the least term, gives the sum of the series. Whence the least term must be equal to the sum of the terms *divided* by this quotient, which is the rule.

$4\times4\times4\times4\times4\times4=4096$, and $4096-1=4095$, then, the ratio less 1, divided by 4095, is $\frac{3}{4095}$, and $\frac{3}{4095}\times\frac{4095}{1}=3$, Ans.

Or, $\frac{4-1}{4^6-1}\times4095=3$.

Problem XIII.

Given the ratio, the number of terms, and the sum of the series, to find the greatest term.

Rule.

1. Subtract that power of the ratio, which is equal to the number of terms less 1, from that power of it, which is equal to the whole number of terms.

2. Divide the remainder by that power of the ratio minus unity which is equal to the number of terms, and the quotient, multiplied by the sum of the series, will give the greatest term.*

Example.

If the ratio be 4, the number of terms 6, and the sum of the series 4095: What is the greatest term?

From $4\times4\times4\times4\times4\times4=4^6=4096$
Subtract $4\times4\times4\times4\times4 \quad =4^5=1024$

$$3072$$

Divide by $4^6-1=4095)3072=\frac{3072}{4095}$ which multiplied by the sum, is $\frac{3072}{4095}\times\frac{4095}{1}=3072$ Ans.

Or, $\frac{4^6-4^{6-1}}{4^6-1}\times4095=3072$

3. The two last problems may be solved by one short operation, thus: Divide the sum by the ratio, and the remainder after the operation will be the least term; then take the quotient from the sum of the series, and the remainder will be the greatest term.†

* This rule is a contraction of the following process. Find by Prob. XII. the least term, and then find by Prob. I. the greatest term. Thus in the example; $\frac{4-1}{4^6-1}\times4095=$ the least term by Prob. XII.; and by Prob. I. $\frac{4-1}{4^6-1}\times4095\times4^{6-1}=\frac{4-1}{4^6-1}\times4095\times4^5=$ the greatest term $=\frac{\overline{4-1}\times4^5}{4^6-1}\times4095=\frac{4^6-4^5}{4^6-1}\times4095=3072$, the greatest term.

† Let the series be $3+12+48+192+768+3072=4095$, or let it be written $3+3\times4+3\times4^2+3\times4^3+3\times4^4+3\times4^5=4095$. This is the same as $3+3\times\overline{4+4^2+4^3+4^4+4^5}=4095$, & it is evident that $4095-3=3\times\overline{4+4^2+4^3+4^4+4^5}$, and that $4095-3$ contains $3\times4+4^2+4^3+4^4+4^5$, a certain number of times exactly. If then both be divided by the ratio 4, we shall have $\frac{4095-3}{4}=3\times\overline{1+4+4^2+4^3+4^4}=3\times341=1023=$ the number of times 4 is contained exactly

For the least term.	For the greatest term.
4)4095(1023 quotient.	From the sum=4095
4	Subtract the quotient=1023
—	——
09	Ans.=3072
8	
—	
15	
12	
—	
3 Ans.	

PROBLEM XIV.

Given the ratio, the last term, and the sum of the series to find the first term.

RULE.

From the sum of the series take the last term, and multiply the remainder by the ratio; then take this product from the sum of the series, and the remainder will be the first term.*

EXAMPLE.

If the ratio be 4, the last term 3072, and the sum of the series 4095; what is the first term?

From the sum=4095
Take the last term=3072
——
Remainder=1023
Multiply by the ratio= 4
——
Subtract 4092 from the sum.
——
And the remainder 3 is the Answer.

PROBLEM XV. and XVI.

Given the number of terms, the last term, and the sum of the series, to find the first term and the ratio.

The solution of these two Problems being very tedious by the Theorems, they may be solved by a very short operation; thus,

in 4095—3. If, however, the first term 3, be not taken from the sum, it is evident that the ratio must be contained in 4095, the same number of times, or 1023 times, with a remainder always equal to the first term, which is in this example 3. And thus also for any other case where the first term is less than the ratio.

Again, $3+3\times4+4^2+4^3+4^4+4^5=4095$, or as it may also be written, $3\times\overline{1+4+4^2+4^3+4^4+4^5}=4095$. But, we have seen that $3\times\overline{1+4+4^2+4^3+4^4}$ is contained in $\frac{4095-3}{4}$ just 1023 times, and $3\times\overline{1+4+4^2+4^3+4^4}$ is the series except the last term 3×4^5. Therefore $4095-1023=3072$, the last term.

* This rule is deduced from Rule 1, Prob. III. and will be evident on attending to that rule.

Divide the sum of the series by the difference between the sum and the last term; the quotient will give the ratio, and the remainder, after the operation, the first term.*

EXAMPLE.

If the number of terms be 4, the last term 54, and the sum of the series 80; required the first term and the ratio?

From the sum=80
Take the last term=54

Divide by the difference=26)80(3 the ratio.
78

The first term= 2

The following Table exhibits a summary view of the doctrine of Geometrical Progression.

CASES OF GEOMETRICAL PROGRESSION.			
Case	Given	Required	Solution
1.	arn	l	ar^{n-1} Prob. I.
		s	$\frac{r^n-1}{r-1}\times a$ Prob. II.
2.	arl	s	$l+\frac{l-a}{r-1}$ Prob. III.
		n	$\frac{L.l-L.a}{L.r}+1$ Prob. IV.

* By the second rule, Prob. XIII. it is found that if the sum of the series be divided by the ratio, the quotient is the difference between the sum of the series and the greatest term, and the remainder is the first term. Therefore, if the sum of the series be divided by the difference between the sum and the last term, the quotient must be the ratio, and the remainder the least term. In the example there taken, the greatest term divided by the ratio, or $\frac{4095}{4}=1023$ with a remainder$=3$ the first term. But as $1023=4095-3072$ or the difference between the sum of the series and the last term, therefore $\frac{4095}{4095-3072}=4$, the ratio, and leaves a remainder $3,=$the first term. The same must hold true in every series, where the ratio is greater than the least term.

Case	Given.	Required	Solution.
3.	ars	l	$\frac{\overline{r-1}\times s+a}{r}$ Prob. V.
		n	$\frac{L.\overline{\overline{r-1}\times s+a}-L.a}{L.r}$ Prob. VI.
4.	als	r	$\frac{s-a}{s-l}$ Prob. VII.
		n	$\frac{L.l-L.a}{L.\overline{s-a}-L.\overline{s-l}}+1$
5.	ans	r	$\frac{rs}{a}-r^n=\frac{s-a}{a}$
		l	$l\times\overline{s-l}\vert^{n-1}=a\times\overline{s-a}\vert^{n-1}$
6.	anl	r	$\frac{l}{a}\Big\vert^{\frac{1}{n-1}}$ Prob. VIII.
		s	$l+\frac{l-a}{\frac{l}{a}\Big\vert^{\frac{1}{n-1}}-1}$ Prob. IX.
7.	rnl	a	$\frac{l}{r^{n-1}}$ Prob. X.
		s	$l+\frac{l-\frac{l}{r^{n-1}}}{r-1}$ Prob. XI.
8.	rns	a	$\frac{r-1}{r^n-1}\times s$ Prob. XII.
		l	$\frac{r^n-r^{n-1}}{r^n-1}\times s$ Prob. XIII.

Case	Given	Required	Solution.
9.	*rls*	*a*	$s-r\times\overline{s-l}$ Prob. XIV.
		n	$\frac{L.l-L.\overline{s-r\times\overline{s-l}}}{L.r}+1$
10.	*nls*	*a*	$\overline{a\times s-a}\vert^{n-1}=l\times\overline{s-l}\vert^{n-1}$
		r	$r^{n}+\frac{s}{l-s}r^{n-1}=\frac{l}{l-s}$

Here $\left\{\begin{array}{l} a\text{=first or least term.} \\ l\text{=last or greatest term.} \\ s\text{=sum of all the terms.} \\ n\text{=number of terms.} \\ r\text{=ratio.} \\ L\text{=logarithm.}\end{array}\right.$

Examples in Geometrical Progression,

To be solved by those formulæ in the Table, of which the rules are not given in the Problems.

1. The least term is 2, the greatest term 4374, and the sum of the series 6560; required the number of terms. Ans. 8.

This is solved by the 2nd rule of Case 4 in the table.

2. If the ratio be 4, the last term 3072, and the sum of the series 4095; what is the first term and the number of terms?

Ans. The first term 3, the number of terms 5.

Wrought by Case 9.

3. Given the number of terms 4, the last term 54, and the sum of the series 80, to find the first term and ratio.

Ans. The first term 2, the ratio 3.

Wrought by Case 9 in the table.

INTEREST.

INTEREST is a premium allowed by the Borrower to the Lender, for the use of his money. It is estimated at a certain number of pounds, dollars, &c. for each hundred pounds, dollars, &c. for a year; and in the same proportion, for a less or greater sum, or for a longer or shorter time. Hence, interest is said to be so much *per cent.* or *per centum, per annum.*

The Principal is the sum lent.
The Rate is the premium per cent. agreed on.*
The Amount is the sum of the principal and interest.
Interest is of two kinds, *Simple* and *Compound*.

SIMPLE INTEREST.

Simple Interest is that which is allowed on the Principal only.

Note. By this Rule, Commission, Brokerage, Insurance, purchasing Stocks, or any thing else, rated at so much per cent. are calculated.

General Rule.

1. Multiply the Principal by the Rate, and divide by 100 (or cut off the two right hand figures in the Pounds) and the quotient, or left hand figures will be the answer in Pounds, &c. the right hand figures being reduced and cut off as at first. If the principal be dollars, the right hand figures will be cents.†

2. For more years than one, multiply the Interest of one year by the number of years.

3. For any number of months take the aliquot parts of a year; and for days, the aliquot parts of 30.

Note. When the rate per cent. per annum is $\left\{\begin{matrix} 9 \\ 8 \\ 6 \\ 4 \\ 3 \\ 2 \end{matrix}\right\}$ multiply the principal by $\left\{\begin{matrix} \frac{3}{4} \\ \frac{2}{3} \\ \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \\ \frac{1}{6} \end{matrix}\right\}$ of the given number of months, and you will have the interest for the given time.

Examples.

1. What is the interest of £573 13s. 9½d. at 6 per cent. per annum? Ans. £34 8s. 5d.

* *Lawful* or *legal* interest is that which is permitted by the laws of the State. It is different under different governments. In England the Rate is *five* per cent. In the New England States, it is *six*, and, in the State of New York it is *seven* per cent. The courts of the United States allow interest according to the practice of the State where the suit was commenced. The rules of the Courts in the States of Massachusetts, Connecticut, and New York, for computing *legal* interest, will be given immediately before Discount.

† This rule is a contraction of a process in the Rule of Three. Thus in the

1st Example. As, £ 100 :: Rate 6 :: Principal £573 13s. 9½d. : the interest for a year. And

$$\frac{6\times £573\ 13s.\ 9\tfrac{1}{2}d.}{100}=£34\ 8s.\ 5d.$$ the interest. The remainder of the rule is obvious.

The reason of the rule in the Note above is, that as 9 is ¾ of 12 months, if the rate be 9 per cent, multiply the principal by ¾ of the months; if the rate be 8 per cent. by ⅔ of the number of months; and if 6, by half the months, &c.

£573 13 9½
6

£34·42 2 9
20

8·42
12

5·13
4

·52

2. What is the interest of £329 17s. 6½d. for 3 years, 7 months, and 12 days, at 5 per cent. per annum. Ans. £59 13s.

£329 17 6½d.			Then,
5	6 mons.	½	16 9 10½ interest of 1 year
			3
16·49 7 8½			
20			49 9 7½ do. of 3 years:
	1 mon.	⅙	8 4 11¼ do. of 6 months
9·87	10 days	⅓	1 7 5¾ do. of 1 month
12	2 days	⅕	9 1¾ do. of 10 days
			1 9¾ do. of 2 days
10·52			
4			Ans. £59 13 do. of 3y. 7m. 12d.
2·10			

Or thus:

		£ s. d.
£5	$\frac{1}{20}$	329 17 6½
6 months	½	16 9 10½
		3
		49 9 7½
1 month	⅙	8 4 11¼
10 days	⅓	1 7 5¾
2 days	⅕	9 1¾
		1 9¾
		£59 13 Ans.

3. What is the interest of 347 dollars 50 cents, at 6 per cent. per annum for a year.

$347·50 Or $100 : 6 :: 347.50c. : $20.85c.
6

20·8500 Ans. $20 85c.

4. What is the interest of $797 13c. at 6 per cent. per annum, for 8 months?

$$\begin{array}{r} \$797{\cdot}13 \\ 4 \\ \hline 31{\cdot}8852 \end{array}$$ Ans. $31 88c. 5$\frac{1}{10}$m.

5. What is the interest of $649 17c. at 6 per cent. per annum, for 15 months? Ans. $48 68c. 7$\frac{3}{4}$m.

6. Required the amount of £725 12s. 6d. at 5 per cent. per annum, for a year? 5=$\frac{1}{20}$|725 12 6

$$\begin{array}{r} 36 \quad 5 \quad 7\frac{1}{2} \end{array}$$

Ans. £761 18 $1\frac{1}{2}$

7. What is the amount of $560 50c. at 6 per cent. for 16 months?

$$\begin{array}{r} \$560 \quad 50 \\ 8 \\ \hline 44{\cdot}84 \\ 560{\cdot}50 \\ \hline \end{array}$$

Ans. $605·34c.

8. What is the interest of $150 75c. for 1 month, at 6 per cent. per annum? $\frac{1}{2}$|150·75

·75375 Ans. 75cts. 3$\frac{3}{4}$ mills.

So that any number of dollars, considered as so many cents, is the interest for 2 months, at 6 per cent. and the half of it is the monthly interest.

COMMISSION, OR FACTORAGE,

Is an allowance of so much per cent. to a Factor or Correspondent, for buying and selling goods.

9. Required the commission on £436 9s. 6d. at 3$\frac{1}{2}$ per cent.

$$\begin{array}{rrr} £436 & 9 & 6 \\ & & 3\frac{1}{2} \\ \hline 1309 & 8 & 6 \\ 218 & 4 & 9 \\ \hline 15{\cdot}27 & 13 & 3 \\ 20 & & \\ \hline 5{\cdot}53 & & \\ 12 & & \\ \hline 6{\cdot}39 & & \\ 4 & & \\ \hline 1{\cdot}56 & & \end{array}$$ Ans. £15 5 $6\frac{1}{4}$.

10. Required the commission on $649 75c. at 1¾ per cent.

½	649·75
¼	324·875
	162·4375
	1137·0625 Ans. $11 37c. 0⅝m.

BROKERAGE

Is an allowance of so much per cent. to a person called a Broker, for assisting merchants in purchasing or selling goods.

11. Required the Brokerage on £911 12s. at 5s. or ¼ per cent.

5s.=¼|911 12

2 27 18
20

5·58 Ans. £2 5s. 6¾d.
12

6·96
4

3·84

12. Required the Brokerage on $876 21c. at 33⅓ cents, or at ⅓ per cent.

⅓|876·21

292·07 Ans. $2 92c. $0\frac{7}{10}$m.

BUYING AND SELLING STOCKS.

Stock is a general name for the capitals of trading companies, or, of a fund established by government, the value of which is variable.

13. Required the amount of £375 15s. bank stock, at £75 per cent?

|50|½|375 15
|25|½|187 17 6
93 18 9
Ans. £281 16 3

Or thus:
|25|¼|375 15
Subtract 93 18 9
As before, £281 16 3

14. Required the amount of $2195 50c. bank stock, at 125 per cent.

|25|¼|2195·50
Add 548·875

Ans. $2744·375

15. What is the value of $6950 at 105 per cent? Ans. $7297 50cts.

16. Value of £225 of stock at 95 per cent.? Ans. £213 15s.

TO CALCULATE INTEREST FOR DAYS.

Rule I.

Multiply the principal by the days, and that product by the rate, and divide the last product by 365×100.*

15. Required the interest of £360 10s. for 175 days, at 6 per cent.

$$\frac{360\cdot5\times175\times6}{100\times365}=£10\cdot17=£10\ 7s.\ 4\tfrac{3}{4}d.$$

Rule for making a divisor for any Rate.

Multiply 365 by 100, and divide by the rate. Thus, for 6 per cent. $\dfrac{365\times100}{6}=6083$ **divisor.**

For 5 per cent. $\dfrac{365\times100}{5}=7300$ **divisor.**

For 7 per cent. $\dfrac{365\times100}{7}=5214$ **divisor, and so for any other rate. Therefore,**

* This rule is the result of a statement in the Double Rule of Three, as follows.

	£	Rate.	Prin.	
	As 100 :	6 ::	360·5 :	the interest for 1 year.
	Days.		Days.	
	And as 365 :	::	175 :	the interest for 175 days.

Wrought according to the Rule we have $\dfrac{360\cdot5\times175\times6}{365\times100}=£10\ 7s.\ 4\tfrac{3}{4}d.$ the interest required.

The rule for finding a divisor for any Rate, is a contraction of this result. For $\dfrac{360\cdot5\times175\times6}{365\times100}=360\cdot5\times175\times\dfrac{6}{365\times100}$. But dividing both parts of the last fraction by 6, the Rate, we have $\dfrac{6}{365\times100}=\dfrac{6}{36500}=\dfrac{1}{6083}$. Therefore $360\cdot5\times175\times\dfrac{6}{365\times100}=360\cdot5\times175\times\dfrac{1}{6083}=\dfrac{360\cdot5\times175}{6083}$. In the same way divisors are formed for any other rate. Hence too, the 2d Rule is obvious, for $\dfrac{360\cdot5\times175}{6083}$ = the interest, and is the product of the principal and days divided by the divisor formed as above.

When the time is given in *months*, the divisor is formed in a similar manner. Suppose in the last example the time had been 11 months. Then, As 100 : 6 :: 360·5 : the interest for a year, and as 12 : :: 11 months : the interest for 11 months. Then $\dfrac{360\cdot5\times11\times6}{12\times100}$ = the interest $=360\cdot5\times11\times\dfrac{6}{1200}=\dfrac{360\cdot5\times11}{200}$. If the Rate were 5, then $\dfrac{5}{1200}=\dfrac{1}{240}$. Hence the rule is evident.

Note. As 365 days : 5 per cent. :: 7300 days : 100 per cent. And as 12 months : 5 per cent. :: 240 months : 100 per cent. Hence it is evident that if the Rate be 5, any principal will gain 100 per cent. that is, will double in 7300 days or 240 months. And at 6 per cent. any sum will double in 6083 days, or 200 months, and at 7 per cent. in $5214\frac{2}{7}$ days, or $171\frac{3}{7}$ months.

RULE II.

Multiply the principal by the days; divide by 6083 for 6 per cent. and 7300 for 5 per cent. (the days in which any sum will double at those rates) and the quotient is the interest. For months, multiply the principal by them, and divide by 200 for 6 per cent. or 240 for 5 per cent. (the months in which any sum will double at those rates) and the quotient is the answer.

Hence, when interest is to be calculated on cash accounts, or accounts current, where partial payments are made, or partial debts contracted; multiply the several balances into the days they are at interest, which should be done at every transaction, and the sum of these products divided by 6083 and 7300 will give the interest at 6 and 5 per cent. For any other rate, make the proper addition or deduction, or find a divisor as before directed.

When partial payments are made at short periods, subtract the several payments from the original sum in their order, placing their dates in the margin.

16. Suppose a bill of $359 was due January 1, 1807; that $75 was paid February 3d, $50 March 5th, $80 April 9th, and June 7th, $145: What interest is due at 5, 6 and 7 per cent.?

Dates.	Bill.	Days.	Products.
January 1.	$350	33	11550
Feb. 3, paid	75		
Balance,	275	30	8250
March 5, paid	50		
Balance,	225	35	7875
April 9, paid	80		
Balance,	145	59	8555
June 7, paid	145		

7300)36230(4·963
Ans. $4 96c. 3m. at 5 per cent.
6083)36230(5·955
Ans. $5 95½c. at 6 per cent.
5214)36230(6·948
Ans. $6 94c. 8m. at 7 per cent.

After the dates are placed in the margin, the number of days in each of those periods is to be computed and marked against its respective sum: lastly, divide the sum of the products by 6083, &c.

Interest on accounts current is calculated nearly in the same manner.

17. Compute the interest at 6 per cent. on the following account to August 10th.

Dr. Mr. A. Jones, his account current, with B. Carr, Cr.

1807.	D.	1807.	D.
Jan. 1, To Cash, - -	560	March 10, By Cash, - -	120
Feb. 10, To do. - -	300	April 25, By do. - -	130
May 15, To do. - -	140	June 16, By do. - -	450
July 25, To do. - -	100	July 21, By do. - -	150

1807.	Ds.		Days.	Products	Dr.	Cr.
Jan. 1,	Dr.	560	40	22400	$560	120
Feb. 10,	Dr.	300			300	130
		—			140	450
	Dr.	860	28	24080	100	150
March 10,	Cr.	120			—	—
		—			1100	850
	Dr.	740	46	34040	850	
April 25,	Cr.	130			—	
		—			250 Balance.	
	Dr.	610	20	12200		
May 15,	Dr.	140			January	30
		—			February	28
	Dr.	750	32	24000	March	31
June 16,	Cr.	450			April	30
		—			May	31
	Dr.	300	35	10500	June	30
July 21,	Cr.	150			July	31
		—			August	10
	Dr.	150	4	600		—
July 25,	Dr.	100			Days	221
		—				
Aug. 10,	Dr.	250	16	4000		
6083)131820(21·672			—	—		
Ans $21 67c. 2m.			221	131820		

Here the sums on either side are introduced according to the order of their dates; those on the Dr. side are added to the former balance, and those on the Cr. side subtracted. Before we calculate the days, we try if the last sum $250 be equal to the balance of the account, which proves the additions and subtractions. And before multiplying we try if the sum of the column of days be equal to the number of days from January 1 to August 10.

When payments are made at considerably distant periods, it is usual to calculate the interest to the date of each payment, and add it to the principal, and then subtract the payment from the amount.

18. A note was given for $540 the 18th August, 1804. and there was paid the 19th of March, 1805, $50, and the 19th of December, 1805, $25; and the 23d of September, 1806, $25; and the 18th of August, 1807, $110: Required the interest, and balance due on the 11th of November, 1807, at 6 per cent.?

A note given 18th August, 1804, for		$540
Interest to 19th March, 1805, 213 days,	$19·352	19·352
		559·352
Paid 19th March, 1805,		50·
Balance due 19th March, 1805,		509·352
Interest to 19th Dec. 1805, 275 days,	23·022	23·022
Balance due 19th Dec. 1805,		532·374
Paid 19th Dec. 1805,		25 00
Balance due 19th Dec. 1805,		507·374
Interest to 23d. Sept. 1806, 278 days,	23·197	23·197
Balance due 23d Sept. 1806,		530·571
Paid 23d Sept. 1806,		25·000
Balance due 23d Sept. 1806,		505·571
Interest to 18th Aug. 1807, 329 days,	27·343	27·343
Balance due 18th Aug. 1807,		532·914
Paid 18th Aug. 1807,		110·
Balance due 18th Aug. 1807,		422·914
Interest to 11th Nov. 1807, 85 days,	5·909	5·909
Balance due 11th Nov. 1807,		428·823
Amount of interest paid,	$98·823	

19. A owes B the following sums, with interest at 6 per cent. per annum: $60 for 7 months, $150 for 9 months, $75·50 for 3 months, $365·25 for 8 months, and 510·20 for 5 months: Required the amount?

$$
\begin{array}{rl}
\$\ 60 & \times 7 = 420 \\
150 & \times 9 = 1350 \\
75{\cdot}50 & \times 3 = 226{\cdot}50 \\
365{\cdot}25 & \times 8 = 2922 \\
510{\cdot}20 & \times 5 = 2551 \\
\hline
1160{\cdot}95 & 200)7469{\cdot}50(\$37{\cdot}347 \text{ Interest.} \\
 & \quad 1160{\cdot}95 \text{ Principal.} \\
\hline
 & \text{Ans. } \$1198{\cdot}297 \text{ Amount.}
\end{array}
$$

20. A note for $1000 is given January 1, 1803, with interest at 6 per cent. per annum; February 19, 1803, $100 are paid; June 7, 1803, $150; April 14, 1804, $37·50; July 11, 1804, $75; Sept. 29, 1804, $250; Dec. 17, 1805, $39; March 4, 1806, $175; Aug. 7, 1806, $105; Oct. 30, 1806, $50; May 12, 1807, $40, and Nov. 17, 1807, $72: How much is due, January 1, 1808?

SIMPLE INTEREST BY DECIMALS.

A TABLE OF RATIOS, FROM ONE POUND, &C. TO TEN POUNDS.

Rate per cent.	ratios.	rate per cent.	ratios.	rate per cent.	ratios.
1	·01	4	·04	7	·07
$1\frac{1}{4}$	·0125	$4\frac{1}{4}$	·0425	$7\frac{1}{4}$	·0725
$1\frac{1}{2}$	·015	$4\frac{1}{2}$	·045	$7\frac{1}{2}$	·075
$1\frac{3}{4}$	·0175	$4\frac{3}{4}$	·0475	$7\frac{3}{4}$	·0775
2	·02	5	·05	8	·08
$2\frac{1}{4}$	·0225	$5\frac{1}{4}$	·0525	$8\frac{1}{4}$	·0825
$2\frac{1}{2}$	·025	$5\frac{1}{2}$	·055	$8\frac{1}{2}$	·085
$2\frac{3}{4}$	·0275	$5\frac{3}{4}$	·0575	$8\frac{3}{4}$	·0875
3	·03	6	·06	9	·09
$3\frac{1}{4}$	·0325	$6\frac{1}{4}$	·0625	$9\frac{1}{4}$	·0925
$3\frac{1}{2}$	·035	$6\frac{1}{2}$	·065	$9\frac{1}{2}$	·095
$3\frac{3}{4}$	·0375	$6\frac{3}{4}$	·0675	$9\frac{3}{4}$	·0975
				10	1

Ratio is the Simple Interest of £1 or $1 for 1 year, at the rate per cent. agreed on, and is found by dividing the rate by 100, and reducing it to a decimal. Thus, $\frac{6}{100}$=·06, and, $\frac{5}{100}$=·05, and so on.

A TABLE for the ready finding of the decimal parts of a year, equal to any number of *days*, or *quarters* of a *year*.

Days.	decimal parts.	days.	decimal parts.	days.	dec. parts.
1	·00274	10	·027397	100	·273973
2	·005479	20	·054794	200	·547945
3	·008219	30	·082192	300	·821918
4	·010959	40	·109589	365	1·000000
5	·013699	50	·136986	$\frac{1}{4}$ of a year=·25	
6	·016438	60	·164383	$\frac{1}{2}$ of a year=·5	
7	·019178	70	·191781	$\frac{3}{4}$ of a year=·75	
8	·021918	80	·219178		
9	·024657	90	·246575		

CASE I.*

The principal, time, and ratio given, to find the interest and amount.

RULE.

Multiply the principal, time and ratio continually together, and the last product will be the interest, commission, brokerage, &c. to which add the principal, and the sum will be the amount.

* This is a contraction of the General Rule for Simple Interest. If the interest on £30 or $30 was required for 2 years at 6 per cent. by the general rule, the interest is $\frac{30\times6}{100}\times2=30\times\frac{6}{100}\times2=30\times{\cdot}06\times2$, which is the product of principal, ratio, and time. And the amount=30+$\overline{30\times{\cdot}06\times2}$=£33·6 or $.

EXAMPLES.

1. Required the amount of £537 10s. at £6 per cent. per annum, for 5 years?

Principal	537·5
Multiply by the ratio=	·06
Product	32·250
Multiply by the time=	5
Interest=	161·250
Add the principal=	537·5
Amount=	698·75
	20
	15·00 Ans. £698 15s.

Or, 537·5×·06×5+537·5=£698 15s.

2. What is the simple interest of £917 16s. at £5 per cent. per annum, for 7 years? Ans. £321 4 7.

3. What is the amount of £391 17s. at £5½ per cent. per annum, for 3¼ years? Ans. £449 3 1¾.

4. What is the amount of £235 3s. 9d. at £4¼ per cent. per annum, from March 5th, 1784, to Nov. 23d, 1784? Ans. £244 0 8½.

5. If my correspondent is to have £2¼ per cent; what will his commission on £785 15s. amount to? Ans. £19 12 10¼.

6. What will be the interest and amount of £445 10s. in 3 years and 129 days, at £8½ per cent. per annum?
Ans. Interest, £126 19 8¼, and the amount=£572 9 8½.

7. If a broker disposes of a cargo for me, to the amount of £637 10s. on commission at £1¼ per cent. and procures me another cargo of the value £817 15s. on commission at £1¾ per cent.; what will his commission, on both cargoes, amount to? Ans. £22 5 7.

8. What is the simple interest of $66·666 for 1¾ years at 7 per cent.? Ans. $8 16c. 6m.

9. Find the amount of $1 for 9 years and 200 days, computing interest at 7 per cent.? Ans. $1 66c. 8m.

10. What is the interest of $236 at 5 per cent. for one year and 300 days?

11. Required the interest on $6485 at 6 per cent. for two years, six months and 20 days.

CASE II.

The amount, time, and ratio given, to find the principal.

RULE.

Multiply the ratio by the time; add unity to the product for a divisor, by which sum divide the amount, and the quotient will be the principal.*

* In the demonstration of the Rule for Case I. it was proved that the amount = the principal added to the product of the principal, ratio, and time, or, taking

EXAMPLES.

1. What principal will amount to £1045 14s. in 7 years, at £6 per cent. per annum?

Ratio=·06
Multiply by the time= 7

Product=·42
Add 1·

Divisor=1·42)1045·7(736.4084+=£736 8 2.

Or, $\frac{1045{\cdot}7}{{\cdot}06\times7+1}$=£736 8 2 Ans.

2. What principal will amount to £3810, in 6 years, at £4½ per cent. per annum? Ans. £3000.

3. What principal will amount to £666 9s. 0¼ in 3¼ years, at £5¼ per cent. per annum? Ans. £563.

4. What principal will amount to £335 7s. 3d. in 3 years and 97 days, at £9½ per cent. per annum? Ans. £255 19 0¾.

CASE III.

The amount, principal, and time given, to find the ratio.

RULE.

Subtract the principal from the amount; divide the remainder by the product of the time and principal, and the quotient will be the ratio.*

EXAMPLES.

1. At what rate per cent. will £543 amount to £705 18s. in 5 years?

From the amount=705·9
Take the principal=543

Divide by 543×5=2715)162.90(·06
162·90

the same example, the amount, $33{\cdot}6=30+\overline{30\times{\cdot}06\times2}$, or which is the same thing, $=\overline{1+{\cdot}06\times2}\times30$. Divide both by the same quantity, $\overline{1+{\cdot}06\times2}$, and the expression will still be equal, and we have $\frac{33{\cdot}6}{1+{\cdot}06\times2}=\frac{\overline{1+{\cdot}06\times2}\times30}{1+{\cdot}06\times2}$; then cancel the equal terms in the last fraction, and $\frac{33{\cdot}6}{1+{\cdot}06\times2}=30$, that is, the amount divided by the product of the ratio and time increased by 1, gives a quotient, which is the principal. The same may be shown in any other example, and, hence the rule is general.

* Under case I. it was shown that the amount, $33{\cdot}6=30+\overline{30\times{\cdot}06\times2}$. Take the principal, 30, from both sides, and $33{\cdot}6-30=30\times{\cdot}06\times2$, or $3{\cdot}6=30\times2\times{\cdot}06$. Divide both parts by the product of time and principal, and $\frac{3{\cdot}6}{30\times2}=\frac{30\times2\times{\cdot}06}{30\times2}$, or $\frac{3{\cdot}6}{30\times2}={\cdot}06$, the ratio, and illustrates the rule.

Or, $\frac{705{\cdot}9-543}{543\times5}={\cdot}06=$ £6 Ans.

2. At what rate per cent. will £391 17s. amount to £449 3s. $1\frac{3}{4}$d. ·74qr. in $3\frac{1}{4}$ years? Ans. £$4\frac{1}{2}$.

3. At what rate per cent. will £413 12s. 6d. amount to £546 4s. $10\frac{1}{4}$d. in $4\frac{3}{4}$ years? Ans. £$6\frac{3}{4}$.

4. At what rate per cent. will £3000 amount to £3810 in 6 years? Ans. £$4\frac{1}{2}$.

CASE IV.

The amount, principal, and rate per cent. given, to find the time.

Rule.

Subtract the principal from the amount; divide the remainder by the product of the ratio and principal; and the quotient will be the time.*

Examples.

1. In what time will £543 amount to £705 18s. at £6 per cent. per annum?

From the amount=705·9
Take the principal=543

Divide by 543×·06=32·58)162·9(5 years, Ans.
162 9

2. In what time will £3000 amount to £3810, at $4\frac{1}{2}$ per cent. per annum? Ans. 6 years.

3. In what time will £391 17s. amount to £449 3s. $1\frac{3}{4}$d. at £$4\frac{1}{2}$ per cent. per annum? Ans. $3\frac{1}{4}$ years.

To find the Interest of any Sum, at 6 per cent. per annum, for any number of months.

Rule.

If the months be an even number, multiply the principal by half that number; and if the months be uneven, halve the even months, to which annex $\frac{5}{10}$; thus the half of 19 is 9·5; and multiply the principal as before, dividing by 100 or cutting off two figures more at the right hand, than there are decimals in both factors, which reduce to farthings, each time cutting off as at first.

4. What is the interest of £345 16s. 6d. for 9 years and 11 months, at 6 per cent. per annum?

* Also, $\frac{3{\cdot}6}{30\times{\cdot}06}=\frac{30\times{\cdot}06\times2}{30\times{\cdot}06}$, and $\frac{3{\cdot}6}{30\times{\cdot}06}=2$, the time, and is an illustration of the Rule for Case IV.

Y. m.
9 11
12

345·825 2)119 months.
59·5

59·5=½ No. of months.

1729125
3112425
1729125

£205·765875=£205 15 3¾ Ans.
Principal=£345 16 6

Amount=£551 11 9¾

A Table of decimal parts for every day in the twelfth part of a year, which consists of 365¼ days.

days.	*dec. pts.*	*days.*	*dec. pts.*	*days.*	*dec. pts.*	*days.*	*dec. pts.*	*days.*	*dec. pts.*
1	·033	7	·230	13	·427	19	·624	25	·821
2	·066	8	·263	14	·460	20	·657	26	·854
3	·098	9	·296	15	·493	21	·690	27	·887
4	·131	10	·328	16	·526	22	·723	28	·920
5	·164	11	·361	17	·558	23	·756	29	·953
6	·197	12	·394	18	·591	24	·788	30	·986

To find the Interest of any Sum, either for Months, or Months and Days at 6 per cent. per annum.

RULE.

Multiply the principal by the number of months, (or months and parts, answering to the given number of days in the table) and cut off one figure at the right hand of the product more than is required by the rule in decimals, and the product will be the interest for the given time, in shillings and decimal parts of a shilling.*

* In the *Note*, under the General Rule for Simple Interest, it is shown that when the Rate is 6 per cent. the product of the principal and *half* the number of months divided by 100, gives the Interest. Whence, the product of the principal and the *number* of months divided by 100, must give *twice* the Interest. Let then the principal £30·5 be put to interest for 17 months. Then $\frac{30·5\times17}{100}$ $=£5·185=2\times£2·5925=$twice the interest, and the interest is 2·5925. Multiply by 20 and the interest will be reduced to shillings and the decimal parts of a shilling, and we have $\frac{30·5\times17\times20}{100}=2\times£2·5925\times20$. Divide by 2, and $\frac{30·5\times17\times10}{100}=£2·5925\times20$, and dividing both parts of the fraction by 10, $\frac{30·5\times17}{10}=£2·5925\times20$, that is, multiply the principal by the number of months and divide the product by 10, or, cut off only one figure more than the rule for

EXAMPLES.

1. What is the interest of £100, for a year?

Principal=100
Mult. by the months=12

Ans. s.120|0=£6

Note. This Table may also be used for the *parts of a year*, in Compound Interest, after having worked for whole years. The shillings, &c. in the principal must first be reduced to the decimal of a pound.

2. What is the interest of £250 10s. for 19 months and 7 days?

Principal= £250·5
Time= 19·23

7515
5010
22545
2505

Ans. s.481·7115
=£24 1 8½

Another Method of calculating Interest for Months, at £6 per cent. per annum.

RULE.

If the principal consists of pounds only, cut off the unit figure, and, as it then stands, it will be the interest for one month in shillings and decimal parts:—If it consist of pounds, shillings, &c. reduce the shillings, &c. to decimals, which, with the unit figure of the pounds, will be decimal parts of a shilling.*

EXAMPLES.

1. What is the interest of £175, for 5 months?

£175=17·5 shill.=interest for 1 month. 17·5
Multiply by the time= 5

2|0)87·5

Ans.=£4 7 6

2. What is the interest of £255 16s. for 7 months?

Shill.
£255 16=25·58 int. for 1 mo.
7

2|0)179 06

Ans. £8 19 0½

decimals requires, and you have the interest in shillings and decimal parts of a shilling. If there be months and days, the days being made decimals from the table, the same rule would manifestly apply.

* This rule is only a contraction of the following process by the Double Rule of Three, to find the interest of any sum, e.g. £36, for 1 month.

As 100 : 6 :: 36
12 : :: 1 month : the interest.

Hence $\frac{6\times36}{12\times100}$=£·18=3·6s. the interest for 1 month. But $\frac{6\times36\times20}{12\times100}$= the interest in shillings, or shillings and decimals, and cancelling the equal parts, we have $\frac{6\times36\times20}{12\times100}=\frac{1\times36\times2}{2\times10}=\frac{1\times36\times1}{1\times10}=\frac{36}{10}$=3·6 shillings. If there be decimals of a pound, the rule would be equally correct.

SIMPLE INTEREST IN FEDERAL MONEY.

PROBLEM I.

When the principal is given in New England pounds, shillings, &c. and the annual interest is required in federal money, at 6 per cent.

RULE.

Reduce the shillings, &c. to their equivalent decimal, by inspection; divide the whole by 5, and the quotient is the annual interest: Or, multiply the principal by 2, and the product (having the unit figure of the pounds cut off) will be the interest as before.*

EXAMPLES.

1. Required the annual interest of £517 3s. 7½d. at 6 per cent.?

3s.=·15	5)517·181
7½d.=·030	D. c. m.
Excess of 12=·001	103·436=103 43 6 Ans.
·181	Or, 517·181
	2
	D. c. m.
	103·4362=103 43 $6\frac{2}{10}$.

2. Required the annual interest of £1, in cents?

Ans. 20 cents.

PROBLEM II.

When the principal is given in New England currency, and the interest and amount are required in federal money at 6 per cent.

RULE.

Reduce the New England money to federal, then divide the principal by 20 and that quotient by 5; add those quotients together, and they are the interest; or add them to the principal, and their sum is the amount.†

EXAMPLES.

1. Required the amount of £425 16s. 8½d. for 1 year, at 6 per cent.?

* This rule is a contraction of the following process. By the General Rule for Simple Interest, (in the first example) the annual interest$=\frac{517{\cdot}181\times6}{100}$ This, multiplied by 20, would be reduced to shillings and decimals of a shilling, and divided by 6, would be reduced to dollars and decimals of a dollar. Then,

$$\frac{517{\cdot}181\times6\times20}{100\times6}=\frac{517{\cdot}181\times1}{5\times1}=\frac{517{\cdot}181}{5}=\$103\ 43c.\ 6\tfrac{2}{10}m.$$

Or $$\frac{517{\cdot}181\times6\times20}{100\times6}=\frac{517{\cdot}181\times1\times2}{10\times1}=\frac{517{\cdot}181\times2}{10}=51{\cdot}7181\times2=\$103{\cdot}4362.$$

† Dividing the principal by 20, gives the interest at 5 per cent. because 5 is one twentieth of a hundred; then dividing this quotient by 5, evidently gives the interest for 1 per cent. Then, as 5+1=6, the sum of the two quotients will be the *interest* at 6 per cent.

Interest at 6 per cent. may often be calculated most easily, by finding the interest at 5 per cent. and adding one fifth of this interest to itself for 6 per cent. And add two fifths of it to itself, and you have the interest at 7 per cent.

Examples.

1. What is the interest of £100, for a year?

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Hence $\frac{6\times36}{12\times100}$=£·18=3·6s. the interest for 1 month. But $\frac{6\times36\times20}{12\times100}$ = the interest in shillings, or shillings and decimals, and cancelling the equal parts, we have $\frac{6\times36\times20}{12\times100}=\frac{1\times36\times2}{2\times10}=\frac{1\times36\times1}{1\times10}=\frac{36}{10}$=3·6 shillings. If there be decimals of a pound, the rule would be equally correct.

SIMPLE INTEREST IN FEDERAL MONEY.

Problem I.

When the principal is given in New England pounds, shillings, &c. and the annual interest is required in federal money, at 6 per cent.

Rule.

Reduce the shillings, &c. to their equivalent decimal, by inspection, divide the whole by 5, and the quotient is the annual interest: Or, multiply the principal by 2, and the product (having the unit figure of the pounds cut off) will be the interest as before.*

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7½d.=·030	——— D. c. m.
Excess of 12=·001	103·436=103 43 6 Ans.
——	Or, 517·181
·181	2
	——— D. c. m.
	103·4362=103 43 $6\frac{2}{10}$.

2. Required the annual interest of £1, in cents?

Ans. 20 cents.

Problem II.

When the principal is given in New England currency, and the interest and amount are required in federal money at 6 per cent.

Rule.

Reduce the New England money to federal, then divide the principal by 20 and that quotient by 5; add those quotients together, and they are the interest; or add them to the principal, and their sum is the amount.†

Examples.

1. Required the amount of £425 16s. 8½d. for 1 year, at 6 per cent.?

* This rule is a contraction of the following process. By the General Rule for Simple Interest, (in the first example) the annual interest$=\frac{517\cdot181\times6}{100}$. This, multiplied by 20, would be reduced to shillings and decimals of a shilling, and divided by 6, would be reduced to dollars and decimals of a dollar. Then,

$$\frac{517\cdot181\times6\times20}{100\times6}=\frac{517\cdot181\times1}{5\times1}=\frac{517\cdot181}{5}=\$103\ 43c.\ 6\tfrac{2}{10}m.$$

$$\text{Or } \frac{517\cdot181\times6\times20}{100\times6}=\frac{517\cdot181\times1\times2}{10\times1}=\frac{517\cdot181\times2}{10}=51\cdot7181\times2=\$103\cdot4362.$$

† Dividing the principal by 20, gives the interest at 5 per cent. because 5 is one twentieth of a hundred; then dividing this quotient by 5, evidently gives the interest for 1 per cent. Then, as 5+1=6, the sum of the two quotients will be the *interest* at 6 per cent.

Interest at 6 per cent. may often be calculated most easily, by finding the interest at 3 per cent. and adding one fifth of this interest to itself for 6 per cent. And add two fifths of it to itself, and you have the interest at 7 per cent.

·8	3)425·835
·034	20)1419·450
·001	5)70·9725
——	14·1945
·835	———— D. c. m.
	1504·6170=1504·61 7. Ans.

2. Required the amount of £112 4s. 6d. for one year?
Ans. $396·52c 7m. 9dec.

Problem III.

When the principal is New England currency, and the monthly interest is required in federal money.

Rule.

Reduce the shillings, &c. to decimals, by inspection, then separate the right hand figure of the pounds with the decimals, divide by 6, and the quotient is the answer in dollars, cents, &c.*

Example.

Required the monthly interest of £425 16s. 8½d. in federal money?

·8	
·034	6)42·5835
·001	———— D. c. m.
——	7·09725=7·09 7¼ Ans.
£·835	

INTEREST ON BONDS AND OBLIGATIONS,

HAVING PARTIAL PAYMENTS ENDORSED.

As there is much diversity of opinion relative to the computation of *lawful* interest in these cases, several Rules will be given, connected with the practice of the Courts of several of the States. The difference of the rules depends on the principle assumed in respect to the *time when interest becomes due.*

Rule I.

The following rule is generally thought to allow too little interest. It is, however, adopted in some of the States.

Find the amount of the principal and interest for the whole time; then find also the amount of each endorsement for the time it has been paid. From the first amount deduct the sum of the amounts of the several endorsements, and the remainder is the balance due.

This rule involves the following points:

1. Interest is not due until the obligation is paid.

* This rule is a contraction of the following process. $\frac{£425·835\times20}{6}=$ the principal in dolls. Then $\frac{425·835\times20\times6}{6\times100\times12}=\frac{425·835\times1\times1}{1\times10\times6}=\frac{425·835}{10\times6}=\frac{42·5835}{6}$, the monthly interest in dollars and decimals of a dollar.

2. Hence, interest must be allowed on the endorsements from the time they were severally made.

Note. A shorter method of computing interest according to this rule, may be seen in examples 16 and 17 of Simple Interest. The absurdity of this rule may be seen in the following manner. Suppose I borrow $100 at 6 per cent. for ten years, and pay six dollars at the end of each year, what will be due at the end of 10 years? The amount of $100 is $160. But the first endorsement of $6 has borne interest for 9 years; the second, for 8 years; the third, for 7 years, and so on; so that six dollars have in fact been drawing interest for *forty five* years, and thus produced $16·20 of interest. This, added to the nine endorsements of $6 each, gives $70·20. That is, while I have paid only the annual interest of $6, the principal has actually been reduced $16·20. By paying $6 annually for 25 years, and computing interest on the several endorsements by this rule, the whole principal would be paid, and the lender would be indebted to the borrower the sum of $2, while in fact the lender had received no part of the sum lent.

RULE II.

The following rule was drafted by the late Judge Sedgwick, and ordered by the Court of Common Pleas for the County of Berkshire in 1791, and is the Rule now used by the Courts of Massachusetts.

"On all contracts carrying interest, on which partial payments may have been made, the principal sum with the interest thereof shall be formed, at the time of payment, into an aggregate sum, from which shall be deducted the sum paid, if one year's interest shall have become due, and if not, then the interest shall be cast to the end of the year, and the aggregate formed as aforesaid, and from the same the payment or payments and the interest thereof from the time or times of payment shall be deducted, and the balance or balances thus formed shall bear interest, and so from time to time, provided, that in *no instance*, interest shall be cast on a greater sum than the principal sum nor on any interest accrued."

[Records of the Court.]

This rule involves the following principles.

1. That, when an obligation has borne interest for one or more years, *interest is not due at intervals of time less than one year.*

2. Interest is to be computed to that endorsement, which, together with the preceding endorsement or endorsements and its or their interest since the time of payment, shall be equal to or exceed the interest on the principal when this endorsement is made, provided one year's interest shall have accrued. The remainder is a new principal, on which interest is to be computed as before.

3. Interest is allowed on all endorsements from the time of their payment, until the year has elapsed, or until an endorsement is made beyond the year, which with the preceding endorsement or endorsements and its or their interest, equals or exceeds the interest due at such endorsement.

4. Interest is not allowed on interest, because the deduction, when made, is intended to pay the interest then due, and the excess, if any, goes to reduce the principal.

5. The design of the rule is to give the interest as nearly annually as the endorsements will admit. But if endorsements are not made on the obligation, the rule implies that the interest is not due until the obligation is paid; and, it is well known, that the Courts will not sustain an action for the payment of the interest from year to year, unless the obligation contains the express condition that the interest shall be *paid annually*.

Note. In the "Scholar's Arithmetick," and in the "Mercantile Arithmetick," the Rule, *said* to be established by the Courts of Massachusetts, is precisely the same as that established in the State of New York, which will be found on a following page. It will be evident from a comparison of the preceding Rule and that of New York, that the computation of interest *ought* to be considerably different in the two States.

RULE III.

The following Rule was established by the Superior Court of Connecticut in 1784.

"Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and, in like manner from one payment to another, till all the payments are absorbed; *provided*, the time between one payment and another be one year or more. But if any payment be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal; and compute the interest on the sum paid, from the time it was paid, up to the end of the year, add it to the sum paid, and deduct that sum from the principal and interest, added as above. If any payments be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed but only on the principal sum, for any period."

This Rule involves the same principles as Rule II. with the following, viz.

That if an endorsement be made after a year's interest has accrued, but which is less than this interest, it shall not bear interest like the endorsements made before a year's interest has become due.

RULE IV.

The following Rule is established for the practice of the Courts in the State of New York.

"The Rule for casting Interest, when partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes towards discharging the principal, and the

subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance of principal as aforesaid." [Johnson's Chancery Reports, Vol. I. page 17.]

This Rule contains the following principles.

1. That Interest is due at any time when a payment is made, if the payment is equal to the interest to that time: if not, the payment is to be added to the following payment or payments, until their sum is equal to or exceeds the interest then accrued, and the balance is a new principal.

2. Interest is not allowed on any endorsement.

3. Interest is not taken on interest, because the interest is due when the endorsement or endorsements are made.

Note. Example 18 in Simple Interest is calculated by this rule.

Example. I.

To be calculated according to the preceding Rules.

For value received I promise to pay James Penny or order one thousand dollars on demand with interest. James Gold.

January 1st, 1815.

The endorsements were,

March 1, 1816, received on the above Note,	75 dolls.
July 1, 1816, - - - - - -	20 do.
Sept. 1, 1817, - - - - - -	20 do.
Nov. 1, 1817, - - - - - -	750 do.
March 1, 1818, - - - - - -	100 do.

What is the balance due July 1st, 1818, interest being allowed at 6 per cent.

By Rule I.

$	
1000	Principal,
210	Interest to July 1, 1818, being 3½ years,
1210	Amount to do.
75	1st endorsement March 1, 1816,
10·50	Interest to July 1, 1818, being 2⅓ years,
85·50	Amount.
20	2nd endorsement July 1, 1816,
2·40	Interest to July 1, 1818,
22·40	Amount.
20	3d endorsement Sept. 1, 1817,
1	Interest to July 1, 1818, being 10 months.
21	Amount.

750	4th endorsement Nov. 1, 1817,
30	Interest to July 1, 1818, being 8 months,
780	Amount.
100	5th endorsement March 1, 1818,
2	Interest to July 1, 1818, being 4 months,
102	Amount.
1010·90	Amount of the several endorsements.
199·10	Balance due July 1, 1818.

By Rule II.

$	
1000	Principal,
70	Interest to 1st endorsement March 1, 1816, being 14 months.
1070	Amount to do.
75	1st endorsement—deduct,
995	New principal March 1, 1816,
99·50	Interest to 4th endorsement Nov. 1, 1817, being 20mo.
1094·50	Amount to do.
20	2nd endorsement July 1, 1816,
1·60	Interest to Nov. 1, 1817, being 16 months,
21·60	Amount to do.
20	3d endorsement Sept 1, 1817,
·20	Interest to Nov. 1, 1817, being 2 months,
20·20	Amount to do.
750	4th endorsement Nov. 1, 1817,
41·80	Sum of the amounts of the two preceding endorsements.
791·80	To be subtracted from 1094·50.
302 70	New principal Nov. 1, 1817,
12·108	Interest to July 1, 1818, being 8 months,
314·808	Amount to do.
100	5th endorsement March 1, 1818,
2	Interest to July 1, 1818, being 4 months,
102	Amount to do. to be deducted from 314·808.
212 808	Balance due July 1, 1818.
199·10	Do. by rule 1.
13 708	Difference.

By Rule III.

$	
1000	Principal,
70	Interest to 1st endorsement,
1070	Amount.
75	1st Endorsement,
995	New Principal March 1, 1816,
99·50	Interest to 4th endorsement, being 20 months,
1094·50	Amount to do.
20	2nd endorsement,
1·60	Interest to 4th endorsement,
21·60	Amount to do.
20	3d endorsement, which does not bear interest, because one year's interest has accrued on new principal.
750	4th endorsement,
791·60	Deduct from 1094·50,
302·90	New principal Nov. 1, 1817,
12·31·6	Interest to July 1, 1818, being 8 months,
315·21·6	Amount to do.
100	5th endorsement,
2	Interest to July 1, 1818, being 4 months,
102	Amount—deduct from 315·21·6.
213·21·6	Balance due July 1, 1818,
212·80·8	Do. by Rule 2.
0·40 8	Difference,
13·11·6	Do. from result of Rule I.

By Rule IV.

$	
1000	Principal,
70	Interest to 1st endorsement, being 14 months.
1070	Amount to do.
75	1st endorsement.
995	New principal March 1, 1816,
19·90	Interest to July 1, 1816,
1014·90	Amount to do.
20	2nd endorsement July 1, 1816.
994·90	New Principal do.

79·59·2	Interest to Nov. 1, 1817,
1074·49·2	Amount to do.
20	3d endorsement,
750	4th do. Nov. 1, 1817,
770	Sum of these endorsements.
304·49·2	New principal Nov. 1, 1817,
6·08·9	Interest to March 1, 1818,
310 58·1	Amount to do.
100	5th endorsement do.
210·58·1	New principal do.
4·21·2	Interest to July 1, 1818,
214·79·3	Balance due do.

15·69·3	Difference between this result	and that by	Rule I.
1·98·5	do.	and	Rule II.
1·77·6	do.	and	Rule III.

$	By Rule IV. at 7 per cent.
1000	Principal,
198·33$\frac{1}{3}$	Interest to 4th endorsement Nov. 1, 1817, being 34m.
1198·33$\frac{1}{3}$	Amount to do.
865	Sum of the first four endorsements.
333·33$\frac{1}{3}$	New principal Nov. 1, 1817,
7·77$\frac{7}{9}$	Interest to March 1, 1818,
341·11	Amount.
100	5th endorsement,
241·11	New principal,
5·62 59	Interest to July 1, 1818,
246·73 59	Balance due do.

EXAMPLE 2.

On Jan. 1, 1816, Samuel Trusty owed me 775l., on which I was to receive interest at 7 per cent. On July 1, he paid me 100l.; on Jan. 1 1817, 25l.; on Sept. 1, 25l.; on March 1, 1818, 25l.; on July 1, 1819, 10l.; on Sept. 1, 390l.; and on Jan. 1, 1821, the balance: What was the balance by the four preceding rules and what is the whole interest paid?

By Rule 1. { Balance due £384 508$\frac{1}{4}$.
Whole interest paid £184·508$\frac{1}{4}$.

By Rule 2. { Balance £410·3444.
Whole interest paid £210·3544.

By Rule 3. { Balance £410·47122¾. Interest £210·4718¼.

By Rule 4. { Balance £418·487+. Interest £245 512+.

EXAMPLE 3.

I gave a promissory note to A. D. March 1, 1817, for $350. On Dec. 1, I paid $150; on April 1, 1818, $35; on June 1, $10; Jan. 1, 1819, $40; Dec. 1, $65; and March 1, 1820, a settlement was made. To whom was a balance due, and how much, computing interest at 6 per cent. by the last three rules? Ans.

DISCOUNT

IS an allowance made for the payment of any sum of money, before it becomes due, and is the difference between that sum, due some time hence, and its present worth.

The *present worth* of any sum or debt, due some time hence, is such a sum, as if put to interest, would in that time and at the rate per cent. for which the discount is to be made, amount to the sum or debt then due.

RULE I.*

As the amount of £100 for the given rate and time is to £100; so is the given sum or debt to the present worth.

* That an allowance ought to be made for paying money before it becomes due, which is supposed to bear no interest till after it is due, is very reasonable; for if I keep the money in my own hands till the debt shall become due, it is plain I may make an advantage of it by putting it out to interest for that time; but if I pay it before it is due, I give that benefit to another; therefore we have only to inquire what discount ought to be allowed. And here, many suppose that, since by not paying till it becomes due they may employ it at interest; therefore, by paying it before due, they shall lose that interest, and for that reason all such interest ought to be discounted; but the supposition is false, for they cannot be said to lose that interest till the time arrives, when the debt becomes due; whereas we are to consider what would be lost, at present, by paying the debt before it becomes due; this can, in point of equity, be no other than such a sum, which being put out at interest till the debt shall become due, would amount to the interest of the debt for the same time. It is besides plain, that the advantage arising from discharging a debt due some time hence, by a present payment, according to the principles above mentioned, is exactly the same as employing the whole sum at interest till the time when the debt becomes due, arrives: for, if the discount allowed for present payment be put out at interest for that time, its *amount* will be the same as the interest of the whole debt for the same time; thus the discount of £106, due one year hence, reckoning interest at £6 per cent. will be £6, and £6 put out to interest at £6 per cent. for one year, will amount to £6 7s. 2¼d. which is exactly equal to the interest of £106, for one year at £6 per cent.

The truth of the rule for working is evident from the nature of Simple Interest; for since the debt may be considered as the *amount* of some principal (called here the present worth) at a certain rate per cent. and for the given time, that amount must be in the same proportion either to its principal or interest,

Subtract the present worth from the given sum, and the remainder will be the discount required.

Or, As the amount of £100 for the given rate and time, is to the interest of £100 for that time: so is the given sum or debt to the discount required.

Or, In *federal money*, divide the given sum by the amount of $100 for the given time and rate; point off from the right of the quotient, two places less than in division of decimals for the present worth.

EXAMPLES.

1. What is the discount of { £635 17s. / $2119 50c. } due 2 years hence at 5½ per cent. per annum?

Interest of £100 per annum=5 10
2 years.

11
Add 100

	£		£ s.		£ s. d.
As £111 :	11	::	635 17 :		63 0 2 disc. Ans.

£	£	£ s.	£ s. d.
Or, As 111 :	100 ::	635 17 :	572 16 9¼ present worth.

£. s. £ s. d. £ s. d.
And 635 17—572 16 9¼=63 0 2¼ discount.

IN FEDERAL MONEY.

$ $ $ c. $ c. m.
As 111 : 11 :: 2119 50 : 210 04 0½=discount. Or,

As 111 : 100 :: 2119 50 : $\frac{2119{\cdot}5\times100}{111}$=$1909 45c. 9½m.= present worth; and 2119·5—1909·4595=210·0405=discount as before.

Or, $\frac{2119{\cdot}5}{111}$=19·094595; and 1909·4595=present worth, as before.

2. What is the present worth of $350 payable in half a year, discounting at 6 per cent. per annum? Ans. $339 80c. 5m.

3. What is the present worth of £65, due 15 months hence, at £6 per cent. per annum? Ans. £60 9s. 3½d.

4. What is the discount on £97 10s. due January 22, this being September 7th, reckoning interest at £5 per cent.?

Ans. £1 15 11.

as the amount of any other sum, at the same rate, and for the same time, to its principal or interest.

In common cases, the interest is taken for the discount, the parties not attending to the real difference between discount and interest. Thus $100 discounted in this way for a year at 6 per cent. or $6 is taken out, and the person receives $94. If he were to lend the $94 on interest for a year at the same rate, he would receive of interest $5 64cts. or 36 cents less than the above discount, which is, in fact, discounting at 6$\frac{18}{47}$ per cent. or nearly 6·4 per cent. instead of 6 per cent.

In Bank Discount, the interest is considered as the discount.

5. What ready money will discharge a debt of $1595 due 5 months and twenty days hence, at 6 per cent.?

Ans. $1541 32c. 6m.

6. Bought a quantity of goods for $250, ready money, and sold them for $300 payable 9 months hence: What was the gain, in ready money, supposing discount to be made at 6 per cent.?

Ans. $37 8c. 1¼m.

7. What is the present worth of $960, payable as follows; viz. ⅓ at 3 months, ⅓ at 6 months, and the rest at 9 months, supposing the discount to be made at 6 per cent.? Ans. $936 70c.

Rule II.

As any sum of money, at 6 per cent. per annum, will double, at simple interest, in 200 months; therefore,

To 200 add the number of months for which the discount is required, by which sum divide the product of the money and time, (in months,) and the quotient will be the discount.

Examples.

1. What is the discount of $150 75c. for a year?

```
 200   150·75
+ 12    ×12
 ---   ------       $ c. m.
212)1809·00(8·533=8·53 3 discount, Ans.
    1696
    ----
     1130
     1060                     150·75
     ----                      8·533
      700                     ------
      636    Present worth   142·217
      ---
       640
       636
       ---
         4
```

2. What is the present worth of $426 55c. at 6 per cent. to be paid 8 months hence? Ans. $410 14c. 5m.

3. What is the discount of £361 15s. 6d. to be paid 1 year and 8 months hence? Ans. £32 17s. 9¼d.

ABBREVIATIONS IN DISCOUNT.

Any principal to be discounted for one year, at any of the following rates, (or for any rate and time, whose product is equal to any of the following rates) being multiplied by the multiplier, (if any.) and divided by the corresponding divisor, the quotient will be the discount.

	Rates.*	
At	$1\frac{1}{4}$	$\div 81$ (or by 9 and 9)
	2	$\div 51$
	$2\frac{1}{2}$	$\div 41$
	4	$\div 26$
	5	$\div 21$ (or by 7 and 3)
	6	$\div 53$ and $\times 3$
	$7\frac{1}{2}$	$\div 43$ and $\times 3$
	8	$\div 27$ and $\times 2$ (or $\times 2$ and $\div 9$ and 3)
	$8\frac{1}{3}$	$\div 13$
	10	$\div 11$
	12	$\div 28$ and $\times 3$ (or $\times 3$, and $\div 7$ and 4)
	$12\frac{1}{2}$	$\div 9$

EXAMPLES.

1. How much must I abate of £5394 10s. due 3 years hence, at $2\frac{2}{3}$ per cent. per annum?

$2\frac{2}{3}$
$\times 3$
——
8, therefore, $\times 2$, and $\div 27$

£5394 10s.
2
——
$27 \div 3 = 9$)10789 0
——
3)1198 15 $6\frac{2}{3}$
——
£399 11 10 Ans.

2. What is the discount of \$546 62c. 5m. for $8\frac{1}{3}$ years, at 1 per cent. per annum, (or for 1 year, at $8\frac{1}{3}$ per cent. per annum?)

\$ c. m.
13)546·62 5
——
Ans. \$42·04 8

3. What is the discount of \$125 at $1\frac{1}{4}$ per cent. per annum, for four years, (or, at 4 per cent. per annum, for $1\frac{1}{4}$ years?)

Ans. \$5 95c. 2m.

* These contractions are obvious from any example, wrought according to the General Rule. Thus, let the sum to be discounted be 300 dollars.

1. At $1\frac{1}{4}$ per cent. Then, by the Rule,

$101\frac{1}{4} : 1\frac{1}{4} :: \$300$: discount, or,

$\frac{405}{4} : \frac{5}{4} ::$

$405 : 5 :: 300 : \frac{5 \times 300}{405} = \frac{300}{81}$, and is the Rule.

81 : 1 :: 300 : the discount required.

2. At 2 per cent. Then,

102 : 2 :: 300 : discount, or,

$51 : 1 :: 300 : \text{discount} = \frac{300}{51}$.

3. At $2\frac{1}{2}$ per cent. Then,

102·5 : 2·5 :: 300 : discount, or

41 : 1 :: 300 : Answer required.

4. At 8 per cent. Then,

108 : 8 :: 300 : discount, or,

27 : 2 :: 300 : Answer required.

In the same way, all the contractions be made. The contractions are made on the two terms of the proportion which are invariable, when the *rate* is given.

DISCOUNT BY DECIMALS.

The sum to be discounted, the time and the ratio given, to find the present worth.

RULE.

Multiply the ratio by the time, add unity to the product for a divisor; by which sum divide the sum to be discounted, and the quotient will be the present worth.*

Subtract the present worth from the principal, or sum to be discounted, and the remainder will be the discount.

Or, as the amount of £1 for the given time, is to £1, so is the interest of the debt for the said time, to the discount required.

Subtract the discount from the principal, and the remainder will be the present worth.

EXAMPLES.

1. What is the present worth of 600l. due 3 years hence, at 6l. per cent. per annum?

First Method.

$$\begin{array}{rl} \text{Ratio}= & \cdot06 \\ \text{Multiply by the time}= & 3 \\ \hline \text{Product}= & \cdot18 \\ \text{Add} & 1\cdot \\ \hline \text{Divisor}= & 1\cdot18)600(508\cdot4745 \text{ present worth.} \end{array}$$

$$\text{Or, } \frac{600}{\overline{\cdot06\times3}+1}=£\,508\ \ 9s.\ 5\tfrac{3}{4}d.\ \text{Ans.}$$

Present worth=508·4745=£508 9s. 5¾d. which, subtracted from the principal, will give the discount=£91 10s. 6¼d.

* As the sum to be discounted is, in fact, the *amount* of some principal at the given rate and time; to find the principal, which is now the *present worth*, you have only to employ the rule for Case 2, in Simple Interest by decimals. This is the rule in the text. Thus in Ex. 1, by said Case 2, $\frac{600}{1+\cdot06\times3}$=the principal, in this case the *present worth*. The remainder of the rule is evident from what has been said under Discount, Rule 1.

Note 1. In the method used in Simple Interest by Decimals, you may easily find rules for obtaining either of the four terms, *present worth*, *ratio*, *time*, or *sum to be discounted*, when the other *three* are given.

Note 2. When the ratio is ·06, or six per cent. per annum, and the given time is expressed in months, if the debt be divided by unity added to half as many hundredths of an unit as there are months in the given time, the quotient will be the present worth. Thus for 3 years or 36 months, the divisor, we have just seen to be $1+\cdot06\times3,=1+\cdot18$, or 1 added to half as many hundredths as there are months; if the time be 3½ years or 42 months, the divisor is $1+\cdot06\times3\cdot5,=1+\cdot21$; if 10 months, then $1+\cdot06\times\frac{10}{12}=1+\cdot05$, as before; if 2 months, then $1+\cdot06\times\frac{2}{12},=1+\cdot01$; if 1 month, then $1+\cdot06\times\frac{1}{12},=1+\cdot005$, and so on.

Second Method.

```
       Ratio=·06
Multiply by  3    As 1·18 : 1 :: 108
          ───                    1
           ·18                  ───
      Add 1·           1·18)108·00(91·5254
          ───
```

Amount of 1l. for the given time $=1{\cdot}18$

And $600\times{\cdot}06\times3=108$=interest of the debt for the given time.— Discount=91·5254=£91 10s. 6d. which taken from the principal will leave the present worth=£508 9s. 6d.

2. What is the present worth of $558 62c. 5m. due 2 years hence, at 4½ per cent. per annum?

First Method.

```
      Ratio=·045
     ×Time=  2·
            ────
             ·09
           +1·
           ────
Divisor=1·09)558·625(512·5=present worth.
              545
              ───
               136
               109
               ───
                272
                218
                ───
                545
                545
                ───
                ...
```

Or, $\dfrac{558\ 625}{{\cdot}045\times2+1}$=$512·5 Ans.

And $558 625—$512·5=$46 12c. 5m.=discount. Or, As $1·09 (=amount of $1 for the given time) : $1 :: $50·27625 (= interest of the debt for the given time) : $46·125=discount as above. And, $558·625—$46·125=$512·5=present worth, as above.

3. Required the present worth and discount of $4125 50c. at 6¾ per cent. per annum, due 18 months hence?

Ans. { present worth $3746 19c. 7½m.
{ discount 379 30 2½.

4. What ready money will discharge a debt of 1354l. 8s. due 3 years, 3 months, and 12 days hence, at 5¾l. per cent. per annum?
Ans. £1135 7s. 9d.

DUTIES.

DUTIES are assessed upon articles imported into the country, at a certain rate per pound, hundred, ton, gallon, &c. without respect to the value of the articles; or upon articles according to their actual cost. The latter are called *ad valorem* duties. The duties are computed in the former case, on the most obvious principles, as will be seen in the following

EXAMPLES.

1. Required the duty on 987lb of chocolate at 3 cents per pound.

3

$29·61 cents, Ans.

2. If the duty on molasses is 5 cents on a gallon when imported in an American vessel, and 10 per cent. more in a foreign vessel, what is the duty on 3950 gallons in both vessels?

Ans. $197·50, and $217·25.

3. Required the duty on 10Cwt. 3qrs. 14lb of cordage at $2·25 per Cwt. in an American vessel, and at 10 per cent. more in a foreign vessel? Ans. $24·47 nearly, and $26·81.

4. What is the duty on 6hhds. of brown sugar, weighing 53Cwt. 2qrs. 20lb tare 12lb per 100, at 2½cts. per pound in an American vessel, and at 10 per cent. more in a foreign vessel?

Ans. $132, and $145·20.

Duties *ad valorem* are estimated by adding 20 per cent. to the actual cost of the goods, &c. when imported from or beyond the Cape of Good Hope, and 10 per cent. when imported from any other places. Insurance, commission, &c. do not belong to the actual cost.

The duties are computed in the following obvious manner. When the cost is reduced to Federal Money, add the per cent. to the cost, and then find the duty per cent. ad valorem.

EXAMPLES.

1. What will be the duty on an invoice of goods, which cost £786 15s. sterling, at 15 per cent. ad valorem when imported in an American vessel, or at 10 per cent. more when imported in a foreign vessel from England?

	£	s.	$
	786	15=	3493·17
Add 10 per cent.=			349·317
			3842·487
10 per cent.=			384·2487
5 do. =			192·12435
15 per cent.=			$576·37·305 Ans. in Am. vessel.
16½ per ct.=			$634·01 Ans. in For. ship.

2. What is the duty on goods, which cost in India, $2780·50, imported in an American ship, at $12\frac{1}{2}$ per cent. ad valorem?

Ans. $417·075.

BARTER

IS the exchanging of one commodity for another, and teaches traders to proportion their quantities without loss.*

CASE I.

When the quantity of one commodity is given, with its value, or that of its integer, that is, of 1lb. 1cwt. 1yd. &c. as also the value of the integer of some other commodity, to be given for it, to find the quantity of this; or, having the quantity thereof given, to find the rate of selling it.

Rule.

Find the value of the given quantity by the concisest method, then find what quantity of the other, at the rate proposed, you may have for the same money: *Or*, if the quantity be given, find from thence the rate of selling it. *Or*, As the quantity of one article is to its price, so, *inversely*, is the quantity of the other to its price. *Or*, as the price of one article is to its quantity; so *inversely*, is the price of the other to its quantity.

Examples.

1. How much tea at 9s. 6d. per ℔ must be given in barter for 156 gallons of wine, at 12s. $3\frac{1}{2}$d. per gallon?

		Galls.
3d.	$\frac{1}{4}$	156
$\frac{1}{2}$	$\frac{1}{6}$	12
		1872
		39
		6 6
9s. 6d.=114d		1917 6
		12
		23010

d. ℔ d. ℔ oz.

As 114 : 1 :: 23010 :: 201 $13\frac{54}{114}$ Ans.

price. quan. price. quan.

s. d. gals. s. d. ℔ oz.

Or, As 12 $3\frac{1}{2}$: 156 : 9 6 : 201 $13\frac{54}{114}$ Ans. as before.

* The Rules in Barter are only applications of the Rule of Three, and are easily understood.

2. How much cloth, at 15s. 8d. per yard, must be given for 5cwt. 3qrs. 19lbs. of steel, at 5 guineas per cwt? Ans. 52yds. 3qrs. 2n.

3. Suppose A has 350 yards of linen, at 25c. per yard, which he would barter with B for sugar, at $5 per cwt. How much sugar will the linen come to? Ans. 17cwt. 2 qrs.

4. A has broadcloths at $44 per piece, and B has mace, at $1 42c.* per ℔: How many pounds of mace must B give A for 35 pieces of cloth? Ans. 1084½℔.

5. A has 7½cwt. of sugar at 12 cents per ℔ for which B gave him 12½cwt. of flour: What was the flour rated at per ℔?

Ans. 7c. 2m.

CASE II.

If the quantites of two commodities be given, and the rate of selling them, to find, in case of inequality, how much of some other commodity must be given.

RULE.

Find the separate values of the two commodities; subtract the less from the greater, and the difference will be the amount of the third commodity, whose quality and rate may be easily found.

EXAMPLES.

1. Two merchants barter; A has 30cwt. of cheese, at 23s. 6d. per cwt. and B has 9 pieces of broadcloth, at 3l. 15s. per piece: Which must receive money and how much?

Ans. B must pay A £1 10s.

2. A and B would barter; A has 150 bushels of wheat, at $1 25c. per bushel, for which B gives 65 bushels of barley, worth 62½c. per bushel, and the balance in oats at 37½c. per bushel: What quantity of oats must A receive from B? Ans. 391⅔ bushels.

CASE III.

Sometimes, in bartering, one commodity is rated above the ready money price; then, to find the bartering price of the other, say,

As the ready money price of the one, is to its bartering price; so is *that* of the other, to *its* bartering price: Next, find the quantity required, according to either the bartering or ready money price.

EXAMPLES.

1. A has ribbands at 2s. per yard ready money; but in barter he will have 2s. 3d. B has broadcloths at 32s. 6d. per yard ready money; at what rate must B value his cloth per yard, to be equivalent to A's bartering price, and how many yards of ribband, at 2s. 3d. per yard, must then be given by A for 488 yards of B's broadcloth?

Ans. B's broadcloth, at £1 16s. 6¼d. per yd. 7930 yds. ribband.

2. A and B barter; A has 150 gallons of brandy, at $1 37½c. per gallon ready money, but in barter he will have $1 50c.; B has

linen at 44c. per yard ready money; how must B sell his linen per yard in proportion to A's bartering price, and how many yards are equal to A's brandy?

Ans. barter price is 48c. and he must give A 468yds. 3qrs.

3. P and Q barter; P has Irish linen, at 60c. per yard, but in barter he will have 64c. Q delivers him broadcloth at $6 per yard, worth only $5 50c. per yard: Pray which has the advantage in barter, and how much linen does P give Q for 148 yards of broadcloth?

c. c. $ c. $ c.
As 60 : 64 :: 5 50 : 5 86$\frac{2}{3}$; therefore Q by selling at $6 has the advantage. Then,

$ yds. c. yds. qrs.
As 6 : 148 :: 64 : 1387 2 linen, Ans.

4. A has 200 yards of linen, at 1s. 6d. ready money per yard, which he barters with B, at 1s. 9d. per yard, taking buttons at 7$\frac{1}{2}$d. per gross, which are worth but 6d.: How many gross of buttons will pay for the linen, who gets the best bargain, and by how much, both in the whole, and per cent.?

Yd. d. Yds. d. d. Gross. d. Gross. Yd. d. Yds. £
As 1 : 21 :: 200 : 4200. As 7$\frac{1}{2}$: 1 :: 4200 : 560. As 1 : 18 :: 200 : 15 [value of A's linen.

gr. d. gr. £
As 1 : 6 :: 560 : 14 value of B's goods. So that B gains 1l. of A.

£ £ £ £ s. d.
As 14 : 1 :: 100 : 7 2 10 per cent.

5. A has linen cloth, at 30c. per yard, ready money, in barter 36c. B has 3610 yards of ribband, at 22c. per yard ready money, and would have of A $200 in ready money, and the rest in linen cloth; what rate does the ribband bear in barter per yard, and how much linen must A give B?

Ans. The rate of ribband is 26c. 4m. per yard, and B must receive 1980$\frac{2}{3}$ yards of linen, and $200 in cash.

LOSS AND GAIN

IS an excellent rule, by which merchants and traders discover their profit, or loss per cent. or by the gross: It also instructs them to raise or fall the price of their goods, so as to gain or lose so much per cent, &c. The rules are only particular applications of the Rule of Three.

CASE I.

To know what is gained or lost per cent.

RULE.

First see what the gain or loss is, by subtraction; then, as the price it cost, is to the gain or loss: so is 100l. to the gain or loss per cent.

Or, in *federal money*, annex two cyphers to the gain or loss, and divide by the cost for the gain or loss per cent.

EXAMPLES.

1. If I buy serge at 90c. per yard, and sell it again at $1 2c. per yard: What do I gain per cent. or in laying out $100?

Sold for $1·02 As 90 : 12 :: 100 : $13\frac{1}{3}$ per cent. gain, Ans.
Cost ·90

Gain ·12 per yard.

Or, 1·02—·90=·12=gain per yard; and $\frac{12·00}{·9}$=$13\frac{1}{3}$ per cent. gain, [as before.

N. B. The first questions in the several cases, serve to elucidate each other.

2. If I buy serge at $1 2c. per yard, and sell it again at 90c. per yard: What do I lose per cent. or in laying out $100?

Cost 1·02 As 1 02 : 12 :: 100 : 11 76 5 per cent. loss, Ans.
Sold for ·90

Loss ·12 Or, $\frac{12.00}{1·02}$=11·765 per cent. loss, Ans. as before.

3. If I buy a cwt. of tobacco for £9 6s. 8d. and sell it again at 1s. 10d. per lb do I gain or lose, and what per cent.?

lb 112

		£	s.	d.
2d. | $\frac{1}{12}$ | £11 4 value at 2s. per lb.	Sold for	10	5	4
0 18 8 value at 2d. per lb.	Cost	9	6	8
10 5 4 value at 1s. 10d. per lb.		0	18	8 gained in the gross.

£ s. d. s. d. £ £
As 9 6 8 : 18 8 :: 100 : 10 Ans. 10 per cent. gain.

4. A draper bought 60 yards of cloth at $4 50c. per yard, and 38 yards of cloth at $2 50c. per yard, and sold them, one with another, at $4 25c. per yard: Did he gain or lose, and what per cent.

60 yards at $4 50c. per yard = $270
38 yards at 2 50 per yard = 95

98 yards cost - - - - - - 365
which subtract from 98yds. at $4 25c.=416·50

gain in the gross = 51·50

Then, as $365 : $51·50c. :: $100 : $\frac{5150·00}{365}$=$14·11c. gain per cent. Ans.

5. Bought sugar at $6\frac{1}{2}$d. per lb and sold it at £2 3s. 9d. per cwt. What was the gain or loss per cent.?

lb d. lb £ s. d.
As 1 : $6\frac{1}{2}$ :: 112 : 3 0 8

Prime cost £3 0 8 per cwt. £ s. d. s. d. £ £ s. d.
Sold at 2 3 9 per cwt. as 3 0 8 : 16 11 :: 100 : 27 17 8¼
[loss per cent. Ans.

Lost £0 16 11 in the whole.

6. At 4s. 6d. in the pound profit: How much per cent.?

£ s. d. £ £ s.
As 1 : 4 6 :: 100 : 22 10 Ans.

7. If I buy candles at 1s. 6d. per lb and sell them again at 2s. per lb and allow 3 months for payment: What do I gain per cent.?

d. d. £ £ s. d. Mo. £ Mo. £ s.
As 18 : 24 :: 100 : 133 6 8; then by discount, As 12 : 6 :: 3 : 1 10
£ s. £ s. £ s. d. £ s. d.
Then, as 101 10 : 1 10 :: 133 6 8 : 1 19 4¾, which taken from £133 6s. 8d. leaves £131 7s. 3¼d. therefore, Ans. £31 7s. 3¼d.

8. If I buy cloth at 13s. per yard, on 8 months credit, and sell it again at 12s. ready money, do I gain, or lose, and what per cent.?

Ans. lost £4 per cent. or 6d. in the yard.

9. If I buy gloves at $1 25c. per pair: How long credit must I have, to gain $13 per cent. when I sell them at $1 36c. per pair?

$ c. $ c. c. $ $ c.
Sold at 1·36 As 1·25 : ·11 : 100 : 8·80 gain per cent. rdy. mo.
Prime cost 1·25 $ $ c. $ c.
Then, 13—8·80=4·20 Now,
Gained ·11 per pair. $ Mo. $ c. Mo. days.
As 6 : 12 :: 4·20 : 8 12 Ans.

In casting up the amount of goods bought, imported or exported: to the prime cost of such goods we must add all the charges upon them, in order to fix the price they stand us in.

10. Suppose I import from France, 12 bales of cloth, containing 10 pieces each, which, with the charges there, amounted to $360: I pay duty here 92c. per piece, for freight $12 and portage $1 25c.; What does it stand me in per piece, and how must I sell it per piece to gain $10 per cent.?

Ans. $4 43 3 the price at which it must be sold per piece.

CASE II.

To know how a commodity must be sold, to gain or lose so much per cent.

Rule.

As £100 is to the price; so is £100 with the profit added, or loss subtracted, to the gaining or losing price. Or,

In *federal money*, multiply 100 dollars added to the gain, or less by the loss per cent. by the cost; and pointing off the two right hand figures of the product gives the answer.

Examples.

1. If I buy a quantity of serge at 90c. per yard: How must I sell it per yard to gain $13\frac{1}{3}$ per cent.?

$ $ c. c. $ c.
As 100 : 113 $33\frac{1}{3}$ :: 90 : 1 2 Ans.

$ c. c. $
Or, 113 33⅓×90=102; and pointing off two right hand places, $1 02, Ans. as before.

2. If a barrel of powder cost £4, how must it be sold to lose £10 per cent.?

£ £ £	Or thus:
As 100 : 4 :: 90	90
4	4
100)360(3	£3|60
300	20
60	s.12|00
20	
100)1200(12 Ans. £3 12s.	

3. Bought cloth, at $2 50c. per yard, which not proving so good as I expected, I am content to lose 17½ per cent. by it: How must I sell it per yard? Ans. $2 6c. 2¼m.

4. If 120lb of steel cost £7, how must I sell it per lb to gain £15¾ per cent.? Ans. 1s. 4d. per lb.

5. A gentleman bought 10 tons of iron for £200, the freight and duties came to £25, and his own charges to £8 6s. 8d.; How must he sell it per lb to gain £20 per cent. by it?

£ £ £ s. d. £ s. d. £ s. d. £ s. d. £
As 100 : 20 :: 233 6 8 : 46 13 4 Then, 233 6 8+46 13 4=280.
Tons. £ lb. d.
As 10 : 280 :: 1 : 3 per cent. Ans.

6. If a bag of cotton, weighing 8cwt. 0qrs. 20lb cost $45 55c. how must it be sold per cwt. to lose $8 per cent.?
Ans. $5 12c. 3m.

7. Bought fish in Newburyport, at 10s. per quintal, and sold it at Philadelphia, at 17s. 6d. per quintal; now, allowing the charges at an average, or one with another, to be 2s. 6d. per quintal, and considering I must lose £20 per cent. by remitting my money home; what do I gain per cent.?

Selling price 17 6 Philadelphia currency, per quintal.
Charges 2 6 ditto.

15 0 ditto.

£ s. £ s.
As 100 : 15 :: 80 : 12 New England currency.
Sold at 12s. per quintal.
Bought at 10s. per quintal.

Gained 2s. per quintal.

s. s. £ £
As 10 : 2 :: 100 : 20 per cent. gained, Ans.

8. Bought 50 gallons of brandy, at 75c. per gallon, but, by accident, 10 gallons leaked out: At what rate must I sell the remainder per gallon, to gain upon the whole prime cost, at the rate of 10 per cent.? Ans. $1 3c. 1¼m.

CASE III.

When there is gain or loss per cent. to know what the commodity cost.

RULE.

As £100 with the gain per cent. added, or loss per cent. subtracted, is to the price; so is £100 to the prime cost. Or,

In *federal money*, divide the price with two cyphers annexed by $100 added to the gain, or less by the loss, per cent. for the answer.

EXAMPLES.

1. If 1 yard of cloth be sold, at $1 2c. and there is gained 13⅓ per cent. what did the yard cost?

$\$$ $\$$ c. $\$$ c.

As $\overline{100+13\frac{1}{3}}$: 1 2 :: 100 : 90 prime cost, Ans.

Or, $\frac{102 \cdot 00}{113 \cdot 33\frac{1}{3}} = \cdot 9$, Ans. as before.

2. If 12 yards of cloth are sold at 15s. per yard, and there is £7 10s. loss per cent in the sale: What is the prime cost of the whole.

Yds. s. Yds. £   £ s. £   £   £ s. d.

As 1 : 15 :: 12 : 9   As 92 10 : 9 :: 100 : 9 14 7 Ans.

3. If 40lb of chocolate be sold at 25c. per lb and I gain 9 per cent. what did the whole cost me? Ans. $9 17c. 4m.+

4. If 19½cwt. sugar be sold at $14 50c. per cwt. and I gain $15 per cent.: What did it cost per cwt.? Ans. $12 60c. 8m.

CASE IV.

If by wares sold at such a rate, there is so much gained or lost per cent. to know what would be gained or lost per cent. if sold at another rate.

RULE.

As the first price is to £100 with the profit per cent. added, or loss per cent. subtracted; so is the other price to the gain or loss per cent. at the other rate.

N. B. If your answer exceed 100, the excess is your gain per cent. but if it be less than 100, the deficiency is your loss per cent.

EXAMPLES.

1. If cloth, sold at $1 2c. per yard, be 13⅓ profit per cent. what gain or loss per cent. shall I have, if I sell the same at 90c. per yard?

$\$$ c. $\$$ c. $\$$

As 1 2 : 113⅓ :: 90 : 100

And, 100—100=0, Ans. I neither gain, nor lose.

2. If cloth, sold at 4s. per yard, be £10 per cent. profit: What shall I gain or lose per cent. if sold at 3s. 6d. per yard?

s. £ s. d.
As 4 : 110 :: 3 6
12 12
— —
48 42
110
——
48)4620(96¼ Ans. I lost £3¾ per cent. by the last sale.
432
——
300
288
——
12

£ £ £
Then, 100—96¼=3¾

3. If I sell a gallon of wine for \$1 50c. and thereby lose 12 per cent.: What shall I gain or lose per cent. if I sell 4 gallons of the same wine for \$6 75c.? Ans. 1 per cent. loss.

4. I sold a watch for 50l. and by so doing, lost 17l. per cent. whereas in trading I ought to have cleared 20l. per cent. How much was it sold under its real value? Ans. 22l. 5s. 9¼d.

EQUATION OF PAYMENTS

IS the finding a time to pay, at once, several debts, due at different times, so that no loss shall be sustained by either party.

Rule I.*

Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the equated time, or that required.

* This rule is founded upon a supposition that the sum of the interests of the several debts, which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms. Some, who defend this principle, have endeavoured to prove it to be right by this argument; that what is gained by keeping some of the debts after they are due, is lost by paying others before they are due; but this cannot be the case; for though, by keeping a debt after it is due, there is gained the interest of it for that time; yet, by paying a debt before it is due, the payer does not lose the interest for that time, but the discount only, which is less than the interest, and therefore the rule is not accurately true; however, in most questions, which occur in business, the errour is so trifling, that it will always be made use of as the most elligible method.

From the principle assumed in this rule, the rule may be derived in the following manner. Thus in Example 1, where 8 months is found to be the equated time, let the interest be supposed at any rate, as 6 per cent. Then the first payment is to be at interest for 8—6 or 2 months, and by the rule for interest, $\frac{100\times6\times\overline{8-6}}{100}$ =its interest. The second sum is to be on interest 8—7 or 1 month, and

EXAMPLES.

1. A owes B $380 to be paid as follows, viz. $100 in 6 months. $120 in 7 months, and $160 in 10 months: What is the equated time for the payment of the whole debt?

$$\begin{array}{r} 100\times 6= 600 \\ 120\times 7= 840 \\ 160\times 10=1600 \\ \hline \end{array}$$

$$100+120+160=380)3040(8 \text{ months, Ans.}$$
$$\underline{3040}$$

2. A owes B 104l. 15s. to be paid in $4\frac{1}{2}$ months, 161l. to be paid in $3\frac{1}{2}$ months, and 152l. 5s. to be paid in 5 months: What is the equated time for the payment of the whole?

Ans. 4 months and 8 days.

3. There is owing to a merchant 998l. to be paid, 178l. ready money, 200l. at 3 months, and 320l. in 8 months; I demand the indifferent time for the payment of the whole? Ans. $4\frac{1}{2}$ months.

4. The sum of $164 16c. 6m. is to be paid, $\frac{1}{2}$ in 6 months, $\frac{1}{3}$ in 8 months, and $\frac{1}{6}$ in 12 months: what is the mean time for the payment of the whole? Ans. $7\frac{2}{3}$ months.

RULE II.

See, by rule 1st, at what time the first man, mentioned, ought to pay in his whole money; then, as his money is to his time, so is the other's money, to his time, inversely, which, when found, must be added to, or subtracted from, the time at which the second ought to have paid in his money, as the case may require, and the sum, or remainder, will be the true time of the second's payment.

EXAMPLES.

1. P is indebted to Q $150 to be paid, $50 at 4 months, and $100 at 8 months: Q owes P $250 to be paid at 10 months: It is agreed

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$\frac{120\times 6\times \overline{8-7}}{100}$ = its interest to the equated time. The sum of the interest of these two payments is, by the assumed principle, to be equal to the interest of the third payment, or £160 for 10—8 or 2 months, which is $\frac{160\times 6\times \overline{10-8}}{100}$.

Then, $\frac{100\times 6\times \overline{8-6}}{100}+\frac{120\times 6\times \overline{8-7}}{100}=\frac{160\times 6\times \overline{10-8}}{100}$. Now, as the rate per cent. and 100 will be factors common to every term in every case, they may be expunged from every term, and then we have,

$100\times \overline{8-6}+120\times \overline{8-7}=160\times \overline{10-8}$. From this equivalent expression, it is easy to find the equated time; for, $100\times 8-100\times 6+120\times 8-120\times 7=160\times 10-160\times 8$, or $100\times 8+120\times 8+160\times 8=100\times 6+120\times 7+160\times 10$, or,

$8\times \overline{100+120+160}=100\times 6+120\times 7+160\times 10$, and

$8=\frac{100\times 6+120\times 7+160\times 10}{100+120+160}$, which is the rule. The same may be shown in every similar case, and the general rule inferred.

This rule is manifestly incorrect. The *true* rule will be given in Equation of Payments by Decimals.

between them that P shall make present pay of his whole debt, and that Q shall pay his so much the sooner, as to balance that favour; I demand the time at which Q must pay the $250 reckoning simple interest.

$50\times4=200$
$100\times8=800$

$50+100=15|0)100|0(6\frac{2}{3}$ months, P's equated time.
90
—
10
—

D. mo. D. mo. mo. mo. mo.
As 150 : $6\frac{2}{3}$ :: 250 : 4. Then, 10—4=6 time of Q's payment.

2. A merchant has 120l. due to him, to be paid at 7 months; but the debtor agrees to pay $\frac{1}{2}$ ready money, and $\frac{1}{3}$ at 4 months; I demand the time he must have to pay in the rest, at simple interest, so that neither party may have the advantage of the other?

Debt £120

$\frac{1}{2}$=60 must be paid down.
$\frac{1}{3}$=40 must be paid at 4 months.
$\frac{1}{6}$=20 unpaid.

Now, as he pays 60l. 7 months, and 40l. 3 months before they are respectively due, say, as the interest of 20l. for 1 month, is to 1 month, so is the sum of the interest of 60l. for 7 months, and of 40l. for 3 months, to a fourth number, which, added to the 7 months, will give the time for which the 20l. ought to be retained.

Ans. 2 years and 10 months.

3. A merchant has $1200 due to him, to be paid $\frac{1}{6}$ at 2 months, $\frac{1}{3}$ at 3 months, and the rest at 6 months; but the debtor agrees to pay $\frac{1}{2}$ down: How long may the debtor detain the other half, so that neither party may sustain loss?

Now as $\frac{1}{2}$ was paid $4\frac{1}{3}$ months before it was due, it is reasonable that he should detain the other $\frac{1}{2}$, $4\frac{1}{3}$ months after it became due, which added, gives $8\frac{2}{3}$ months, the true time for the second payment. Equated time=$4\frac{1}{3}$ months.

EQUATION OF PAYMENTS BY DECIMALS.

Rule.*

1. To the sum of both payments add the continual product of the first payment, the ratio, and the time between the payments, and call this the first number.

* Suppose a sum of money be due immediately, and another at the expiration of a certain given time forward, and it is proposed to find a time, so that neither party shall sustain loss.

Now, it is plain that the equated time must fall between the two payments; and that what is gotten by keeping the first debt after it is due, should be equal to what it lost by paying the second debt before it is due; but the gain arising

2. Multiply twice the first payment by the ratio, and call this the second number.

3. Divide the first number by the second, and call the quotient the third number.

4. Call the square of the third number the fourth number.

5. Divide the product of the second payment and time between the payments by the product of the first payment and the ratio, and call the quotient the fifth number.

6. From the fourth number take the fifth, and call the square root of the difference the sixth number.

7. Then the difference of the third and sixth numbers is the equated time, after the first payment.

EXAMPLES.

There are $100 payable in 2 years, and $106 at 6 years hence; what is the equated time, allowing simple interest, at 6 per cent. per annum?

1st payment=100 1st payment 100
Ratio=·06 Multiply by 2

6·00 200
Time between the payments=4ys. Mult. by the ratio=·06

24 12·00=2d num.
Add both payments= { 100, 106 }

Div. by the 2d num.=12)230=1st number.

19·166+=3d number.
19·166+

3d number squared=367·335556=4th number.
2d payment=106
Multiplied by the time= 4

1st payment mult. by the ratio=6)424= { prod. of the 2d payment and time between the payments.

70 666+=5th number.
From the 4th number=367·335556
Take the 5th number= 70·666666

296·668890(17·224sqr.root=6th num.
Carried up.

from the keeping of a sum of money after it is due, is evidently equal to the *interest* of the debt for that time: And the loss, which is sustained by the paying of a sum of money before it is due, is evidently equal to the *discount* of the debt for that time: Therefore it is obvious that the debtor must retain the sum immediately due, or the first payment, till its *interest* shall be equal to the *discount* of the second sum for the time it is paid before due; because in that case the gain and loss will be equal, and consequently neither party can be a loser.

From the 3d number=19·166 Brought up.
Take the 6th number=17·224

1·942=equated time from the first payment; therefore 3·942 years =3y. 11m. 9d.=whole equated time.

$$\text{Or, } \frac{100+106+\overline{100\times\cdot06\times4}}{100\times2\times\cdot06}-\left|\left(\frac{100+106+\overline{100\times\cdot06\times4}}{100\times2\times\cdot06}\right)^{2}-\frac{106\times4}{100\times\cdot06}\right|^{\frac{1}{2}}=1\cdot942.$$

2. There are \$100 payable one year hence, and \$106 to be paid six years hence; what is the equated time, computing interest at 6 per cent.? Ans.

3. A debt of \$1000 is to be paid, one half in three years and the other half in 6 years; what is the equated time for paying both, computing interest at 7 per cent? Ans.

EXCHANGE.

THE object of Exchange is to ascertain what sum of money ought to be paid in one country for a sum of different denominations or of different relative value received in another, according to the course of exchange.

The *par of exchange* respects the intrinsic value of the money of different countries compared with each other. Thus a pound sterling is equal to 4 dolls. and 44 cents in the United States; the mark banco of Hamburgh, to 33½ cents; 40 marks banco to £3 sterling. If the exchange be made at the intrinsic value of the money of different countries, it is said to be *at par*; but if the money of one country be estimated at less or more than its intrinsic value, the exchange is said to be *above par*, or *below par*.*

Owing to changes in the course of trade, to demand for money, to variations in the relative value of gold and silver, &c. the relative value of the money of two countries is liable to frequent changes. Hence the *course of exchange*, or the *current price of exchange*, must vary with these circumstances, and be sometimes *above*, and sometimes *below*, *par*. Tables of the course of exchange are published daily in the great commercial cities.

* The Rules under Reduction of Coins are founded on the par of exchange. For the reduction of the Money, and Measures of most commercial countries to Federal and Sterling Money, and American Measures, see also the Tables of Money, Length, Capacity and Weight.

1. OF GREAT BRITAIN.*

The denominations are pounds, shillings, and pence.

EXAMPLES.

1. What is the amount in Federal Money of a Bill of Exchange on a merchant at Liverpool of £133 sterling, sold in New York at ½ per cent. advance?

£ $ c.
133=591·11$\frac{1}{9}$
2·95$\frac{10}{18}$=½ per cent.

Amount $594·06$\frac{2}{3}$ Ans.

2. In Aug. 1821, Bills on London, bore at Boston a premium of 8½ per cent.; what is the amount of a bill of exchange of £250, at this rate, in Federal Money, and what is the value of a pound sterling at this course of exchange? Ans. Amount $1205 55$\frac{5}{9}$cts.
Value of a pound sterling $4 82$\frac{2}{9}$cts.

3. A Bill of Exchange on London of £90 sterling, was sold at New York, at 36 shillings New York currency per pound sterling; what was its amount in the currency of New York, and how much *above* or *below par*?

£ s. £ £
As 1 : 36 :: 90 : 162 N. Y. currency.

Now £9 sterling=£16 N. Y. currency, or 20s. sterling=35$\frac{5}{9}$ N. Y. currency. But 36—35$\frac{5}{9}$=$\frac{4}{9}$s. N. Y. currency, the gain on every pound sterling, or £2 N. Y. in the whole.

Then, as 35$\frac{5}{9}$s. : $\frac{4}{9}$ :: 100 : 1¼ per cent. above par.
Or, 162—2 : 2 :: 100 : 1¼ do.

4. The invoice of goods, amounting to £170 10s. sterling, is sold at New York at 25 per cent. advance;† what is the amount in Federal Money? Ans.

* The Rules on which the operations of Exchange are performed, are obvious from the rules for Reduction of Coins, and the Rule of Three.

† To reduce sterling money to the currency of New England, when there is a certain per cent. advance, merchants use the following method.

For 12½ per cent. advance, multiply the sterling by 1½

20	1$\frac{3}{5}$
25	1$\frac{2}{3}$
31¼	1¾
50	2
62½	2$\frac{1}{6}$
75	2$\frac{1}{3}$
87½	2½
100	2$\frac{2}{3}$
125	3
150	3$\frac{1}{3}$
175	3$\frac{2}{3}$
200	4

These multipliers are thus formed. Let the advance be 25 per cent. on £100; then, as 25=¼ of a hundred, $100\times\frac{100}{4}=\frac{500}{4}$=the sum with the advance. This is to be reduced to New England currency by increasing it by one third of itself. Thus $\frac{500}{4}\times\frac{500}{3\times4}=\frac{2000}{12}$=166$\frac{2}{3}$ pounds; which is evidently the same as to multiply 100 by 1$\frac{2}{3}$. In the same way may the other multipliers be found.

5. A Bill of Exchange of £75 16s. is sold at Boston at 26s. New England currency per pound sterling; what is the value in Federal Money of a pound sterling at this rate of exchange? Ans.

2. OF FRANCE.

The money of account is livres, sols, and deniers.

12 deniers make	1 sol or shilling.
20 sols	1 livre or pound.

The livre is estimated at $18\frac{1}{2}$ cents in the U. S.

The crown of exchange is 3 livres, or livres tournois, and is equal to $55\frac{1}{2}$ cents.

The present money of account is francs and centimes or hundredths. 80 francs=81 livres, or a franc=$\frac{81}{80}$ livre.

1. To reduce francs to livres, or the contrary, multiply the francs by 81 and divide the product by 80 for livres; or multiply the livres by 80 and divide the product by 81 for francs.

Thus 2156 francs$=\frac{2156\times 81}{80}=$2183 livres 19 sols. And 2341 livres$=\frac{2341\times 80}{81}=$2312 francs, $09\frac{71}{81}$ centimes.

2. To reduce livres to dollars and cents; multiply the livres by the cents in a livre at the course of exchange.

Examples.

1. If the livre be 20 cents in exchange, what is the amount of 2150 livres in Federal money, and what is the per cent. above par at this exchange?

Ans. Amount is $430. And above par $8\frac{4}{37}$ per cent.

2. If the livre be 18 cents in exchange, required the amount of 3580 livres 16 sols, in dolls. and cents, and the rate per cent. below par.

Ans. $644{\cdot}54\frac{4}{10}$ cents, and $2\frac{26}{37}$ per cent. below par.

3. If a crown be valued in exchange at 18d. sterling, required the livres in £100 sterling, and the amount also in Federal money at par. d. liv. £ liv.

As 3 livres=1 crown, 18 : 3 :: 100 : 4000 and $4000\times 18\frac{1}{2}=$ $740.

4. In 2583 francs, how many dollars?

$2583\times 55\frac{1}{2}=$1433dolls. $56\frac{1}{2}$ cents.

5. A bill of exchange on a merchant in New York of $730 65cts. was bought at Paris at $1\frac{1}{2}$ per cent. advance; what is the amount in francs, and what was the estimated value of a franc at this exchange? Ans.

3. OF SPAIN.

4 Maravadies make	1 quarto.
$8\frac{1}{2}$ quartos=34 marav.	1 rial plate.
8 rials plate	1 piastre or current dollar.
375 maravadies	1 ducat of exchange.

Hard or *plate* dollars are $88\frac{4}{17}$ per cent. above *current* dollars or money of vellon, or

100 rials plate $=188\frac{4}{17}$ rials vellon.
17 do. $=32$ do.

The rial plate is 10 cents, and the rial vellon 5 cents in the U. States.

To reduce rials plate to rials vellon, or the contrary, multiply the rials plate by 32 and divide the product by 17, for rials vellon; or multiply the rials vellon by 17 and divide the product by 32, for rials plate.

1. Thus 1100 rials plate$=\frac{1100\times32}{17}$rials vellon$=2070\frac{10}{17}$ Ans.

And 100 rials vellon$=\frac{100\times17}{32}$ rials plate$=53\frac{1}{8}$ rials plate. Ans.

Note. The rules to reduce rials plate or vellon to Federal Money are obvious and need no examples.

2. In the sale of a bill of exchange of 1563 rials plate, the rial plate was estimated at $9\frac{5}{8}$ cents; how much per cent. was the rial below par and how much the loss?

Ans $4\frac{1}{4}$ per cent. $\$6{\cdot}94\frac{5}{8}$ the loss.

3. If the piastre be valued in exchange at 81 cents, what is the per cent. above par on a bill of 1672 piastres 5 rials plate, and what is the advance on the bill in Federal Money? Ans.

4. OF HAMBURGH.

12 deniers=2 grotes make	1 shilling lubs, or stiver.
16 shilling lubs=32 grotes	1 mark banco.*
3 marks	1 rix dollar.
Or, 12 grotes or pence Flemish make	1 shilling Flemish.
20 shillings Fl.$=7\frac{1}{2}$ marks	1 pound Flemish.

A mark is $\frac{1}{3}$ of a dollar, or $33\frac{1}{3}$ cents in the U. States, and the Rix dollar is equal to the Spanish dollar, or 100 cents.

The mark is $2\frac{2}{3}$ shillings Flemish.

The Bank money of Hamburgh is superior to the currency at a variable rate per cent.

1. To reduce marks banco to dollars, divide the marks by 3.

Thus 3437 marks$=\frac{3437}{3}$dolls.$=\$1145\ 66\frac{2}{3}$cts.

2. To reduce pounds Flemish to dollars, multiply the pounds by 5, and divide the product by 2 for dollars. Thus, to reduce 175 pounds Fl. and 10 shillings to dollars, $\frac{175{\cdot}5\times5}{2}$ dolls.$=\$438{\cdot}75$cts.

3. To reduce Hamburgh money to sterling, use the following proportion; As, the value of a pound sterling at Hamburgh is to 1 pound, so is the Hamburgh sum to the sterling required.

1. When the pound sterling is 33 shillings Flemish, what is the value of £1567 10s. Fl. in sterling money?

s.	£	£ Fl.	£ sterling.
As 33 :	1 ::	1567·5 :	950

* *Banco* is money placed in banks of deposit, and is not to be drawn out, but is transferred from one person to another for the payment of contracts.

2. Reduce 2560 marks 8 stivers to sterling, at the rate of 33½ shillings Fl. per pound sterling. Ans. £204·16·9·6d. sterling.

3. When the pound sterling is 34 shillings Flemish, what is the per cent. below par? Ans. 4⅔ per cent.

4. To reduce current to Bank money, use the following proportion. As 100 marks with the rate added is to 100 bank money, so is current sum to the bank money required.

1. Reduce 360 marks current to bank money, when rate or agio is 20 per cent.

As 100+20 : 100 :: 360 : 300 bank money, Ans.

2. When the rate or agio is 18½ per cent. what is the value of 3759 marks 8 stivers current in bank money? Ans.

3. If 375 marks current are estimated at 320 marks bank, what is the rate per cent? Ans.

OF CALCUTTA.

12 pice make 1 anna,
16 annas 1 rupee.

The Bengal rupee is estimated at 50 cents in the United States; in exchange it is usually 3 or 4 cents less.

100 sicca rupees are equal to 116 current rupees.

1. Reduce 187 rupees 8 annas to federal money at 46½ cents per rupee. Ans. $89 06¼ cents.

2. Reduce $367½ to rupees, the rupee being valued at 48 cents, Ans. 763 rupees, 15 annas, and 4 pice.

Note. From the exchange value of the money of different countries, and from the Table of Money of commercial countries, immediately before the "Chronological Problems," the student will be able to derive particular rules for making all the exchanges of money, which may be necessary in business.

POLICIES OF INSURANCE.

INSURANCE is an assurance or security by a contract, to indemnify, for a specified sum, the insured for such losses as the property may be exposed to, for a certain time.

The *insurer* or *underwriter*, is the party that is bound to indemnify for the loss sustained.

The *premium* is the compensation paid by the insured for the insurance.

The *policy* is the document by which the contract of insurance is made.

Goods are said to be *covered*, when their value and the premium and other charges are insured.

If the loss do not exceed *five per cent.* the underwriter is free, and the loss is borne by the insured. *Particular average*, is the proportioning of such losses as arise from ordinary accidents at sea,

among the proprietors of the property which suffers the injury. *General average*, is the proportion to be paid by all the owners of ship and cargo, for losses necessary to preserve the rest, such as cutting away masts, &c. throwing part of the cargo overboard, and the like. As this is done for the common good, it is to be borne by the owners of the ship and cargo, in proportion to the value of the property possessed by them severally.

In computing general average for masts, &c. to replace those cut away, one third is usually deducted from the expense, because the new articles may be supposed better than the old

Unless the property is *covered* the insured is not indemnified, in case of total loss, but in the proportion contained in the *policy;* and, in case of a partial loss, the insured is to be indemnified only in the same proportion.

Note. General average is computed by the Rule for Single Fellowship. See examples 19 and 20 under that rule.

CASE I.

When the premium, at a certain rate per cent, for insuring a sum, is required, the operation is the same as in interest, or commission.

Examples.

1. What is the premium upon 537l. 15s. 9d. at 6½ per cent.?

	£	s.	d.
	537	15	9
			6½
	3226	14	6
½=	268	17	10½
	34|95	12	4½
	20		
	19|12		
	12		
	1|48		
	4		
	1|94		

Ans. £34 19s. 1¼d. nearly.

2. What is the premium upon $375, at 7½ per cent.?

Ans. $28·125.

CASE II.

To find the sum for which a policy should be taken out to cover a given sum.

Rule. Take the premium from 100l. or $100, and say, As the remainder is to 100, so is the sum adventured to the policy.* Or,

* It is plain, that the *policy* should be equal to the insurance and the sum insured. Hence at 8 per cent. a policy of £100 would secure only £92. In order to recover £92, therefore, the policy must be taken out for £100. Hence the rule is

In *decimals*, take the premium from 100, annex two cyphers to the sum to be covered, and divide by the remainder for the policy.

EXAMPLES.

1. It is required to cover 759l. premium 8 per cent.: For what sum must the policy be taken?

$$\begin{array}{l} 100 \\ \underline{\,8} \\ 92 : 100 :: 759 \\ \underline{100} \\ 92)75900(825 \text{ Ans. } £ \\ \underline{736} \\ 230 \\ \underline{184} \\ 460 \quad \text{Or, } \frac{75900}{92} = £825, \text{ Ans. as before.} \\ \underline{460} \end{array}$$

2. A merchant sent a vessel and cargo to sea, valued at $5760: What sum must the policy be taken out for, to cover this property, premium $19\frac{1}{2}$ per cent.? Ans. $7155 28c.

CASE III.

When a policy is taken out for a certain sum in order to cover a given sum.

To find the premium, say, as the policy is to the covered sum; so is 100l. (or $100) to a fourth number, which, being taken from 100, will leave the premium. Or,

In *decimals*, divide the sum covered, with two cyphers annexed, by the policy; subtract the quotient from 100, the remainder is the premium.

EXAMPLES.

1. If a policy be taken out for 1250l. to cover 500l. What is the premium per cent.?

obvious. The difference between 100 and the rate per cent. will be the first term, 100 the second, and the sum to be insured the third term of a proportion, and the rule is merely a particular application of the Rule of Three. In the first example, the proportion would stand thus, 100—8 : 100 :: 759 : the policy= $\frac{100\times759}{100-8}$=£825. Now the premium on £825, is, $\frac{8\times825}{100}$=£66, and 66+759=£825, the policy. The rule for *decimals* is evidently a contraction of this rule.

In Case III. the last three terms in the preceding proportion are given to find the *rate*. Those three terms evidently give the difference between 100 and the rate, and the rule is obvious.

In Case IV. the first two terms and the last term of the preceding proportion are given, to find the third term or sum covered, and the reason of the operation is plain from the consideration of that proportion.

1250 : 500 :: 100
100

1250)50000(40 and £100—40=£60, Ans.
50000

Or, $\frac{50000}{1250}$=40, &c. as before.

2. If a policy be taken out for $781·25, to cover $625 : Required the premium per cent. ?

$ c. $ $ $ c.
As 781·25 : 625 :: 100 : 87·50. And, 100—87·5=12·5, or $12\frac{1}{2}$ [per cent. premium, Ans.

Or, $\frac{62500}{781 \cdot 25}$=87·5, &c. as before.

CASE IV.

When the policy for covering any sum and the premium per cent. are given, to find the sum to be covered.

Rule.

Deduct the premium per cent. from 100, and say, As 100 is to the remainder, so is the policy to the sum required to be covered.

Or, In *decimals*, Multiply the policy by the remainder found as before, and point off two right hand places in the product for the answer.

Examples.

1. If a policy be taken out for 1250l. at 60 per cent. : What is the adventure or sum to be covered ?

100
60

100 : 40 :: 1250
40
£
100)50000(500 Ans.

Or, $1250 \times \overline{100-60}$=50000, and, pointing off two places, 500·00 Ans. as before.

2. If a policy be taken out for $781 25c. at $12\frac{1}{2}$ per cent. required the sum covered ?

As 100 : $100-12\frac{1}{2}$ :: 781·25 : $\frac{781 \cdot 25 \times \overline{100-12\frac{1}{2}}}{100}$=$625, Ans.

Or, $781 \cdot 25 \times \overline{100-12 \cdot 5}$=62500 ; and 625·00, Ans. as before.

CASE V.

When a given sum is adventured several voyages round from one place to another, either at the same, or different risks, from place to place, and it is required to take out a policy for such a sum as will cover the adventure all round, supposing the risk out and home to be equal and tantamount to the several given risks.

Rule.

1. Raise 100l. or $100 to that power denoted by the number of risks, and multiply the said power by the sum adventured, (or to be covered) for a dividend.

2. Subtract the several premiums, each, from 100l. and multiply the several remainders continually together for a divisor, and the quotient, arising from this division, will give the policy to cover the adventure the voyage round.*

Example.

A merchant adventured \$1500 from Boston to Philadelphia, at 3 per cent. from thence to Guadaloupe, at 4, from thence to Nantz, at 5, and from thence home at 6 per cent.; For what sum must he take out a policy to cover his adventure the vogage round, supposing the risk to be equal out and home, and tantamount to the several given risks?

$$\frac{100 \times 100 \times 100 \times 100 \times 1500}{\overline{100-3} \times \overline{100-4} \times \overline{100-5} \times \overline{100-6}} = \$1803{\cdot}835, \text{ Ans.}$$

CASE VI.

When a given sum is adventured several voyages round, as in the last case, either at the same, or different risks, from port to port, and the premium for the voyage round is required, tantamount to the several given rates per cent.

* It is evident that the *policy* to be taken out for the first voyage becomes the sum for which a policy is to be taken out for the second voyage, and so on. Hence the examples of this case are to be solved by the rule for Case II. making the sum in the policy for the first voyage, the sum for which a policy is to be taken out for the second voyage. Therefore the operation on the given example would be as follows.

$100-3 : 100 :: 1500 :$ policy for 1st voyage$=\dfrac{100 \times 1500}{100-3}$. Now as $\dfrac{100 \times 1500}{100-3}$ is the sum to be insured on the second voyage, we have,

$$100-4 : 100 :: \frac{100 \times 1500}{100-3} : \text{2nd policy} = \frac{100 \times 100 \times 1500}{\overline{100-3} \times \overline{100-4}}.$$

$$\text{And } 100-5 : 100 :: \frac{100 \times 100 \times 1500}{\overline{100-3} \times \overline{100-4}} : \text{3d policy} = \frac{100 \times 100 \times 100 \times 1500}{\overline{100-3} \times \overline{100-4} \times \overline{100-5}}.$$

$$\text{And } 100-6 : 100 :: \frac{100 \times 100 \times 100 \times 1500}{\overline{100-3} \times \overline{100-4} \times \overline{100-5}} : \text{4th policy} =$$

$$\frac{100 \times 100 \times 100 \times 100 \times 1500}{\overline{100-3} \times \overline{100-4} \times \overline{100-5} \times \overline{100-6}} = \frac{100^4 \times 1500}{\overline{100-3} \times \overline{100-4} \times \overline{100-5} \times \overline{100-6}},$$

which is the Rule. The same may be shown by the Double Rule of Three, thus,

$$\left.\begin{array}{l} 100-3 : 100 :: 1500 : \\ 100-4 : 100 :: \quad : \\ 100-5 : 100 :: \quad : \\ 100-6 : 100 :: \quad : \end{array}\right\} \text{the policy} = \frac{100^4 \times 1500}{\overline{100-3} \times \overline{100-4} \times \overline{100-5} \times \overline{100-6}} = \$1803 \text{ 83c. 5m.}$$

It is plain that however numerous the voyages, the power of 100 must be equal to their number, and that the divisor must always be the continued product of the differences between 100 and the several rates of insurance. If the rate of insurance had been the same on each of the voyages, then the policy$=\dfrac{100^4 \times 1500}{\overline{100-6}^4}$ if the rate had been 6 per cent.

Rule.*

1. Find the sum for which the policy must be taken, by the last case.

2. Multiply the sum adventured by 100, and divide that product by the policy.

3. Take the quotient from 100, and the remainder will be the premium per cent. on the policy, tantamount to the several premiums given in the question.

Example.

A merchant adventured $1500 from Boston to Philadelphia, at 3 per cent.: from thence to Guadaloupe, at 4; thence to Nantz, at 5; and thence home, at 6 per cent.: What will be the premium, tantamount to those given in the question, on a policy for covering the first adventure, the whole voyage, supposing the risks out and home equal?

In Case V. we found the policy, which would cover the adventure the voyage round, to be $1803·835. Then $100-\frac{1500\times100}{1803\cdot835}=$ 16·844=the premium per cent. on the policy the voyage round, and tantamount to the several given premiums.

CASE VII.

If a policy be taken out for a given sum, to cover a certain adventure, from one port to another, on to several ports, at equal premiums from one place to the other, to find what that equal premium is.

Rule.†

1. Involve 100 to that power denoted by the number of risks, and multiply this power by the sum adventured, (or covered.)

* When the policy is found by Case V. the operation becomes the same as that directed by the Rule, Case III. which has been proved. The operations may be shortened in many cases, by keeping the terms separate in the first part of the process. Thus—by Case V. the *policy* in this example=

$$\frac{100^4\times1500}{\overline{100-3}\times\overline{100-4}\times\overline{100-5}\times\overline{100-6}}.\ \text{Then, by Case II.}$$

$$\frac{100^4\times1500}{\overline{100-3}\times\overline{100-4}\times\overline{100-5}\times\overline{100-6}}:1500::100:\left\{\begin{array}{l}\text{100 diminished by}\\ \text{the premium}=\end{array}\right.$$

$$\frac{1500\times100\times\overline{100-3}\times\overline{100-4}\times\overline{100-5}\times\overline{100-6}}{100^4\times1500}=$$

$$\frac{\overline{100-3}\times\overline{100-4}\times\overline{100-5}\times\overline{100-6}}{100^3}=\$83\cdot156,\text{ and }100-83\cdot156=\$16\cdot844.$$

† By the last remark in the demonstration of the Rule Case V. when the insurance is the same on each of several voyages, the *policy* is equal to the product of the sum to be insured and 100 raised to a power whose index is the number of voyages, divided by the difference between 100 and the rate of insurance raised to the same power. Hence this product divided by the *policy* must give a quotient equal to the difference between 100 and the rate of insur

2. Divide the last product by the policy.

3. Extract that root of the quotient denoted by the number of risks.

4. Take this root from 100, and the remainder will be the equal premium from one port to the other.

EXAMPLE.

A merchant adventured \$1500 from Boston to Philadelphia, thence to Guadaloupe, thence to Nantz, and thence home; to cover which all round he took out a policy for \$1803·835; and the premium was equal from one place to the other: what was the premium per cent.?

$$100-\sqrt[4]{\frac{100\times100\times100\times100\times1500}{1803{\cdot}835}}=4{\cdot}507 \text{ per cent. Answer.}$$

CASE VIII.

When an adventure is insured out and home at one risk, at a given rate per cent. and the voyage terminates short of what was at first intended: To find what the underwriter must receive per cent.

RULE.

1. If just half the voyage is performed, it must be considered as *two* equal risks: If one third, then, as *three* equal risks; if but one fourth, then, as *four* risks, and so on; and by Case 2d must be found the amount which will cover the adventure the voyage round.

2. Involve 100 to that power denoted by the number of risks, and multiply this power by the sum adventured.

3. Divide this product by the aforesaid amount.

4. Extract that root of the quotient denoted by the number of risks.

5. Take this root from 100, and the remainder will be the sum per cent. which the underwriter must receive.

EXAMPLE.

A merchant covers \$200 at 6 per cent. from Newburyport to the West Indies and home again; but the voyage terminating in the West Indies, what must the insurer receive per cent.?

100
6
——
94 : 100 :: 200 : 212·765957=amount to cover \$200 voyage round.

$100\times100\times200=2000000$ and $\frac{2000000}{212{\cdot}765957}=9400$, and $100-\sqrt{9400}$ $=3{\cdot}0465$ to be paid the insurer per cent. upon the above amount.

ance raised to a power whose index is the number of years. If that root of the quotient, indicated by the number of years, be extracted you will have the difference between 100 and the rate per cent. and this difference taken from 100 gives the rate.

COMPOUND INTEREST

IS that which arises from the interest being added to the principal, and (continuing in the hands of the borrower) becoming part of the principal, at the end of each stated time of payment.

METHOD I.

RULE.*—Find the *amount* of the given principal, for the time of the first payment, by Simple Interest: next, find the interest of that sum, or principal, and add it as before, and thus proceed for any number of years, still accounting the last amount as the principal for the next payment. The given principal being subtracted from the last amount, the remainder will be the compound interest.

In *federal money*, multiply the principal by the rate for the first time of payment, setting the product two places more to the right than the multiplicand, and the decimal point in the product under that in the multiplicand; then find the amount, and proceed as above.

Note. It is not usually necessary to carry the work beyond mills; therefore, when the figure next beyond mills, at the right, exceeds 5, increase the number of mills 1; when it does not exceed 5, it may be omitted. The result will be exact enough for common purposes.

EXAMPLES.

1. What will £480 amount to in 5 years, at 6 per cent. per annum?

Principal 480
Rate of interest 6

28|80
20

16|00

	£	s.	d.
Prin. for the 2d year	508	16	0
Interest for ditto	30	10	6½
Prin. for the 3d year	539	6	6½
			6
	32|35	19	3
	20		

	£	
Principal for the 1st year	480	0
Interest of ditto	28	16
Principal for the 2d year	508	16
		6
	30|52	16
	20	
	10|56	
	12	
	6|72	
	4	
	2|88	

Carried up.

* It may be observed that all the computations, relating to Compound Interest, are founded upon a series of terms, increasing in Geometrical Progression, wherein the number of years assigns the index of the last and highest term: Therefore, as one pound is to the amount of one pound, for any given time, so is any proposed principal, or sum, to its amount for the same time.

Brought up. 1|97 Principal for the 3d year £539 6 6¼
12 Interest for ditto 32 7 2¼

2|31 Principal for the 4th year 571 13 8¾
4

1|24

	£	s.	d.		£	s.	d.
Prin. for the 4th year	571	13	8¾	Prin. for the 4th year	571	13	8¾
			6	Interest for ditto	34	6	0¼
	34|30	2	4½	Prin. for the 5th year	605	19	9
	20						6
	6|02				36|35	18	6
	12				20		
	0|28				7|18		
	4				12		
	1|14				2|22		

	£	s.	d.
Principal for the 5th year	605	19	9
Interest for ditto	36	7	2
Amount for 5 years	642	6	11
Subtract the first principal	480	0	0
Compound interest for 5 years	162	6	11

In *federal money*, thus: The principal being $1600 for five years.

Principal for the 1st year	$1600·
Rate of interest	6
Interest 1st year	96·00
Amount 1st and prin. 2d year	1696·
	6
Interest 2d year	101·76
Amount 2d year, prin. 3d	1797·76
	6
Interest 3d year	107·8656
Amount 3d, principal 4th	1905·6256
	6
Interest 4th year	114·337536
Amount 4th, principal 5th year	2019·963136
	6

Brought over.	Interest 5th year	121·19778816
	Amount for 5 years	2141·16092416
	Subtract 1st principal	1600·
	Compound Interest for 5 years=	541·16092416

Or thus:

1st principal	$1600·
	6
Interest	96·00
2d principal	1696·
	6
Interest	101·76
3d principal	1797·76
	6
Interest	107·866
4th principal	1905·626
	6
Interest	114·338
5th principal	2019·964
	6
Interest	121·198
Amount	2141·162
1st principal	1600·
Compound Interest	541·162 nearly, as before.

2. What is the compound interest of $740 for 6 years, at 4 per cent. per annum? Ans. $196 33c. 6m.

3. What will £400 amount to in 5 years, at £4 per cent. per annum? Ans. £486 13s. 2½d.

4. What will £150 amount to in a year, at 2 per cent. per month? Ans. £190 4s. 5d.

5. What is the compound interest of $500 at 2 per cent. a month for one year? Ans. $134 12c. 1m.

6. What is the amount of $100 at 6 per cent. compound interest for 3 years?

7. What is the compound interest of $100 at 7 per cent. for 3 years?

Method II.

When the rate is at 5 per cent. per annum.

1. Divide the principal by 20, and this quotient, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner find the amount for every succeeding year.

When the rate is at 6 per cent. per annum.

1, Divide the principal by 20, and that quotient by 5: these quotients, added to the principal, will be the amount for the first year, and the principal for the second.

2. In like manner obtain the amount for every succeeding year.

Examples.

1. What is the amount of £480 at 6 per cent. per annum, for 5 years?

```
20)480
 5) 24
     4 16
  ---------
20)508 16    amount of 1st year.
 5) 25  8  9½
     5  1  9
  ---------------
20)539  6  6½  ditto of 2d.
 5) 26 19  3¾
     5  7 10¼
  ---------------
20)571 13  8½  ditto of 3d.
 5) 28 11  8¼
     5 14  4
  ---------------
20)605 19  8¾  ditto of 4th.
 5) 30  5 11¾
     6  1  2¼
  ---------------
 £642   6 10¾  do. of 5th. Ans.
```

2. Of the same sum at 5 per cent. per annum, for 5 years.

```
   £
20)480
   24
  ---
20)504         amount of 1st year.
   25  4
  -------
20)529  4      ditto of 2d.
   26  9  2¼
  ------------
20)555 13  2¼
   27 15  7¾
  ------------
20)583  8 10   ditto of 4th.
   29  3  5¼
  ------------
 £612 12  3¼  do. of 5th. Ans.
```

Note. The same may be done in *federal money*, but the first method is generally more easy.

COMPOUND INTEREST BY DECIMALS.

A Table of the Amount of £1 or $1, at ½ per cent. per month, as practised at the Banks.

Months.	£ or $ Dec. parts.	Months.	£ or $ Dec. parts.	Months.	£ or $ Dec. parts.
1	1·005	5	1·025	9	1·045
2	1·01	6	1·03	10	1·05
3	1·015	7	1·035	11	1·055
4	1·02	8	1·04	12	1·06

A Table of the Amount of £1 or $1, from 1 Day to 31 Days, at 6 per cent. per annum.

Days.	£ or $ Dec. parts.	Days.	£ or $ Dec. parts.	Days.	£ or $ Dec. parts.
1	1·00016	12	1·00197	22	1·00361
2	1·00032	13	1·00213	23	1·00378
3	1·00049	14	1·0023	24	1·00394
4	1·00065	15	1·00246	25	1·0041
5	1·00082	16	1·00263	26	1·00427
6	1·00098	17	1·00279	27	1·00443
7	1·00115	18	1·00295	28	1·0046
8	1·00131	19	1·00312	29	1·00476
9	1·00147	20	1·00328	30	1 00493
10	1·00164	21	1·00345	31	1·00509
11	1·00180				

These tables are formed by adding the interest of £1 or $1, to £1 or $1, for the given rate and time. Thus, by rule for Simple Interest, the interest of £1 or $1 for 1 day, is, ·00016438+, and the amount is 1·00016438+.

CASE I.*

When the principal, the rate of interest, and time, are given, to find either the amount or interest.

RULE.

1. Find the amount of £1 or $1 for one year at the given rate per cent.

2. Involve the amount, thus found, to such power, as is denoted by the number of years; or, in Table I. at the end of Annuities,

* The reason of the rule may be seen by the following process. If the rate be 6 per cent. the amount of £1 or $1 for 1 year, is, by the rule for Simple Interest by Decimals, 1·06. This is the principal for the second year, and its amount is by the same rule, $1{\cdot}06+1{\cdot}06\times{\cdot}06=\overline{1+{\cdot}06}\times1{\cdot}06=1{\cdot}06\times1{\cdot}06=\overline{1{\cdot}06}^2$. That is, the amount of £1 or $1 for *two* years is equal to the *square* of the amount of £1 or $1 for one year. This is the principal for the third year, and its amount is, $\overline{1{\cdot}06}^2+\overline{1{\cdot}06}^2\times{\cdot}06=\overline{1+{\cdot}06}\times\overline{1{\cdot}06}^2=1{\cdot}06\times\overline{1{\cdot}06}^2=1{\cdot}06^3$, that is the *amount* for *three* years is the *cube* of the amount for 1 year. In the same way it may be shown, that the amount for *four* years is the *fourth* power of the amount of £1 or $1 for 1 year; for *five* years, is the *fifth* power, and so on. The same would be true, whatever be the rate per cent. Now, whatever be the principal, the amount must be so much greater than the amount of £1 or $1 for the same time and rate. Therefore, the *amount* for any principal will be found by multiplying the amount of £1 or $1, at the given rate and time by the principal and is the rule. Let the principal be $100 or £100, the rate 5 per cent. and the time 5 years. Then, $\overline{1{\cdot}05}^5\times100=$ the amount. And $\overline{1{\cdot}05}^5\times 100-100=$ the interest.

If the rate of interest be determined to any other time than a year, as ½, ¼, &c. the rule is the same.

If the compound interest, or amount of any sum, be required for the parts of a year, it may be determined as follows:

under the rate, and against the given number of years, you will find the power.*

3. Multiply this power by the principal, or given sum, and the product will be the amount required, from which if you subtract the principal, the remainder will be the interest.

EXAMPLES.

1. What is the compound interest of £600 for 4 years, at 6 per cent. per annum?

1·06= { amount of £1 for 1 year, at 6 per cent. per annum.
Multiply by 1·06

1·1236=2d power.
Multiply by 1·1236

1·26247696=4th power.
Multiply by 600=principal.

757·48617600=amount.
Subtract 600

157·486176=£157 9s. 8¼d.=interest required.

BY TABLE I.

Tabular amnt. of £1 for 4 years, at 6 per cent. per ann.=1·2624769
Multiply by the principal= 600

Amount=757·4861400

2. What is the amount of $1500 for 12 years, at 3½ per cent. per annum?

$1·035=amount of $1 for 1 year at 3½ per cent. per annum.
And, $1·035^{12}$×1500=$2266 60c. nearly, Ans.

Another method of working compound interest for *years*, *months*, and *days*, which is much more concise than the preceding method.

I. *When the time is an aliquot part of a year.*

RULE 1. Find the amount of £1 for 1 year, as before, and that root of it, which is denoted by the aliquot part, will be the amount of £1 for the time sought.

2. Multiply the amount, thus found, by the principal, and it will be the amount of the given sum required.

II. *When the time is not an aliquot part of a year.*

RULE 1. Reduce the time into days, and the 365th root of the amount of £1 for 1 year is the amount for 1 day.

2. Raise this amount to that power, whose index is equal to the number of days, and it will be the amount of £1 for the given time.

3. Multiply this amount by the principal, and it will be the amount of the given sum required.

* The amounts of £1 or $1 in this table, are so many powers of the amount of £1 or $1 for 1 year; whose indices are denoted by the number of years.

Note. When the given time consists of years and months, or years, months, and days; first seek the amount of £1 or $1 in the table of years, then in the table of months, &c. multiply these several amounts and the principal continually together, and the last product will be the amount required.

Thus, if the amount of £480 in 5½ years, at 6 per cent. per annum, were required; the amount of £1 for 5 years=£1·33822, ditto for 6 months=£1·02956. Now, 1·33822×1·02956×480=£661·2341 Answer.

Rule.

To the logarithm of the principal, found in any Table of logarithms, add the several logarithms, answering to the number of *years*, *months* and *days* found in the following tables, and their sum will be the logarithm of the amount for the given time, which being found in any table of logarithms, the natural number corresponding thereto will be the answer.*

LOGARITHMICK TABLES, AT SIX PER CENT. PER ANNUM, FOR YEARS, MONTHS AND DAYS.

Years.	Dec. pts.	Y.	Dec. pts.	Y.	Dec. pts.	Y.	Dec. pts.	Months	Dec. pts.
1	·025306	11	·278366	21	·531426	31	·784586	1	·002166
2	·050612	12	·303672	22	·556732	32	·809792	2	·004321
3	·075918	13	·328978	23	·582038	33	·835098	3	·006466
4	·101224	14	·354284	24	·607344	34	·860404	4	·0086
5	·12653	15	·37969	25	·63265	35	·88571	5	·010724
6	·151836	16	·404896	26	·657956	36	·911016	6	·012837
7	·177142	17	·430202	27	·683262	37	·936322	7	·01494
8	·202448	18	·455508	28	·708568	38	·961628	8	·017033
9	·227754	19	·480814	29	·733974	39	·986934	9	·019116
10	·25306	20	·50612	30	·75938	40	1·01224	10	·021189
								11	·023252
Days.		D.		D.		D.		D.	
1	·000071	8	·000571	14	·000999	20	·001426	26	·001852
2	·000143	9	·000642	15	·00107	21	·001497	27	·001923
3	·000215	10	·000713	16	·001142	22	·001568	28	·001994
4	·000287	11	·000785	17	·001213	23	·001639	29	·002065
5	·000358	12	·000857	18	·001284	24	·00171	30	·002136
6	·000429	13	·000928	19	·001355	25	·001781	31	·002207
7	·0005								

What is the amount of 132l. 10s. at 6 per cent. per annum, for 9 years, 8 months, and 15 days?

To the log. of £132·5=2·122216

$$\text{Add}\begin{cases}\text{Log. for 9 years} = \cdot 227754\\ \text{ditto for 8 months} = \cdot 017033\\ \text{ditto for 15 days} = \cdot 00107\end{cases}$$

2·368073

Because 8 months are past, deduct 4 per cent. upon the logarithm of 15 days } = ·0000428

Remains 2·3680302, the nearest to which, in the table of logarithms, is 2·368101, and the natural number answering thereto is 233·4=£233 8s. Ans.

* Although there is a small errour in the logarithm for days, yet they are exact enough for common use. And if after the first month you deduct ½ per cent. for each month past (that is, ½ per cent. after 1 month, 1½ per cent. after 3 months, &c.) from the logarithm of the number of days, it will give the true answer.

Note, That, after 1 month, ½ per cent. on the logarithm of 1 day is ·00000355, on 2 days, is ·00000715: After 2 months, 1 per cent. on the logarithm of 1 day, is ·0000071, on 2 days, ·0000143: After 10 months, 5 per cent. on the logarithm for 1 day, is ·0000355, on 6 days, is, ·0000215, &c.

CASE II.

When the amount, rate and time, are given, to find the principal.

Rule.

Divide the amount by the amount of £1 or $1 for the given time, and the quotient will be the principal.*

Or, If you multiply the present value of £1 or $1 for the given number of years, at the given rate per cent. by the amount, the product will be the principal or present worth.†

Examples.

1. What is the present worth of 757l. 9s. $8\frac{1}{2}$d. due 4 years hence, discounting at the rate of 6l. per cent. per annum?

By Table I.

Divide by the tabular amount of 1l. for 4 years, } =1·2624769)757·4861400(£600 Ans.

By Table II.

Mult. by the present worth of 1l. for 4 years, at 6 per cent per ann. } Amount=757·48614
= ·79 20936

Ans. 599·999923582704+=£600.

2. What principal must be put to interest 6 years, at $5\frac{1}{2}$ per ct. per annum, to amount to $689·4214033809453125? Ans. $500.

CASE III.

When the principal, rate and amount, are given, to find the time.

Rule.

Divide the amount by the principal: then divide this quotient by the amount of £1 or $1 for 1 year, this quotient by the same, till nothing remain, and the number of the divisions will show the time.‡

Or, Divide the amount by the principal, and the quotient will be the amount of £1 or $1 for the given time, which seek under the given rate in Table 1, and, in a line with it, you will see the time.

* By Case I. the *amount* is equal to the principal multiplied by that power of the amount of £1 or $1 for 1 year at the given rate, which is indicated by the number of years: therefore, if the amount be divided by this power of the amount of £1 or $1 for 1 year, the quotient must be the principal. Thus, in the example in the proof of Case I. $\overline{1·05}^5 \times 100$=the amount; therefore, $\frac{\overline{1·05}^5 \times 100}{\overline{1·05}^5}$ = 100, the principal.

† See Table II. shewing the present value of £1, discounting at the rates of 4, $4\frac{1}{2}$, &c. per cent. the construction of which is thus:

Amount. Pres. worth. Amount. Pres. worth.
As 1·06 : 1 :: 1 : ·9433962, and so on, for any other rate per cent. and time.

‡ By the example in the proof of Case I. $\overline{1·05}^5 \times 100$=the amount; divide this by the principal, 100, and the quotient will be $\overline{1·05}^5$. This quotient divided by the ratio, and this quotient by the ratio, and so on, will be exhausted by five divisions, which shows the number of years.

among the proprietors of the property which suffers the injury. *General average*, is the proportion to be paid by all the owners of ship and cargo, for losses necessary to preserve the rest, such as cutting away masts, &c. throwing part of the cargo overboard, and the like. As this is done for the common good, it is to be borne by the owners of the ship and cargo, in proportion to the value of the property possessed by them severally.

In computing general average for masts, &c. to replace those cut away, one third is usually deducted from the expense, because the new articles may be supposed better than the old.

Unless the property is *covered* the insured is not indemnified, in case of total loss, but in the proportion contained in the *policy*; and, in case of a partial loss, the insured is to be indemnified only in the same proportion.

Note. General average is computed by the Rule for Single Fellowship. See examples 19 and 20 under that rule.

CASE I.

When the premium, at a certain rate per cent, for insuring a sum, is required, the operation is the same as in interest, or commission.

EXAMPLES.

1. What is the premium upon 537l. 15s. 9d. at 6½ per cent.?

	£	s.	d.
	537	15	9
			6½
	3226	14	6
½=	268	17	10½
	34|95	12	4½
	20		
	19|12		
	12		
	1|48		
	4		
	1|94		

Ans. £34 19s. 1½d. nearly.

2. What is the premium upon $375, at 7½ per cent.?

Ans. $28·125.

CASE II.

To find the sum for which a policy should be taken out to cover a given sum.

RULE. Take the premium from 100l. or $100, and say, As the remainder is to 100, so is the sum adventured to the policy.* Or,

* It is plain, that the *policy* should be equal to the insurance and the sum insured. Hence at 8 per cent. a policy of £100 would secure only £92. In order to recover £92, therefore, the policy must be taken out for £100. Hence the rule is

In *decimals*, take the premium from 100, annex two cyphers to the sum to be covered, and divide by the remainder for the policy.

EXAMPLES.

1. It is required to cover 759l. premium 8 per cent.: For what sum must the policy be taken?

```
100
  8
---
 92 : 100 :: 759
               100
              ----- £
           92)75900(825 Ans.
              736
              ---
               230
               184
               ---
               460
               460
```

Or, $\frac{75900}{92}$=£825, Ans. as before.

2. A merchant sent a vessel and cargo to sea, valued at $5760: What sum must the policy be taken out for, to cover this property, premium 19½ per cent.? Ans. $7155 28c.

CASE III.

When a policy is taken out for a certain sum in order to cover a given sum.

To find the premium, say, as the policy is to the covered sum; so is 100l. (or $100) to a fourth number, which, being taken from 100, will leave the premium. Or,

In *decimals*, divide the sum covered, with two cyphers annexed, by the policy; subtract the quotient from 100, the remainder is the premium.

EXAMPLES.

1. If a policy be taken out for 1250l. to cover 500l. What is the premium per cent.?

obvious. The difference between 100 and the rate per cent. will be the first term, 100 the second, and the sum to be insured the third term of a proportion, and the rule is merely a particular application of the Rule of Three. In the first example, the proportion would stand thus, 100—8 : 100 :: 759 : the policy= $\frac{100\times759}{100-8}$=£825. Now the premium on £825, is, $\frac{8\times825}{100}$=£66, and 66+759=£825, the policy. The rule for *decimals* is evidently a contraction of this rule.

In Case III. the last three terms in the preceding proportion are given to find the *rate*. Those three terms evidently give the difference between 100 and the rate, and the rule is obvious.

In Case IV. the first two terms and the last term of the preceding proportion are given, to find the third term or sum covered, and the reason of the operation is plain from the consideration of that proportion.

The *amount* is the sum of the annuities for the time it has been forborne, with the interest due on each.

CASE I.

To find the amount of an annuity at Simple Interest.

Rule.

Multiply the sum of the natural series of numbers, 1, 2, 3, 4, &c. to the number of years less 1, by the interest of the annuity for one year, and the product will be the interest which is due on the annuity.

Multiply the annuity by the time, and the sum of the two products, will be the amount.*

Examples.

1. What is the amount of an annuity of £100 for four years, computing interest at 6 per cent.?

$1+2+3=6$, sum of the natural series to the number of years less 1.

6l. interest of annuity for 1 year.

$6\times6=36$l. the whole interest.

$100\times4=400$l. product of annuity and time.

Ans. 436l. amount.

2. If a pension of $20 be continued unpaid for six years, what is its amount at 6 and 7 per cent.?

Ans. At 6 per cent. $138. At 7 per cent. $141.

3. If an annuity of $20 to be paid half each half year is forborne for six years; what is its amount at 6 per cent.?

Ans. $159 60c.

4. If a pension of £33 is forborne for 12 years, at 7 per cent. what is the amount? Ans.

CASE II.

To find the present worth of an annuity at Simple Interest.

Rule.

Let the present worth of each year be found by itself, discounting from the time it is due; then, the sum of all these will be the present worth.†

* It is plain that upon the first year's annuity there will be due so many year's interest, as the given number of years less one, and gradually one year less upon each succeeding year, to that preceding the last, which has but one year's interest, and the last bears none. There is, therefore, due in the whole as many years' interest of the annuity as the sum of the series, 1, 2, 3, &c. to the number of years diminished one. It is evident then, that the whole interest due must equal this sum of the natural series multiplied by the interest for one year; and that the amount will be all the annuities or the product of the annuity and time added to the whole interest. This is the rule.

† This rule depends on the principles of discount. The annuity may be considered for each year, as a debt, due 1, 2, 3, &c. years hence, of which the pres-

EXAMPLES.

1. Find the present worth of an annuity of $100 continued five years at six per cent.

As 106 : 100 :: 100 :	$ 94·3396,	the present worth for 1 year.
112 : 100 :: 100 :	89·2857,	2 years.
118 : 100 :: 100 :	84·7457,	3 years.
124 : 100 :: 100 :	80·6451,	4 years.
130 : 100 :: 100 :	76·9230,	5 years.
	$425·9391,	present worth required.

2. Find the present worth of an annuity of 75l. continued for 4 years at 7 per cent. ? Ans.

3. What is the present worth of a pension of $20 to be continued for 6 years, at 6 per cent. ? Ans.

ANNUITIES OR PENSIONS, IN ARREARS, AT COMPOUND INTEREST.

CASE I.

When the annuity, or pension, the time it continues, and the rate per cent. are given, to find the amount.

RULE I.*

1. Make 1 the first term of a Geometrical Progression, and the amount of £1 or $1 for 1 year at the given rate per cent. the ratio.

2. Carry the series to so many terms as the number of years, and find its sum.

ent worth is to be found. Hence the sum of the present worth for the several years, must be the present worth for the whole.

This rule is very absurd in practice. It is obvious on inspecting the operation of Ex. 1. that the difference between the present worth of the several years is continually diminishing. Whence, after a certain number of years, the present worth of an annuity of $100 would produce more than $100 interest in one year, which is greater than the annuity to be purchased.

* I. From the nature of an annuity, as explained in the proof of the rule, Case I. of Annuities at Simple Interest, there is due one year's interest less than the number of years the annuity has been continued. Now, by Case I. of Compound Interest, the amount of £1 or $1 at the given rate, is equal to that power of the amount for one year, which is indicated by the number of years. This amount is obtained for one less than the number of years, by forming the geometrical series as directed in the Rule, or beginning with unity. Thus in Ex. 1, the series is, 1, 1·06, $1{\cdot}06^2$, $1{\cdot}06^3$, and the last term is the amount of £1 or $1 for one less than *four*, the number of years. The sum of this series is the amount at Compound Interest, of an annuity of £1 or $1 for four years. The amount of any other annuity for the same time and rate, will be as much greater or less, as the annuity is greater or less than £1 or $1, that is, the amount of the annuity of £1 or $1 must be multiplied by the annuity to obtain its amount. Hence, the rule is manifestly correct. In Ex. 1, the above series amounts, by Prob. III. of Geometrical Progression, to $\frac{1{\cdot}06^4-1}{{\cdot}06}$, and this multiplied by the

3. Multiply the sum thus found by the given annuity, and the product will be the amount sought.

Rule II.

Or, multiply the amount of £1 or $1 for 1 year into itself so many times as there are years less by 1; then multiply this product by the annuity; and subtract the annuity therefrom. Lastly, divide the remainder by the ratio less 1, and the quotient will be the amount.

Examples.

1. What will an annuity of 60l. per annum, payable yearly, amount to in 4 years, at 6l. per cent.?

First Method.

$$1+1{\cdot}06+\overline{1{\cdot}06}|^2+\overline{1{\cdot}06}|^3=4{\cdot}374616=\text{sum}.$$

Multiply by 60=annuity.

262·476960
20

9·53920
12

6·4704
4

1·8816 Ans. £262 9s. 6¼d.

$$\text{Or, } \overline{1+1{\cdot}06+\overline{1{\cdot}06}|^2+\overline{1{\cdot}06}|^3}\times 60=£262\ 9s.\ 6\tfrac{1}{4}d.$$

Second Method.

$$1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06=1{\cdot}26247$$

Multiply by 60 annuity.

75·74820
Subtract 60
Carried up.

annuity, 60, gives the amount required, $=\frac{1{\cdot}06^4-1}{{\cdot}06}\times 60=262{\cdot}47696$.

II. The second rule is derived from the expression, $\frac{1{\cdot}06^4-1}{{\cdot}06}\times 60$; for it is also, $\frac{1{\cdot}06^4\times60-1\times60}{{\cdot}06}=$ the above amount, and is the rule.

Because the amounts of annuities, at the same rate and for the same time, are as the annuities, if the amount be divided by the amount of £1 or $1 for the same time and rate, the quotient will be the annuity. This is the 2d Rule under Case II. And the 2d Rule of Case III. is readily inferred from the same principle.

Brought up.

Divide by 1·06—1=·06)15·7482(262·47=262l. 9s. 4½d. Ans.

```
              12
              --
              37
              36
              --
               14
               12
               --
                28
                24
                --
                 42
                 42
```

$$\text{Or, } \frac{1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times60-60}{1{\cdot}06-1}=£262{\cdot}47.$$

Or, by Table III.*

Multiply the tabular number under the rate, and opposite to the time, by the annuity, and the product will be the amount.

2. What will an annuity of 60l. per annum amount to in 20 years, allowing 6l. per cent. compound interest?

Under 6l. per cent. and opposite 20, in table 3d, you will find,

Tabular number=36·78559

Multiply by 60=annuity.

2207·13540=2207l. 2s. 8¼d. Ans.

3. What will a pension of $75 per annum, payable yearly, amount to in 9 years at 5 per cent. compound interest?

Ans. $826 99 2$\frac{3}{10}$m.

4. If a salary of 100l. per annum, to be paid yearly, be forborne 5 years, at 6l. per cent. What is the amount? Ans. 563l. 14s. 2d.

5. What will wages of $25 per month, amount to in a year, at ½ per cent. per month? Ans. $308 38c. 9m.

CASE II.

When the amount, rate per cent. and time are given, to find the annuity, pension, &c.

Rule I.

Multiply the whole amount by the amount of 1l. or $1 for a year, from which subtract the whole amount, divide the remainder by that power of the amount of 1l. or $1 for a year, signified by the number of years, made less by unity, and the quotient will be the answer.

* Table 3 is calculated thus: Take the first year's amount, which is 1l. multiply it by 1·06+1=2·06=second year's amount, which also multiply by 1·06+1=3·1836=third year's amount, &c. and in this manner proceed in calculating tables at any other rates.

RULE II.

Or, find the amount of an annuity of 1l. or $1 for the given time and rate (by Case 1;) divide the given sum by this amount; and the quotient will be the annuity required.

EXAMPLES.

1. What annuity, being forborne 4 years, will amount to £262·47696, at 6l. per cent. compound interest?

262.47696=amount.
Multiply by 1·06=amount of 1l. for 1 year.

```
          157486176
         262476960
         ---------
         278·2255776
Subtract 262·47696
         -----------
·26247696)15·7486176(£60 Ans.
          15·7486176
          ----------
                   0
```

```
         1·06
         1·06
         ----
          636
         1060
       ------
       1·1236
         1·06
       ------
        67416
       112360
     --------
     1.191016
         1·06
     --------
      7146096
     11910160
   ----------
   1·26247696
Subtract 1·
   ----------
Divisor=·26247696
```

$$\text{Or, } \frac{262{\cdot}47696\times1{\cdot}06-262{\cdot}47696}{1{\cdot}06\times1{\cdot}06\times1{\cdot}06\times1{\cdot}06-1}=60.$$

Or, thus.

Amount of an annuity of 1l. for 4 years at 6 per cent. per annum $=4.374616$ (by Case 1;) and $\frac{262.47696}{4{\cdot}374616}=$£60 Ans.

Or, by Table III. the amount of 1l. is found to be 4·374616; and the answer is found, as before.

2. What annuity, being forborne 20 years, will amount to $2207·1354, at 6 per cent. compound interest?

Amount of an annuity of $1 for 20 years at 6 per cent. per annum=36·78559. And,

```
36·78559)2207·13540($60, Ans.
         2207·1354
         ---------
                 0
```

CASE III.

When the annuity, amount and ratio are given, to find the time.

RULE I.

Multiply the amount by the ratio, to this product add the annuity, and from the sum subtract the amount; this remainder being

divided by the annuity, the *quotient* will be that power of the ratio signified by the time, which being divided by the amount of 1l. for 1 year, and this *quotient* by the same, till nothing remain, the number of those divisions will be equal to the time. Or, look for this number under the given rate in table 1, and in a line with it, you will see the time. Or,

Rule II.

Divide the amount by the annuity; from the quotient subtract 1; from the remainder subtract the ratio; from successive remainders subtract the square, cube, &c. of the ratio, till nothing remain; and the whole number of the subtractions will be the answer. Or, find the quotient in Table III. under the rate, and in a line with it stands the answer.

Examples.

1. In what time will 60l. per annum, payable yearly, amount to £262·47696, allowing 6l. per cent. compound interest, for the forbearance of payment?

	262·47696=amount.		
Multiply by	1·06=ratio.		
	157486176		
	262476960		
	278·2255776		
Add	60· =annnuity.		Or thus:
		Annuity=60)262·47696=amt.	
	338·2255776		
Subtract	262·47696		4·374616
		1. Subtract	1·
Divide by	60)75·7486176		
			3·374616
Divide by	1·06)1·26247696	2. Subtract	1·06 =ratio.
Divide by	1·06)1·191016		2·314616
		3. Subtract	1·1236 $=\overline{\text{ratio}}\|^{2}$.
Divide by	1·06)1·1236		
			1·191016
Divide by	1·06)1·06	4. Subtract	1·191016 $=\overline{\text{ratio}}\|^{3}$.
	1	Ans. 4 years.	

The number of divisions by 1·06, being 4, gives the number of years = 4, the answer.

Or, looking into Table III. under the rate, 6, the quotient 4·374616, stands against 4 years, Ans. as before.

Or, in Table I. under the given rate, you will find 1·262476, and in a line under years, you will find 4.

2. In what time will an annuity of $60 payable yearly, amount to $2207·1354, allowing 6 per cent. for the forbearance of payment? Ans. 20 years.

PRESENT WORTH OF ANNUITIES, &c. AT COMPOUND INTEREST.

CASE I.

When the annuity, &c. rate and time are given to find the present worth.

RULE I.*

1. Divide the annuity by the amount of $1 or £1 for 1 year, and the quotient will be the present worth of 1 year's annuity.

* This rule depends on the rule for finding the present worth in Discount at Compound Interest. For each year the present worth is to be found by that rule. Then, the sum of the present worth for the several years, must evidently be the present worth of the whole, and is the rule.

Or, suppose the annuity to be 1l. or $1 at 6 per cent. then $\frac{1}{1{\cdot}06}$ is the present worth for one year; $\frac{1}{1{\cdot}06^2}$ for two years; $\frac{1}{1{\cdot}06^3}$ for three years; $\frac{1}{1{\cdot}06^4}$ for four years, and so on. Then the sum, or $\frac{1}{1{\cdot}06}+\frac{1}{1{\cdot}06^2}+\frac{1}{1{\cdot}06^3}+\frac{1}{1{\cdot}06^4}$, will be the whole present worth. Let any annuity be substituted for the numerator of these several fractions, and you have the rule in the text.

By Note 2, Prob. I. of Geometrical Progression, the sum of the series, $\frac{1}{1{\cdot}06}+\frac{1}{1{\cdot}06^2}+\frac{1}{1{\cdot}06^3}+\frac{1}{1{\cdot}06^4}$, is $\overline{1-\frac{1}{1{\cdot}06^4}}\times\frac{1}{{\cdot}06}$, or $\frac{1}{{\cdot}06}-\frac{1}{{\cdot}06}\times\frac{1}{1{\cdot}06^4}$. Now if the annuity were to continue forever, or the number of years were infinite, then the index of the denominator of the last expression would be infinite, and the value of the fraction would be infinitely diminished or become nothing, and $\frac{1}{{\cdot}06}$ would be the present worth of an annuity of 1l. or $1 to continue forever at 6 per cent. Hence, if an annuity is a *perpetuity*, or is to continue forever, its present worth is found by dividing the annuity by the ratio, or the interest of 1l. or $1 for a year at the given rate.

The present worth of an annuity of $1 to continue forever at 5 per cent. is $\frac{1}{{\cdot}05}=\frac{100}{5}=$ $20, and an annuity of $100 at 5 per cent. to continue forever, would now be worth $2000, and at 7 per cent. $$1428\frac{4}{7}$.

Rule II. is derived from the expression $\overline{1-\frac{1}{1{\cdot}06^4}}\times\frac{1}{{\cdot}06}$, when the annuity is $1 or 1l. and the rate 6 per cent. That is when the annuity is $1 or 1l. divide the annuity by that power of the ratio indicated by the number of years, and subtract the quotient from the annuity; the remainder divided by the ratio of the series less 1, will be the present worth. But the present worth of annuities varies as the annuity. Hence the rule is manifest.

Note. Another rule for obtaining the present worth may be derived from the preceding. Thus, the sum of the series, $\frac{1}{1{\cdot}06}$, $\frac{1}{1{\cdot}06^2}$, $\frac{1}{1{\cdot}06^3}$, $\frac{1}{1{\cdot}06^4}$, is, by Note 2, of Prob. I. of Geometrical Progression, $\overline{1-\frac{1}{1{\cdot}06^4}}\times\frac{1}{1{\cdot}06-1}$, which is also $\frac{1{\cdot}06^4-1}{1{\cdot}06^4}\times\frac{1}{1{\cdot}06-1}=\frac{1{\cdot}06^4-1}{1{\cdot}06^5-1{\cdot}06^4}=$ the present worth of 1l. or $1 for four years at 6 per cent. That is, divide the difference between unity and that power

2. Divide the annuity by the square of the ratio, and the quotient will be the present worth for two years.

3. In like manner, find the present worth of each year by itself, and the sum of all these will be the present value of the annuity sought.

RULE II.

Or, divide the annuity, &c. by that power of the ratio signified by the number of years, and subtract the quotient from the annuity; this remainder being divided by the ratio less 1, the quotient will be the present worth.

EXAMPLES.

1.* What ready money will purchase an annuity of 60l. to continue 4 years, at 6l. per cent. compound interest?

First Method.

Ratio = 1·06)60·00000(56·603=present worth for 1 year.
$\overline{\text{Ratio}}|^2$= 1·1236)60·00000(53·399= do. for 2 years.
$\overline{\text{Ratio}}|^3$= 1·191016)60·00000(50·377= do. for 3 years.
$\overline{\text{Ratio}}|^4$=1·26247696)60·00000(47·525= do. for 4 years.

207·904=£207 18s. 0¾d. Ans.

Second Method.

4th power of the ratio } =1·26247696)60·0000000(47·525

From 60
Subtract 47·525 Or, $\frac{60}{\overline{\cdot06}|^4}$=47·525 60—47·525=12·475

Divis. 1·06—1=·06)12·475 And $\frac{12\cdot475}{\cdot06}$=207·916·

2 07·916=£207 18s. 3¾d. Ans.

of the ratio which is indicated by the number of years, by the difference between that power of the ratio which is one greater than the number of years and that power of the ratio which is equal to the number of years, and the quotient is the present worth of 1l. or $1. Then, as annuities are as their present worth, multiply this quotient by the given annuity, and the product is its present worth. The rules for the next Case are derived directly from this rule, and need no further illustration.

* The *amount* of an annuity may also be found for years and parts of a year, thus:

1. Find the amount for the whole years, as before.
2. Find the interest of that amount for the given parts of a year.
3. Add this interest to the former account, and it will give the whole amount required.

The *present worth* of an annuity for years and parts of a year may be found thus:

1. Find the present worth for the whole years, as before.
2. Find the present worth of this present worth, discounting for the given parts of a year, and it will be the whole present worth required.

Questions in this case may also be answered by first finding the amount of the given annuity by Case I. of annuities in arrears, page 315, and then the present worth, or principal, by Case II. of Compound Interest, page 311.

By Table III.

Under 6l. per cent. and opposite 4, we find
4·37461=amount of 1l. annuity for 4 years.
Multiply by 60=annuity.

262·47660=amount of 60l. for 4 years.
Then, opposite 4 years, and under 6l. per cent. in Table 2d.
We have ·792093
Multiply by 262·7466

```
          4752558
         4752558
        3168372
       5544651
      1584186
     4752558
    1584186
---------------
208.1197426338= £208 2s. 4½d.
```

Or, opposite 4 years, and under 6l. per cent. in Table 1st, we have 1·26247=the amount of 1l. for 4 years:
Then, 262·7466÷1·26247=208·1209= £208 2s. 5d. Ans.

By Table IV.*

Multiply the tabular number, under the rate, and opposite the time, into the annuity, and the product will be the present worth.
Thus, in Example 1st. What ready money will purchase £60 annuity, to continue 4 years, at £6 per cent. compound interest?
Under 6l. per cent. and even with 4 years,
We have 3·4651=present worth £1 for 4 years.
Multiply by 60=annuity.

Ans.=207·9060= £207 18s. 1¾d.

2. What is the present worth of an annuity of \$60 per annum, to continue 20 years, at 6 per cent. compound interest?
Ans. \$688·65 (nearly.)

CASE II.

When the present worth, time, and rate are given, to find the annuity, rent, &c.

Rule.

1. From that power of the ratio, denoted by the number of years plus 1, subtract that power of it denoted by the number of years.

* Table 4th is thus made: Divide £1 by 1·06=·94339 the present worth of the first year, which, divided by 1·06, is equal to ·88999, which, added to the first year's present worth, is = 1·83339, the second year's present worth, then ·88999, divided by 1·06, and the quotient added to 1·83339, gives 2·6701 for the third year's present worth, &c.

2. Divide the remainder by that power of the ratio, signified by the time made less by unity.

3. Multiply the present worth into this quotient, and the product will be the annuity, pension, rent, &c.

Or, 1. Multiply that power of the ratio, denoted by the number of years plus 1, by the present worth.

4. Multiply that power of the ratio, denoted by the time, by the present worth, and subtract this product from the former.

5. Divide the remainder by that power of the ratio, denoted by the time made less by unity, and the quotient will be the annuity.

EXAMPLES.

1. What annuity, to continue 4 years, will £207·904 purchase, compound interest, at £6 per cent.?

First Method.

From 1·06×1·06×1·06×1·06×1·06=1·3382255776
Subt. 1·06×1·06×1·06×1·06 =1·26247696

Divide by $\overline{1\cdot06}|^4$—1=·26247696)·0757486176(·2885898
·2885898
Multiply by 207·9 present worth.

25973082
20201286
57717960

Ans. 59·99781942=£60.

Second Method.

From 1·06×1·06×1·06×1·06×1·06×207·9=278·217097573
Take 1·06×1·06×1·06×1·06×207·9 =262·468959984

Divide by $\overline{1\cdot06}|^4$—1=·26247696)15·748137589(59·998=601.

BY TABLE V.*

Multiply the tabular number corresponding with the rate and time, by the purchase money, and the product will be the annuity.

Under £6 per cent. and opposite 4 years, you will find
·28859=annuity which £1 will purchase in 4 years.
Multiply by 207·9

259731
202013
577180

59·997861=£60.

2. What salary, to continue 20 years, will $688 65c. purchase, at 6 per cent. compound interest? Ans. $60.

* Table 5th is made in this manner: Divide £1 by the present worth of £1 for 1 year, and the quotient will be the annuity, which £1 will purchase for 1 year: divide £1 by the present worth of £1 for 2 years, and the quotient will be the annuity, which £1 will purchase for 2 years, &c.

CASE III.

When the annuity, present worth and ratio, are given, to find the time.

RULE.*

Divide the annuity by the product of the present worth and ratio subtracted from the sum of the present worth and annuity, and the quotient will be that power of the ratio, denoted by the number of years, which, being divided by the ratio, and this quotient by the same, till nothing remain, the number of divisions will show the time: Or, the above quotient being sought in Table 1st under the given rate, in a line with it, you will see the time.

EXAMPLES.

1. For how long may an annuity of £60 per annum be purchased for £207·906336762, at £6 per cent. compound interest?

Multiply 207·906336762	To 207·906336762=present worth,
by 1·06	Add 60· =annuity.
1247438020572	From 267·906336762
2079063367620	Subt. 220·380716967
220·38071696772	47·525619795=divisor.

47·525619795)60·000000000(1·26247696

Divide by 1·06)1·26247696

1·06)1·191016

1·06)1·1236

1·06)1·06

1 } The number of divisions =time=4 years.

$$\text{Or, } \frac{60}{207{\cdot}906336762+60-207{\cdot}906336762\times1{\cdot}06}=1{\cdot}26247696,$$

which being sought in Table 1, under the given rate, in a line with it, is 4=4 years.

2. How long may a lease of $300 yearly rent, be had for $2132·341 allowing 5 per cent. compound interest, to the purchaser? Ans. 9 years.

ANNUITIES, LEASES, &c. TAKEN IN REVERSION AT COMPOUND INTEREST.

CASE I.

When the annuity, time and ratio, are given, to find the present worth of the annuity in reversion.

RULE I.

1. Divide the annuity by that power of the ratio denoted by the time of its continuance.

* This rule is derived directly from Rule II. Case I.

2. Subtract this quotient from the annuity: divide by the ratio less 1, and the quotient will be the present worth, to commence immediately.

3. Divide this quotient by that power of the ratio denoted by the time of reversion, (or, time to come, before the annuity commences) and the quotient will be the present worth of the annuity in reversion.

Rule II.

Or, 1. Multiply the annuity by that power of the ratio denoted by the time of its continuance, minus unity, for a dividend.

2. Multiply that power of the ratio denoted by the time of its continuance, that power of it denoted by the time of reversion, and the ratio less 1, continually together for a divisor, and the quotient arising from the division of these two numbers will be the present worth of the annuity in reversion.

Rule III.

Find the present worth of the annuity for the number of years before it is to begin and is to be continued; find also the present worth of the annuity for the number of years before the annuity commences: and the difference between the present worth for these two periods, is the present worth of the annuity in reversion.*

Examples.

1. What is the present worth of 60l. payable yearly, for 4 years; but not to commence till two years hence, at 6l. per cent.?

First Method.

Ratio=1·06	Or, in Table 4th, find the present
1·06	value of 1l. at the given rate, both for
——	the time in being and the time in re-
636	version added together, and subtract
1060	the present worth of the time in be-
——	ing from the other, multiply the re-
2d. power=1·1236	mainder by the annuity, and the pro-
Carried over. 1·1236	duct will be the answer.

* Rule III. is merely an expression of this truth, viz. the present worth of an annuity continued for the sum of the years before the annuity is to commence, and is to continue after it begins, is evidently too much by the present worth of the same annuity for the time of reversion or the time before the annuity is to commence.

Rule I. The first two steps are Rule II. of Case I. to find the present worth of Annuities &c. at Compound Interest, for the time of reversion and continuance of the annuity. But as this present worth would evidently be too much, it must be discounted at compound interest for the time of reversion, which in the first example is 2 years. The third step in Rule I. is, therefore, merely the rule for Discount at Compound Interest. Hence the process is obvious.

Brought over. 1·1236 Pres. worth of the time in being and reversion } =4·91732

64416 Present worth of the time in being } =1·8333

33708

22472 3·08402

11236 60

11236

£185·04120

Div. by 4th pow.=1·26247696)60·000000000000(47·525619794281

Subtract the quotient=47·525619794281

Divide by 1·06—1=·06)12·474380205719

Divide by 1·06×1·06=1·1236)207·9063367619(185·035899=185l. 0s. 8½d.=the present worth of the annuity in reversion.

Or, $\frac{60}{1\cdot26247696}=47\cdot5256$ $\frac{60-47\cdot5256}{1\cdot06-1}=207\cdot906$

And $\frac{207\cdot906}{1\cdot1236}=185\cdot035899$

Second Method.

·26247696=4th power—1

Multiply by 60=annuity.

15·74861760=dividend. ·08511115)15·74861760(185·036. [Ans.

1·26247696=4th power.

1·1236=2d power.

757486176

378743088

252495392

126247696

126247696

1·418519112256 Or, $\frac{\overline{1\cdot06|^4-1}\times60}{\overline{1\cdot06|^4}\times\overline{1\cdot06|^2}\times\overline{1\cdot06-1}}=185\cdot036$

·06=ratio—1

·08511114673536=divisor.

Third Method.

$60\times\overline{1-\frac{1}{1\cdot06^6}}\times\frac{1}{\cdot06}=295\cdot038+$ the present worth for 6 years.

$60\times\overline{1-\frac{1}{1\cdot06^2}}\times\frac{1}{\cdot06}=110\cdot002+$ the present worth for 2 years.

£185·035+=the present worth in reversion.

EXAMPLE 2.

What is the present worth of a reversion of a lease of \$60 per annum, to continue 20 years, but not to commence till the end of 8 years, allowing 6 per cent to the purchaser?

Ans. \$431·782 (nearly.)

3. An annuity of $1 in reversion is to commence at the end of 20 years, and is to continue 15 years; what is its present worth at 4 per cent.? Ans. $5·0743 nearly.

4. An annuity of $1 in reversion is to commence after 5 years, and to continue forever; what is its present worth at 6 per cent.? Ans. $12 45c. 43.

An annuity, several times in reversion, and rate being given, to find the several present values.

Find the present value of £1 or $1 by Table 4, at the given rate, and for the several given times, which, being severally multiplied by the annuity, the products will be the several present values of that annuity, for the several times given; subtract the several present values, the one from the other, and the several remainders will answer the question.

5. A has a term of 6 years in an estate at 60l. per annum. B has a term of 14 years in the same estate, in reversion, after the 6 years are expired; and C has a further term of 16 years, after the expiration of 20 years. I demand the present values of the several terms at 6 per cent.?

	£	s.	d.	
Pres. value of £1 for 36y.=14·61722×60=	877	0	$7\frac{3}{4}$	
Ditto of ditto for 20 years =11·46992×60=	688	3	$10\frac{3}{4}$	
Ditto of ditto for 6 years = 4·91732×60=	295	0	$9\frac{1}{4}$	=A's term.

Therefore, 877 0 $7\frac{3}{4}$—688 3 $10\frac{3}{4}$=£188 16 9 C's term, and 688 3 $10\frac{3}{4}$—295 0 $9\frac{1}{4}$=£393 3 $1\frac{1}{2}$=B's term.

6. For a lease of certain profits for 7 years, A offers to pay $300 gratuity, and $300 per annum, B offers $800 gratuity and $250 per annum, C bids $1300 gratuity and $200 per annum, and D bids $2500 for the whole purchase, without any yearly rent; which is the best offer, computing at 6 per cent.?

	$
By Table 4, the present worth of $300 per annum for 7 years, at 6 per cent is	1674·714
To which add	300·
Value of A's offer=	1974·714
Present worth of $250 per annum for 7 years=	1395·595
To which add	800·
Value of B's offer=	2195·595
Present worth of $200 per annum for 7 years=	1116·476
To which add	1300·
Value of C's offer=	2416·476
D's offer=	2500·

Hence it appears that D's offer is the best.

The above questions may be answered by the 4th and 2d Tables.

Take question 1st. for Example.

1. Multiply the tabular number in Table 4, corresponding to the rate and the time of continuance, into the annuity, and the product will be the present worth, to commence immediately.

2. Multiply this present worth by the tabular number in Table 2, corresponding to the rate and the time of reversion, and the product will be the present worth of the annuity in reversion.

In Table 4th we have 3·4651
Multiply by 60=annuity.

207·9060
In Table 2d we have ·889996

1247436
1871154
1871154
1871154
1663248
1663248

185·035508376=pres. worth of the reversion.

CASE II.

When the present worth of the reversion, rate and time are given, to find the annuity.

Rule 1. Multiply that power of the ratio signified by the time of reversion, by the present worth, and the product will be the amount of the present worth for the time before the annuity commences.

2. Multiply that power of the ratio signified by the time of continuance plus 1, by the last product.

3. Multiply that power of the ratio, signified by the time, by the aforesaid product, and this last product, divided by that power of the ratio denoted by the time minus unity, will give the annuity.

Or, Divide the continual product of the present worth, that power of the ratio denoted by the time of continuauce, that power of it denoted by the time of reversion, and the ratio minus 1, by that power of the ratio denoted by the time of continuance minus 1, and the quotient will be the annuity.

Examples.

1. What annuity, to be entered upon 2 years hence, and then to continue 4 years, may be purchased for $185·035899, at 6 per ct.?

First Method.

1·06×1·06=1·1236=2d power of the ratio.
Multiply by 185·036=present worth.

67416
33708
561800
89888
11236

207·9064496 amount for the time of reversion.

Brought up. 207·9064496 amount for the time of reversion.
Multiply by 1·33822 =5th power of the ratio.

```
        415812      4th power of the ratio=1·26247
       415812                 Multiply by 207·906
     1663248                          ---------
     623718                             757482
    623718                            11362230
   207906                              883729
  -----------                         2524940
From 278·22396732                  ------------
Take 262·47508782                  262·47508782
```

Divide by $\overline{1\cdot06}|^4-1=\cdot26247$)15·7488750(60 the annuity required.
Or, 185·036×1·1236=207·906

$$\text{Then, }\frac{207\cdot906\times1\cdot33822-207\cdot906\times1\cdot26247}{1\cdot26247-1}=\$60\text{ Ans.}$$

Second Method.

185·036=present worth of the reversion.
1·26247=4th power of the ratio.

```
     1295252
     740144
    370072
   1110216
   370072
  185036
  --------
  233·6024
   1·1236=2d. power of the ratio.
  --------
  14016144
  7008072
 4672048
2336024
2336024
----------
262·47565664
   ·06=ratio—1.
```

Or by Table 4th, divide the present worth of the reversion by the difference between the present worth of $1 for the time both in being and reversion, and the time in being, and the quotient will be the annuity.

4·91732= { pr. worth of $1 for the time in being & revsn.
1·3333 = { present worth of $1 for the time in being.

3·08402)185·0412(60 Ans.

$\overline{1\cdot06}|^4-1=\cdot26247$)15·7485393984(60.

$$\text{Or, }\frac{185\cdot036\times1\cdot26247\times1\cdot1236\times1\cdot06-1}{1\cdot26247-1}=60.$$

2. The present worth of a lease of a house is £431 15s. 7d. 2·7819qrs. taken in reversion for 20 years; but not to commence till the end of 8 years, allowing £6 per cent. to the purchaser: What is the yearly rent? Ans. £60.

PURCHASING ANNUITIES FOREVER, OR FREEHOLD ESTATES, AT COMPOUND INTEREST.

CASE I.

When the annuity, or yearly rent, and the rate are given, to find the present worth or price.

RULE.*

As the rate per cent. is to £100 or $100 so is the yearly rent, to the value required.

Or, Divide the yearly rent by the ratio less 1, and the quotient will be the value required.

EXAMPLES.

1. What is the worth of a freehold estate of £60 per annum, allowing 6l. per cent. to the purchaser?

```
£   £     £
6 : 100 :: 60                 Or, 1·06—1=·06)60·00
    60                                       ------
    ---                                       1000
6)6000
  ----
£1000 Ans.
```

2. An estate brings in yearly $75: What will it sell for, allowing the purchaser 5 per cent. compound interest? Ans. $1500.

CASE II.

When the price, or present worth, and rate are given, to find the annuity, or yearly rent.

RULE.

As £100 or $100 is to the rate so is the present worth to its rent.

Or, Multiply the present worth by the ratio less 1, and the product will be the yearly rent.

EXAMPLES.

1. If a freehold estate be bought for £1000 allowing £6 per cent. to the purchaser: What is the yearly rent?

```
 £    £    £
100 : 6 :: 1000
              6
           ----             Or, 1000×·06=£60.
    100)6000(£60 Ans.
        600
        ---
          0
```

2. If an estate be sold for $1500 and 5 per cent. allowed to the buyer; what is the yearly rent? Ans. $75.

* The reason of this rule is obvious; for since a year's interest of the price, which is given for it, is the annuity, there can neither more nor less be made of that price, than of the annuity, whether it be employed at simple or compound interest. It has also been proved under Case I. of the Present Worth of Annuities &c. at Compound Interest. Case II. and III. follow directly from the rule for Case I. and their rules are hence manifest.

CASE III.

When the present worth, or price, and yearly rent, are given, to find the rate.

Rule.

As the present worth is to the rent; so is £100 or $100 to the rate.

Or, Divide the rent by the present worth; add 1 to the quotient, and the sum will be the ratio of the rate per cent.

Or, Divide the sum of the present worth and rent by the present worth, and the quotient will be the ratio.

Examples.

1. If an estate of £60 per annum be bought for £1000 what rate of interest was allowed the purchaser for his money?

```
 £     £     £
1000 : 60 :: 100          Or, 1000)60·00(·06+1=1·06
             100                   60·00
          ------
1000)6000(£6 Ans.
                           Or, to 1000=present worth.
                              Add   60=rent.
                                  ------
                             1000)1060(1·06
                                  1000
                                  ------
                                    6000
                                    6000
```

2. An estate of $75 per annum was purchased for $1500 what rate of interest had the buyer for his money? Ans. 5 per cent.

To find at how many years' purchase an estate may be bought.

CASE I.

When the rate of interest is given, to find the number of years.

Rule.

Divide £100 or $100 by the rate, and the *quotient* will be the years.

Examples.

1. How many years' purchase should a gentleman offer for the purchase of an estate, to have 6 per cent. for his money?

$$6)100$$

$$16\cdot666+=16\tfrac{2}{3}\text{ years.}$$

2. How many years' purchase is an estate worth, allowing 5 per cent. to the purchaser? Ans. 20 years.

CASE II.

When the number of years' purchase, at which an estate is bought, or sold, is given, to find the rate of interest.

RULE.

Divide £100 or $100 by the number of years, and the quotient will be the rate.

EXAMPLES.

1. A gentleman gives $16\frac{2}{3}$ years' purchase for a farm; what interest is he allowed? $16\frac{2}{3}$=16·666+)100·000(6 per cent. Ans.

2. A gentleman gives 20 years' purchase for an estate; what interest has he? Ans. 5 per cent.

PURCHASING FREEHOLD ESTATES IN REVERSION.

CASE I.

The rate and rent of a freehold estate being given, to find the present worth of reversion.

RULE.*

1. Find the present worth of the annuity or rent, (by Case 1, of purchasing Freehold Estates, page 330,) as though it were to be entered on immediately.

2. Divide the last present worth by that power of the ratio denoted by the time of reversion (by Discount by Compound Interest) and the quotient will be the answer required.

Or, 1. Having found the present value of the estate, supposing it to be immediate: Multiply the annuity, or rent, by the present worth of 1l. or $1 corresponding with the time of reversion and rate in Table 4th, and the product will be the present worth of the annuity, or rent, for the time of reversion; or the value of the present possession.

2. Subtract the value of the possession from the value of the estate, and the remainder will be the value of reversion.

EXAMPLES.

1. Suppose a freehold estate of 60l. per annum to commence 2 years hence, be put up to sale; what is its value, allowing the purchaser 6l. per cent.?

First Method.

1·06—1=·06)60·00=rent per annum.

1000=present worth, if entered on immediately.

* By the first step, the present worth is found for the present time; but as the estate is not to be entered on for a certain time, discount for that time must be allowed at Compound Interest. This is the second step, and the propriety of the rule is manifest. Case II. needs no illustration.

$\overline{1\cdot06}|^2=1\cdot1236)1000\cdot000(889\cdot996=$£ 889 19s. 11d.=present worth of 1000l. for 2 years, or the whole present worth required.

Second Method.

$1\cdot06-1=\cdot06)60\cdot00$

1000=present worth, for immediate possession.

In Table 4th. we have 1·33339=value of 1l. for 2 years.

Multiply by 60=rent.

110·00340=value of possession.

From 1000·0000

Subtract 110·0034

889·9966=value required.

2. Suppose an estate of $75 per annum, to commence 10 years hence, were to be sold, allowing the purchaser 5 per cent; what is its worth? Ans. $920 87c. 1m. (nearly.)

CASE II.

The value of a Reversion, the Time prior to its Commencement, and rate of Interest given, to find the Annuity or Rent.

Rule.

1. Multiply the price of the reversion by that power of the amount of 1l. or $1 for 1 year, denoted by the time of reversion, and the product will be its amount, (by Case 1 of Compound Interest.)

2. Find the interest of the amount (by Case 1st Simple Interest) and it will be the annuity, or yearly rent.

Examples.

1. A freehold estate is bought for £889·9966 which does not commence till the end of 2 years; the buyer being allowed 6l. per cent. for his money; I desire to know the yearly income?

889·9966=price of the reversion.

Multiply by $\overline{1\cdot06}|^2=1\cdot1236$ denoted by the time of reversion.

```
   53399796
  26699898
 17799932
 8899966
8899966
------------
1000·00017976=amount of the reversion.
 ·06
```

Ans. £60·00

2. If a freehold estate, to commence 10 years hence, be sold for $920 87c. 1m. allowing the purchaser 5 per cent.; what is the yearly income? Ans. $75.

TABLE I.

SHEWING THE AMOUNT OF £1 OR $1 FROM 1 YEAR TO 50.

years.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.
1	1·0300000	1·0350000	1·0400000	1·0450000
2	1·0609000	1·0712250	1·0816000	1·0920250
3	1·0927270	1·1087178	1·1248640	1·1411661
4	1·1255088	1·1475230	1·1698585	1·1925186
5	1·1592740	1·1876863	1·2166529	1·2461819
6	1·1940523	1·2292553	1·2653190	1·3022601
7	1·2298738	1·2722792	1·3159317	1·3608618
8	1·2667700	1·3168090	1·3685690	1·4221006
9	1·3047731	1·3628973	1·4233118	1·4860951
10	1·3439163	1·4105987	1·4802842	1·5529694
11	1·3842338	1·4599697	1·5394540	1·6228530
12	1·4257608	1·5110686	1·6010322	1·6958814
13	1·4685337	1·5639560	1·6650735	1·7721961
14	1·5125897	1·6185945	1·7316764	1·8519449
15	1·5579674	1·6753488	1·8009435	1·9352824
16	1·6047064	1·733986	1·8729812	2·0223701
17	1·6528476	1·7946755	1·9479005	2·1133768
18	1·7024330	1·8574892	2·0258161	2·2084787
19	1·7535060	1·9225013	2·1068491	2·3078603
20	1·8061112	1·9897888	2·1911231	2·4117140
21	1·8602945	2·0594314	2·2787680	2·5202411
22	1·9161034	2·1315115	2·3699187	2·6336520
23	1·9735865	2·2061144	2·4647155	2·7521663
24	2·0327941	2·2833284	2·5633041	2·8760138
25	2·0937779	2·3632449	2·6658363	3·0054344
26	2·1565912	2·4459585	2·7724697	3·1406790
27	2·2212890	2·5315671	2·8833685	3·2820095
28	2·2879276	2·6201719	2·9987033	3·4296999
29	2·3565655	2·7118779	3·1186514	3·5840364
30	2·4272624	2·8067937	3·2433975	3·7453181
31	2·5000803	2·9050314	3·3731334	3·9138574
32	2·5750827	3·0067075	3·5080587	4·0899810
33	2·6523352	3·1119423	3·6483811	4·2740301
34	2·7319053	3·2208603	3·7943163	4·4663615
35	2·8138624	3·3335904	3·9460889	4·6673478
36	2·8982783	3·4502661	4·1039325	4·8773784
37	2·9352266	3·5710254	4·2680898	5·0968604
38	3·0747834	3·6960113	4·4388134	5·3262192
39	3·1670269	3·8253717	4·6163650	5·5658990
40	3·2620377	3·9592597	4·8010206	5·8164645
41	3·3598988	4·0975337	4·9930614	6·0782054
42	3·4606958	4·2412579	5·1927833	6·3517246
43	3·5645167	4·3897020	5·4004952	6·6375522
44	3·6714522	4·5433415	5·616515	6·9362421
45	3·7815957	4·7023585	5·8411756	7·248373
46	3·8950436	4·8669411	6·0748236	7·5745497
47	4·0118949	5·0372840	6·3168166	7·9154045
48	4·1322518	5·2135889	6·5694892	8·2715977
49	4·2562193	5·3960645	6·8322688	8·6438196
50	4·3830059	5·5849268	7·1055596	9·0327915

TABLE I.—CONTINUED.

SHEWING THE AMOUNT OF £1 OR $1 FROM 1 YEAR TO 50.

years.	5 per cent.	5½ per cent.	6 per cent.	7 per cent.
1	1·0500000	1·0550000	1·0600000	1·07000
2	1·1025000	1·1130250	1·1236000	1·14490
3	1·1576250	1·1742413	1·1910160	1·22504
4	1·2155062	1·2388245	1·2624796	1·31079
5	1·2762815	1·3069598	1·3382256	1·40255
6	1·3400956	1·3788426	1·4185191	1·50073
7	1·4071004	1·4546789	1·5036302	1·60578
8	1·4774554	1·5346862	1·5938480	1·71818
9	1·5513282	1·6190939	1·6894789	1·83845
10	1·6288946	1·7081440	1·7908476	1·96715
11	1·7103393	1·8020919	1·8982985	2·10485
12	1·7958563	1·9012069	2·0121964	2·25219
13	1·8856491	2·0057732	2·1329282	2·40984
14	1·9799316	2·1160907	2·2609039	2·57853
15	2·0789281	2·2324756	2·3965581	2·75903
16	2·1828745	2·3552617	2·5472716	2·95216
17	2·2920183	2·4848011	2·6927727	3·15881
18	2·4066192	2·6214652	2·8543391	3·37293
19	2·5269502	2·7656458	3·0255995	3·61652
20	2·6532977	2·9177563	3·2071355	3·86968
21	2·7859625	3·0782329	3·3995636	4·14056
22	2·9252607	3·2475357	3·6035374	4·43040
23	3·0715237	3·4261502	3·8197496	4·74052
24	3·2250999	3·6145885	4·0489346	5·07236
25	3·3863549	3·8133910	4·2918707	5·42743
26	3·5556726	4·0231279	4·5493829	5·80735
27	3·7334563	4·2443999	4·8223459	6·21386
28	3·9201291	4·4778419	5·1116867	6·64883
29	4·1661356	4·7241232	5·4183879	7·11425
30	4·3219423	4·9839499	5·7434912	7·61225
31	4·5380394	5·2580671	6·0881007	8·14571
32	4·7649414	5·5472608	6·4533867	8·71527
33	5·0031885	5·8523600	6·8405899	9·32533
34	5·2533479	6·1742398	7·2510253	9·97811
35	5·5160152	6·5138230	7·6860868	10·6765
36	5·7918101	6·8720832	8·147252	11·4239
37	6·0814069	7·2500478	8·6360871	12·2236
38	6·3854772	7·6488004	9·1542523	13·0792
39	6·7047511	8·0694844	9·7035074	13·9948
40	7·0399887	8·5133060	10·2857178	14·9744
41	7·3919881	8·9815378	10·9028608	16·0226
42	7·7615875	9·4755224	11·5570325	17·1442
43	8·1496669	9·9966761	12·2504547	18·3443
44	8·5571502	10·5464933	12·9854317	19·6284
45	8·9850077	11·1265504	13·7626109	21·0024
46	9·4342581	11·7385217	14·5883673	22·4726
47	9·9059710	12·3841404	15·4636693	24·0457
48	10·4012696	13·0652681	16·3914894	25·7289
49	10·9213331	13·7838579	17·3749788	27·5299
50	11·4673697	14·5410000	18·2174775	29·4570

TABLE II.

SHEWING THE PRESENT VALUE OF £1 OR $1, DUE AT THE END OF ANY NUMBER OF YEARS, FROM 1 TO 40.

ys.	4 per ct.	4½ per ct.	5 per ct.	5½ per ct.	6 per ct.	7 per ct.
1	·961538	·956938	952381	·947867	·943396	·934579
2	·924556	·91573	·90703	·898513	·889996	·873438
3	·888996	·876297	·863838	851728	·839619	·816297
4	·854804	·838561	·822702	·807397	·792093	·762895
5	·821927	·802451	·783526	·765392	·747258	·712986
6	·790314	·767896	·746215	·725587	·70496	·666042
7	·759918	·734828	·710681	·687869	·665057	·622749
8	·730690	·703185	·676839	·652125	·627412	·582009
9	·702587	672904	·644609	·618253	·591898	·543933
10	675564	·643928	·613913	·586153	·558394	·508349
11	·649581	·616199	·584679	·573733	562787	·475092
12	·624597	·589664	·556837	·526903	·496969	·444012
13	·600574	·564271	·530321	·49958	·468839	414964
14	·577475	·539973	·505068	·473684	·442301	.387817
15	·555264	·516720	481017	·449141	417265	·362446
16	·533908	·494469	·458311	·425979	·393647	338734
17	·513373	·473176	·436297	·40383	·371364	·316574
18	·493628	·4528	·415521	·382932	350343	·295864
19	·474642	·433302	·395734	·363123	·330513	·276508
20	·456387	·414643	·376889	·344346	·311804	·258419
21	·438833	·396787	·358942	·326568	·294155	·241513
22	·421955	·379701	·34185	·309677	·277505	·225713
23	·405726	·36335	·325571	.293684	·261797	·210947
24	·390121	·347703	·310068	·278523	·246978	·197146
25	·375117	·332731	·305303	·26915	·232998	·184249
26	·360689	·318402	·281241	·250525	·21981	·172195
27	·340816	·304691	·267848	·237603	·207368	·160930
28	·333477	·291571	·255094	·225362	·19563	·150402
29	·320651	·279015	·242946	·213715	·184556	·140562
30	·308309	·267	·231377	·202743	·17411	·131367
31	290460	·255502	220359	·192307	·164255	·122773
32	285058	·2445	·209866	·182411	·154957	·114741
33	·274094	·233971	·199872	·173029	·146186	·107234
34	·263552	·223896	·190355	164133	·137912	·100219
35	·254415	·214251	·18129	·155692	·130105	·093663
36	·243669	·205028	·172057	·147399	·122741	·087535
37	·234297	·196299	·164436	·140114	·115793	·081808
38	·225285	·18775	·156605	·132893	·109182	·076456
39	.216671	·179665	·149148	·126075	·103002	071455
40	·208289	·171929	·142046	·119608	·09717	066780

TABLE III.

SHEWING THE AMOUNT OF £1 OR $1 ANNUITY FOR ANY NUMBER OF YEARS, FROM 1 TO 40.

ys.	4 pr. cent.	4½ pr. cent.	5 pr. cent.	5½ pr. cent.	6 pr. cent.	7 pr. cent.
1	1·	1·	1·	1·	1·	1·
2	2·04	2·045	2·05	2·055	2·06	2·07
3	3·1216	3·137025	3·1525	3·16802	3·1836	3·2149
4	4·246464	4·278191	4·310125	4·34226	4·374616	4·43994
5	5·416322	5·47071	5·525631	5·58109	5·637093	5·75073
6	6·632975	6·716892	6·801913	6·888051	6·975318	7·15329
7	7·898294	8·019152	8·142008	8·266894	8·393837	8·65402
8	9·214266	9·380014	9·549109	9·721573	9·897467	10·2598
9	10·582795	10·802114	11·026564	11·256259	11·491315	11·9799
10	12.006107	12·2882·	12·577892	12·875354	13·180794	13·8164
11	13·486351	13·841179	14·206787	14·583498	14·971642	15·7836
12	15·025805	15·464032	15·917126	16·38559	16·86994	17·8884
13	16·626838	17·159913	17·712983	18·286798	18·882132	20·1406
14	18·291911	18·932109	19·598632	20·292572	21·015064	22·5504
15	20·023588	20·784054	21·578563	22·408663	23·275968	25·1290
16	21·824531	22·719337	23·657492	24·64114	25·672527	27·8880
17	23·697512	24·741707	25·840366	26·996402	28·212879	30·8402
18	25·645413	26·855084	28·132385	29·481205	30·905652	33·9990
19	27·671229	29·063562	30·529004	32·102671	33·759901	37·3789
20	29·778078	31·371423	33·065954	34·868318	36·78559	40·9954
21	31·969202	33·783137	35·719252	37·786075	39·992725	44·8651
22	34·2[illegible]797	36·303378	38·505214	40·864309	43·392289	49·0057
23	36·617888	38·93703	41·430475	44·111846	46·995826	53·4361
24	39·082604	41·689196	44·501999	47·537998	50·815576	58·1766
25	41·645908	44·56521	47·727099	51·152588	54·86451	63·2490
26	44·311745	47·570645	51·113454	54·96598	59·156381	68·2490
27	47·084214	50·711324	54·669126	58·989109	63·705763	74·4838
28	49·967583	53·993333	58·402583	63·23351	68·528109	80·6976
29	52·966286	57·423033	62·322712	67·711353	73·639796	87·3465
30	56·084938	61·007067	66·438847	72·435478	79·058183	94·4607
31	59·328335	64·752388	70·76079	77·419429	84·801674	102·073
32	62·701469	68·666245	75·298829	82·677498	90·889775	110·218
33	66·209527	72·756226	80·063771	88·22476	97·343161	118·933
34	69·857908	77·030256	85·066959	94·077122	104·183751	128·258
35	73·652225	81·496618	90·320307	100·251363	111·434776	138·236
36	77·598314	86·163966	95·836323	106·765188	119·120863	148·913
37	81·702246	91·041344	101·628139	113·637274	127·268114	160·337
38	85·970336	96·138205	107·709546	120·887324	135·904201	172·561
39	90·40915	101·464424	114·095025	128·536127	145·058453	185·640
40	95·025516	107·030323	120·799774	136·605614	154·761961	199·635

TABLE IV.

SHEWING THE PRESENT WORTH OF £1 OR $1 ANNUITY, FOR ANY NUMBER OF YEARS FROM 1 TO 40.

ys.	4 per cent.	4½ per ct.	5 per cent.	5½ per ct.	6 per cent.	7 per ct.
1	0,96154	0,95694	0,95238	0,94786	0,94339	0,9345
2	1,88609	1,87267	1,85941	1.8463	1,83339	1,8080
3	2,77509	2,74896	2,72325	2,6979	2,67301	2,6243
4	3,62989	3,58752	3,54595	3,49862	3,4651	3,3872
5	4,45182	4,38997	4.32948	4.25759	4,21236	4,1001
6	5,24214	5,15787	5,07569	4,97699	4,92732	4,7665
7	6,00205	5,8927	5,78637	5,65888	5,58238	5,3892
8	6,73274	6,59589	6,46321	6,30522	6,20979	5,9712
9	7,43533	7,26879	7,10782	6,91786	6,80169	6,5152
10	8,11089	7,91272	7,72173	7.49856	7.36008	7.0235
11	8.76048	8,52892	8,30641	8,04898	7,88687	7,4986
12	9,38507	9,11858	8.86325	8,5707	8,38384	7,9426
13	9,98565	9,68285	9,39357	9,06522	8,85268	8,3576
14	10,56312	10,22282	9,89864	9,53395	9,29498	8,7454
15	11,11839	10,73954	10,37966	9,97824	9,71225	9,1079
16	11,65229	11,23401	10,83777	10,39936	10,10589	9,4466
17	12,16567	11,70719	11,27407	10,79852	10,47726	9,7632
18	12,65929	12,15099	11,68958	11,17687	10,8276	10,059
19	13,13394	12,59329	12,08532	11,53549	11,15811	10,335
20	13,59032	13.00793	12,46221	11,87541	11,46992	10,594
21	14,02916	13,40472	12,82115	12,1976	11,76407	10,835
22	14,45111	13,79442	13,163	12,50299	12,04158	1[illegible],061
23	14,85684	14,14777	13,48807	12,79245	12,30338	11,272
24	15,24696	14,49548	13,79864	13.06682	12,55035	11,469
25	15,62208	14,82821	14.09394	13.3688	12,78335	11,653
26	15,98277	15,14661	14,37518	13,57338	13,00316	11,825
27	16,32959	15,4513	14,64303	13,80702	13,21053	11,986
28	16,66306	15,74287	14,89813	14,02848	13,40616	12,137
29	16,98371	16,02189	15,14107	14,23838	13,59072	12,277
30	17,20202	16,28889	15.37245	14,43733	13.76483	12,409
31	17,58849	16,54439	15,59281	14,6259	13,92908	12,531
32	17,87355	16,78889	15,80268	14,80463	14,08398	12,646
33	18,14764	17,02286	16.00255	14,97404	14,22917	12,753
34	18,4112	17,24676	16,1929	15,13461	14,36613	12,854
35	18,66461	17,46101	16.37419	15,2868	14,49533	12,947
36	18,90828	17,66604	16,54685	15.43105	14,61722	13,035
37	19,14258	17,86224	16,71129	15.56779	14,73211	13,117
38	19,36787	18,04999	16.86789	15,6974	14,84048	13,193
39	19,58448	18,22965	17,01704	15,82024	14,9427	13,264
40	19,79277	18,40158	17,15909	15,93667	15.03913	13,331

TABLE V.

THE ANNUITY WHICH £1 OR $1 WILL PURCHASE FOR ANY NUMBER OF YEARS TO COME, FROM 1 TO 40.

ys.	4 per ct.	4½ per ct.	5 per ct.	5½ per ct.	6 per ct.	7 per ct.
1	1,04	1,045	1,05	1,055	1,06	1,07
2	,5302	,534	,5378	,54162	,54544	,55309
3	,36035	,36377	,36721	,37065	,37411	,38105
4	,27549	,27874	,28201	,28582	,28859	,29523
5	,22463	,22779	,23097	,23487	,23739	,24389
6	,19076	,19388	,19702	,20092	,20336	,20977
7	,16661	,1697	,17282	,17671	,17913	,18556
8	,14853	,15161	,15473	,15859	,16103	,16747
9	,13449	,13757	,14069	,14455	,14702	,15349
10	,12329	,12638	,1295	,13334	,13587	,14238
11	,11415	,11725	,12039	,12424	,12679	,13336
12	,10655	,10967	,11282	,11667	,11927	,12592
13	,10014	,10327	,10645	,11031	,11296	,11965
14	,09467	,09782	,10102	,10489	,10758	,11435
15	,08994	,09311	,09624	,10022	,10296	,10979
16	,08582	,08901	,09227	,0962	,09895	,10586
17	,0822	,08542	,0887	,0926	,09544	,10243
18	,07899	,08224	,08555	,08947	,09235	,09941
19	,07614	,07941	,08274	,08699	,08962	,09676
20	,07359	,07688	,08024	,08427	,08718	,09439
21	,07128	,0746	,078	,08198	,085	,09230
22	,0692	,07254	,07597	,07998	,08303	,09041
23	,06731	,07068	,07414	,07825	,08128	,08880
24	,06559	,06899	,07247	,07653	,07968	,08719
25	,06401	,06744	,07095	,07503	,07825	,08581
26	,06257	,06602	,06956	,07367	,0769	,08457
27	,06124	,06472	,06329	,07242	,0757	,08343
28	,06001	,06352	,06712	,07128	,07459	,08239
29	,05888	,06241	,06604	,07023	,07358	,08145
30	,05783	,06139	,06505	,06926	,07272	,08058
31	,05685	,06044	,06413	,06837	,07179	,07980
32	,05595	,05956	,06328	,06754	,071	,07908
33	,0551	,05874	,06249	,06678	,07027	,07841
34	,05431	,05798	,06175	,06607	,06959	,07779
35	,05358	,05727	,06107	,06541	,06899	,07724
36	,05289	,0566	,06043	,0648	,06839	,07672
37	,05224	,05598	,05984	,06425	,06785	,07624
38	,05163	,0554	,05928	,0637	,06735	,07579
39	,05106	,05485	,05876	,06321	,06689	,07539
40	,05052	,05434	,05828	,06274	,06646	,07501

TABLE VI.

VALUE OF AN ANNUITY OF £1 OR $1, AT DIFFERENT RATES PER CENT. PAYABLE YEARLY, HALF YEARLY, QUARTERLY, DAILY OR MOMENTLY, FOR EVER.

Rate per cent.	Perpetuity payable yearly.	Half yearly.	Quarterly.	Daily.	Perpetuity payable momently.
3	33,3333	33,6022	33,6927	33,8238	33,8308
3½	28,5714	28,7886	28,9083	29,0841	29,2908
4	25,0000	25,2525	25,3807	25,4858	25,4992
4½	22,2222	22,4699	22,5938	22,7175	22,7244
5	20,0000	20,2429	20,3583	20,4763	20,4959
5½	18,1818	18,4298	18,5529	18,6630	18,6812
6	16,6666	16,9147	17,0380	17,0554	17,1635
6½	15,5555	15,6299	15,7550	15,7743	15,7902
7	14,2857	14,5349	14.6575	14,7694	14,7800

Note. This Table is calculated by the Rule, Case I. of "Purchasing Annuities for ever."

TABLE VII.

THE VALUE OF AN ANNUITY OF £1 OR $1, FOR A SINGLE LIFE.

Age.	3 per cent.	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.
10	19,87	18,27	16,88	15,67	14,60	12,80
12	19,60	18,05	16,69	15,51	14,47	12,70
15	19,19	17,71	16,41	15,27	14,27	12,55
18	18,76	17,33	16,10	15,01	14,05	12,48
20	18,46	17,09	15,89	14,83	13,89	12,30
22	18,15	16,83	15,67	14,64	13,72	12,15
25	17,66	16,42	15,31	14,34	13,46	12,00
28	17,16	15,98	14,94	14,02	13,18	11,75
30	16,80	15,68	14,68	13,79	12,99	11,60
33	16,25	15,21	14,27	13,43	12,67	11,35
35	15,86	14,89	13,98	13,17	12,46	11,15
38	15,29	14,34	13,52	12,77	12,09	10,90
40	14,84	13,98	13,20	12,48	11,83	10,70
43	14,19	13,40	12,68	12,02	11,43	10,35
45	13,73	12,99	12,30	11,70	11,14	10,10
48	13,01	12,36	11,74	11,19	10,68	9,75
50	12,51	11,92	11,34	10,82	10,35	9,45
53	11,73	11,20	10,70	10,24	9,82	9,00
55	11,18	10,69	10,24	9,82	9,44	8,70
58	10,32	9,91	9,52	9,16	8,83	8,20
60	9,73	9,36	9,01	9,69	8,39	7,80
63	8,79	8,49	8,20	7,94	7,68	7,20
65	8,13	7,88	7,63	7,39	7,18	6,75
68	7,10	6,91	6,75	6,54	6,36	6,00
70	6,38	6,22	6,06	5,92	5,77	5,50
73	5,25	5,14	5,02	4,92	4,82	4,60
75	4,45	4,38	4,29	4,22	4,14	4,00
77	3,63	3,57	3,52	3,47	3,41	3,30
79	2,78	2,74	2,70	2,67	2,64	2,25
80	2,34	2,31	2,28	2,26	2,23	2,15

This Table is formed by ascertaining from Bills of mortality the *mean length* of the lives of persons of a certain age, and then calculating the value of the annuity for the number of years they may thus be expected to live. This *mean* is called the Expectation of Life, at any given age, which is exhibited in the following table.

TABLE VIII.

EXPECTATION OF LIFE AT SEVERAL AGES.

Age.	Expec.	Age.	Expec.	Age.	Expec.	Age.	Expec.	Age.	Expec.
1	29,80	20	33,62	42	21,65	62	11,68	82	3,31
2	37,92	22	32,46	45	20,10	65	10,20	85	2,64
3	40,18	25	30,83	47	19,07	67	9,20	87	2,40
4	41,32	27	29,74	50	17,55	70	7,80	90	2,12
6	41,77	30	28,12	52	16,55	72	7,00	92	1,50
8	41,34	32	27,04	55	15,10	75	5,84	93	1,14
10	40,25	35	25,34	57	14,12	77	5,15	94	0,90
12	38,57	37	24,30	59	13,15	79	4,40	95	0,66
15	36,64	40	22,82	60	12,66	80	4,00	96	0,50
18	34,77								

CIRCULATING DECIMALS

ARE produced from Vulgar Fractions, whose denominators do not measure their numerators, and are distinguished by the continual repetition of the same figures.

1. The circulating figures are called *repetends*; and, if one figure only repeats, it is called a *single repetend*: As ·1111, &c. ·6666, &c.

2. A *compound repetend* has the same figures circulating alternately: As ·010101, &c. ·379379379, &c.

3. If other figures arise before those which circulate, the decimal is called a *mixed repetend*; thus, ·375555, &c. is a *mixed single repetend*, and ·378123123, &c. a *mixed compound repetend*.

4. A single repetend is expressed by writing only the circulating figure with a point over it; thus, ·1111, &c. is denoted by $\cdot\dot{1}$, and ·6666, &c. by $\cdot\dot{6}$.

5. Compound repetends are distinguished by putting a point over the first and last repeating figures; thus, ·010101, &c. is written $\cdot\dot{0}\dot{1}$, and ·379379379, &c thus, $\cdot\dot{3}7\dot{9}$.

6. *Similar circulating decimals* are such as consist of the same number of figures, and begin at the same place, either before or after the decimal point; thus, $\cdot\dot{3}$ and $\cdot\dot{5}$ are similar circulates; as are also $3\cdot\dot{5}\dot{4}$ and $7\cdot\dot{3}\dot{6}$, &c.

7. *Dissimilar repetends* consist of an unequal number of figures, and begin at different places.

8. *Similar and conterminous circulates* are such as begin and end at the same place; as $47\cdot3\dot{4}57\dot{6}$, $9\cdot7\dot{3}52\dot{8}$ and $\cdot0\dot{5}46\dot{3}$, &c.

REDUCTION OF CIRCULATING DECIMALS.

CASE I.

To reduce a simple Repetend to its equivalent Vulgar Fraction.

RULE.*

1. Make the given decimal the numerator, and let the denominator be a number, consisting of so many nines as there are recurring places in the repetend.

2. If there be integral figures in the circulate, so many cyphers must be annexed to the numerator as the highest place of the repetend is distant from the decimal point.

EXAMPLES.

1. Required the least vulgar fractions equal to $\cdot\dot{3}$ and $\cdot\dot{3}2\dot{4}$.

$\cdot\dot{3}=\frac{3}{9}=\frac{1}{3}$; and $\cdot\dot{3}2\dot{4}=\frac{324}{999}=\frac{12}{37}$. Ans. $\frac{1}{3}$ and $\frac{12}{37}$.

2. Reduce $\cdot\dot{7}$ to its equivalent vulgar fraction. Ans. $\frac{7}{9}$.

3. Reduce $\dot{2}.3\dot{7}$ to its equivalent vulgar fraction. Ans. $\frac{2370}{999}$.

4. Required the least vulgar fraction equal to $\cdot\dot{3}8461\dot{5}$. Ans. $\frac{5}{13}$.

CASE II.

To reduce a mixed Repetend to its equivalent Vulgar Fraction.

RULE.†

1. To so many nines as there are figures in the repetend, annex so many cyphers as there are finite places, (that is, as there are decimal places before the repetend) for a denominator.

* If unity, with cyphers annexed, be divided by 9 *ad infinitum*, the quotient will be 1 continually; that is, if $\frac{1}{9}$ be reduced to a decimal, it will produce the circulate $\cdot\dot{1}$, and since $\cdot\dot{1}$ is the decimal equivalent to $\frac{1}{9}$, $\cdot\dot{2}$ will $=\frac{2}{9}$, $\dot{3}=\frac{3}{9}$, and so on till $\cdot\dot{9}=\frac{9}{9}=1$. Therefore every single repetend is equal to a vulgar fraction, whose numerator is the repeating figure and denominator 9.

Again, $\frac{1}{99}$ or $\frac{1}{999}$ being reduced to decimals, make $\cdot010101$, &c. and $\cdot001001001$, &c. *ad infinitum*$=\cdot\dot{0}\dot{1}$ and $\cdot\dot{0}0\dot{1}$; that is, $\frac{1}{99}=\cdot\dot{0}\dot{1}$, and $\frac{1}{999}=\cdot\dot{0}0\dot{1}$, consequently $\frac{2}{99}=\cdot\dot{0}\dot{2}$, $\frac{3}{99}=\cdot\dot{0}\dot{3}$, &c. and $\frac{2}{999}=\cdot\dot{0}0\dot{2}$, $\frac{3}{999}=\cdot\dot{0}0\dot{3}$, &c. and the same will hold universally.

† In like manner for a mixed circulate; consider it as divisible into its finite and circulating parts, and the same principle will be seen to run through them also; thus the mixed circulate $\cdot1\dot{3}$ is divisible into the finite decimal $\cdot1$, and the repetend $\cdot0\dot{3}$: but $\cdot1=\frac{1}{10}$, and $\cdot0\dot{3}$ would be equal to $\frac{3}{9}$ provided the circulation began immediately after the place of units; but as it begins after the place of tenths, it is $\frac{3}{9}$ of $\frac{1}{10}=\frac{3}{90}$, and so the vulgar fraction$=\cdot1\dot{3}$ is $\frac{1}{10}+\frac{3}{90}=\frac{9}{90}+\frac{3}{90}=\frac{12}{90}$, and is the same as by the rule.

2. Multiply the nines in the said denominator by the finite part, and add the repeating decimals to the product for the numerator.

3. If the repetend begins in some integral place, the finite value of the circulating part must be added to the finite part.

Examples.

1. What is the vulgar fraction equivalent to ·153?

There being 1 figure in the repetend, and 2 finite places, I annex 2 cyphers to 9 for a denominator, viz. 900; then I multiply the 9 in the denominator by the two figures in the finite part, and add the repeating figure for a numerator; thus, $9\times15+3=138$ numerator.

Therefore, $\cdot153=\frac{138}{900}=\frac{23}{150}$ the Ans.

2. What is the least vulgar fraction equal to ·4123? Ans. $\frac{4119}{9990}$.

3. Required the finite number equivalent to 45·78? Ans. $45\frac{78}{99}$.

CASE III.

To make any number of dissimilar repetends similar and conterminous; that is, of an equal number of places.

Rule.*

Change them into other repetends, which shall each consist of so many figures, as the least common multiple of the sums of the several numbers of places, found in all the repetends, contains units.

Examples.

1. Make 6·317; 3·45; 52·3; 191·03; ·057; 5·3 and 1·359 similar and conterminous.

Here, in the first repetend, there are three places, in the second, one, in the third, none, in the fourth, two, in the fifth, three, in the sixth, one, and in the seventh, one.

Now find the least common multiple of these several sums, thus:

3, 1, 2, 3, 1, 1
1, 1, 2, 1, 1, 1

and $2\times3=6$ units; therefore, the similar and conterminous repetends must contain 6 places.†

* Any given repetend whatever, whether single, compound, pure, or mixed, may be transformed into another repetend, which shall consist of an equal or greater number of figures at pleasure; thus, ·3 may be transformed into ·33, or ·333, &c. also ·79=·7979=·797, and so on.

† The learner may observe that the similar and conterminous repetends begin just so far from unity, as is the farthest among the dissimilar repetends; and it is so in all cases.

Dissimilar made similar and conterminous.

$$6\cdot\dot{3}1\dot{7} = 6\cdot31\dot{7}3173\dot{1}$$

$$3\cdot4\dot{5} = 3\cdot45\dot{5}5555\dot{5}$$

$$52\cdot3 = 52\cdot3\dot{0}00000\dot{0}$$

$$191\cdot\dot{0}\dot{3} = 191\cdot03\dot{0}3030\dot{3}$$

$$\cdot\dot{0}5\dot{7} = \cdot057\dot{0}570\dot{5}$$

$$5\cdot\dot{3} = 5\cdot33\dot{3}3333\dot{3}$$

$$1\cdot35\dot{9} = 1\cdot359\dot{9}999\dot{9}$$

2. Make $\cdot\dot{5}3\dot{1}$, $\cdot73\dot{4}\dot{8}$, $\cdot\dot{0}\dot{7}$ and $\cdot0\dot{5}0\dot{3}$ similar and conterminous.

CASE IV.

To find whether the decimal fraction, equal to a given vulgar one, be finite or infinite, and how many places the repetend will consist of.

Rule.*

1. Reduce the given fraction to its least terms, and divide the denominator by 2, 5 or 10, as often as possible.

2. Divide 9999, &c. by the former result, till nothing remain, and the number of 9s used will show the number of places in the repetend; which will begin after so many places of figures as there were 10s, 2s, or 5s, divided by.

If the whole denominator vanish in dividing by 2, 5 or 10, the decimal will be finite, and will consist of so many places as you perform divisions.

* In dividing 1·000, &c. by any prime number whatever, except 2 or 5, the figures in the quotient will begin to repeat over again as soon as the remainder is 1: and since 999, &c. is less than 1000, &c. by 1, therefore 999, &c. divided by any number whatever, will, when the repeating figures are at their period, leave 0 for a remainder.

Now, whatever number of repeating figures we have, when the dividend is 1, there will be exactly the same number, when the dividend is any other number whatever.

Thus, let ·390539053905, &c. be a circulate, whose repeating part is 3905. Now, every repetend (3905,) being equally multiplied, must give the same product: For although these products will consist of more places, yet the overplus in each, being alike, will be carried to the next, by which means, each product will be equally increased, and consequently every four places will continue alike. And the same will hold for any other number.

Now from hence it appears that the dividend may be altered at pleasure, and the number of places in the repetend will still be the same; thus, $\frac{1}{11} = \cdot\dot{0}\dot{9}$; and $\frac{4}{11}$ or $\frac{1}{11} \times 4 = \cdot\dot{3}\dot{6}$, whence the number of places in each are alike.

EXAMPLES.

1. Required to find whether the decimal equal to $\frac{475}{2800}$ be finite or infinite, and if infinite, how many places that repetend will consist of.

$$\text{First } 25\Big)\frac{475}{2800} = \frac{19}{112}\quad 2\Big)112 \overset{(2)}{=} 56 \overset{(2)}{=} 28 \overset{(2)}{=} 14 = 7.$$

Then, $\dfrac{7)999999}{142857}$; therefore, because the denominator 112 did not vanish in dividing by 2, the decimal is infinite, and, as six 9s were used, the circulate consists of 6 places, beginning at the fifth place, because four 2s were used in dividing.

2. Let $\frac{2}{11}$ be the fraction proposed.

3. Let $\frac{3}{5}$ be the fraction proposed.

ADDITION OF CIRCULATING DECIMALS.

RULE.

1. Make the repetends *similar and conterminous*, and find their sum as in common addition.

2. Divide this sum (of the repetends only) by so many nines as there are places in the repetend, and the remainder is the repetend of their sum; which must be set under the figures added, with cyphers on the left hand, when it has not so many places as the repetends.

3. Carry the quotient of this division to the next column, and proceed with the rest as infinite decimals.

EXAMPLES.

1. Let $5\cdot\dot{3}+59\cdot4\dot{3}5\dot{6}+397\cdot\dot{6}+519+\cdot\dot{3}\dot{9}+217\cdot\dot{5}$ be added together.

$$\begin{array}{rcr}
5\cdot\dot{3} & = & 5\cdot3\dot{3}3333\dot{3} \\
59\cdot4\dot{3}5\dot{6} & = & 59\cdot4\dot{3}5635\dot{6} \\
397\cdot\dot{6} & = & 397\cdot3\dot{6}6666\dot{6} \\
519\cdot & = & 519\cdot0\dot{0}0000\dot{0} \\
\cdot\dot{3}\dot{9} & = & \cdot3\dot{9}3939\dot{3} \\
217\cdot\dot{5} & = & 217\cdot5\dot{5}5555\dot{5} \\
\hline
 & & 1199\cdot3851303 \\
\hline
\end{array}$$

$1199\cdot3\dot{8}5130\dot{5}$ the sum.

In this question, the sum of the repetends is 2851303, which divided by 999999, gives 2 to carry to the next column 5,3,0, &c. and the remainder is 851305.

2. Let $3275{\cdot}319+36{\cdot}4\dot{5}+123{\cdot}1\dot{9}+5{\cdot}317\dot{3}+112{\cdot}351\dot{3}+11{\cdot}13\dot{1}$ $+{\cdot}12\dot{5}+29{\cdot}1005\dot{3}$ be added together. Ans. $3593{\cdot}0004\dot{2}$.

SUBTRACTION OF CIRCULATING DECIMALS.

Rule.

Make the repetends *similar* and *conterminous*, and subtract as usual, observing, that if the repetend of the number to be subtracted be greater than the repetend of the number it is to be taken from, then the right hand of the remainder must be less by unity than it would be if the expressions were finite.

Examples.

1. From $57{\cdot}\dot{0}\dot{3}$ take $29{\cdot}7\dot{3}58\dot{7}$

$$57{\cdot}\dot{0}\dot{3} \quad =57{\cdot}0\dot{3}03\dot{0}$$
$$23{\cdot}7\dot{3}58\dot{7}=29{\cdot}7\dot{3}58\dot{7}$$

$$27{\cdot}2\dot{9}44\dot{2} \text{ the difference.}$$

2. From $325{\cdot}1\dot{7}$ take $137{\cdot}581\dot{9}$. Ans. $187{\cdot}595\dot{7}$.

MULTIPLICATION OF CIRCULATING DECIMALS.

Rule.

1. Turn both the terms into their equivalent vulgar fractions, and find the product of those fractions as usual.
2. Turn the vulgar fraction expressing the product, into an equivalent decimal one, and it will be the product required.

Examples.

1. Multiply ${\cdot}\dot{5}\dot{4}$ by ${\cdot}1\dot{5}$. ${\cdot}\dot{5}\dot{4}=\frac{54}{99}=\frac{6}{11}$ and ${\cdot}1\dot{5}=\frac{14}{90}=\frac{7}{45}$

$\frac{6}{11}\times\frac{7}{45}=\frac{42}{495}={\cdot}0\dot{8}\dot{4}$ the product.

2. Multiply $378{\cdot}\dot{5}$ by $23{\cdot}\dot{6}$. Ans. $8959{\cdot}\dot{1}4\dot{8}$.

DIVISION OF CIRCULATING DECIMALS.

Rule.

1. Change both the divisor and dividend into their equivalent vulgar fractions, and find their quotient as usual.
2. Turn the vulgar fraction expressing the quotient, into its equivalent decimal, and it will be the quotient required.

EXAMPLES.

1. Divide $\dot{5}\dot{4}$ by $\cdot 1\dot{5}$.

$\cdot\dot{5}\dot{4}=\frac{54}{99}=\frac{6}{11}$ and $\cdot 1\dot{5}=\frac{14}{90}=\frac{7}{45}$

$\frac{6}{11}\div\frac{7}{45}=\frac{6}{11}\times\frac{45}{7}=\frac{270}{77}=3\frac{39}{77}=3\cdot\dot{5}0649\dot{3}$ the quotient.

2. Divide $345\cdot\dot{8}$ by $\dot{6}$. Ans. $51\dot{8}\cdot8\dot{3}$.

ALLIGATION

IS the method of mixing two or more simples of different qualities, so that the composition may be of a mean or middle quality; It consists of two kinds, viz. Alligation Medial, and Alligation Alternate.

ALLIGATION MEDIAL

Is, when the quantities and prices of several things are given, to find the mean price of the mixture compounded of those things.

RULE.

As the sum of the quantities, or the whole composition, is to their total value; so is any part of the composition to its mean price or value.

EXAMPLES.

1. A Tobacconist would mix 60℔ of tobacco, at 6d. per ℔ with 50℔ at 1s. 40℔ at 1s. 6d. and 30℔ at 2s. per ℔: What is 1℔ of this mixture worth?

	℔		s.	d.		£	s.
	60	at	0	6	is	1	10
	50	—	1	0	—	2	10
	40	—	1	6	—	3	0
	30	—	2	0	—	3	0
Sum of the simples,	180	Total value				10	0

℔ £ ℔
As 180 : 10 : 1
1
—
10
20
—
180)200(1s.
180
—
20
12
—
180)240($1\frac{1}{3}$d. s. d.
180 Ans. 1 $1\frac{1}{3}$ pr. ℔
—
60

2. A farmer would mix 20 bushels of wheat at $1 per bushel, 16 bushels of rye at 75c. per bushel, 12 bushels of barley at 50c.

per bushel, and 8 bushels of oats at 40c. per bushel: What is the value of one bushel of this mixture? Ans. 73c. $5\frac{5}{7}$m.

3. A wine merchant mixes 12 gallons of wine, at 75c. per gallon, with 24 gallons at 90c. and 16 gallons at \$1 10c.: What is a gallon of this composition worth? Ans. 92c. 6m.

4. A goldsmith melted together 8oz. of gold of 22 carats fine, 1lb 8oz. of 21 carats fine, and 10oz. of 18 carats fine: Pray what is the quality, or fineness of the composition?

$$\frac{\overline{8\times22}+\overline{20\times21}+\overline{10\times18}}{8+20+10}=20\tfrac{8}{19}\text{ carats fine, Ans.}$$

5. A refiner melts 5lb of gold of 20 carats fine with 8lb of 18 carats fine: How much alloy must be put to it, to make it 22 carats fine?

$$22-\overline{\overline{5\times20}+\overline{8\times18}\div\overline{5+8}}=3\tfrac{3}{13}$$

Answer: It is not fine enough by $3\frac{3}{13}$ carats, so that no alloy must be added, but more gold.

*ALLIGATION ALTERNATE**

Is the method of finding what quantity of each of the ingredients, whose rates are given, will compose a mixture of a given rate: So that it is the reverse of Alligation Medial, and may be proved by it.

CASE I.

The whole work of this case consists in linking the extremes truly together and taking the differences between them and the mean price, which differences are the quantities sought.

RULE.

1. Place the several prices of the simples, being reduced to one denomination, in a column under each other, the least uppermost, and so gradually downward, as they increase with a line of

* *Demon.* By connecting the less rate with the greater, and placing the difference between them and the mean rate alternately, or one after the other in turn, the quantities resulting are such, that there is precisely as much gained by one quantity as is lost by the other, and therefore the gain and loss, upon the whole, are equal, and are exactly the proposed rate.

In like manner, let the number of simples be what it may, and with how many soever, each one is linked, since it is always a less with a greater than the mean price, there will be an equal balance of loss and gain between every two, and consequently an equal balance on the whole.

It is obvious from the rule, that questions of this sort admit of a great variety of answers; for having found one answer, we may find as many more as we please, by only multiplying or dividing each of the quantities found, by 2, 3, 4, &c. the reason of which is evident; for if two quantities of two simples make a balance of loss and gain with respect to the mean price, so must also the double or triple, the half or third part, or any other ratio of these quantities, and so on *ad infinitum.*

If any one of the simples be of little or no value with respect to the rest, its rate is supposed to be nothing, as water mixed with wine, and alloy with gold and silver.

connection at the left hand, and the mean price at the left hand of all.

2. Connect, with a continued line, the price of each simple, or ingredient, which is less than that of the compound, with one or any number of those which are greater than the compound, and each greater rate or price with one or any number of the less.

3. Place the difference, between the mean price (or mixture rate) and that of each of the simples, opposite to the rates with which they are connected.

4. Then, if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

EXAMPLES.

1. A merchant has spices, some at 1s. 6d. per lb, some at 2s. some at 4s. and some at 5s. per lb: How much of each sort must he mix that he may sell the mixture at 3s. 4d. per lb?

Mean rate 40d.	d.	lb	s. d.	
	18	20 at	1 6	per lb
	24	8 —	2 0	
	48	16 —	4 0	
	60	22 —	5 0	

40d.	d.	lb	s. d.	
	18	8 at	1 6	per lb
	24	20 —	2 0	
	48	22 —	4 0	
	60	16 —	5 0	

d. 40	d.		lb	s. d.	
	18	20+ 8	28 at	1 6	per lb
	24	8	8 —	2 0	
	48	16+22	38 —	4 0	
	60	22	22 —	5 0	

d. 40	d.		lb	s. d.	
	18	20	20 at	1 6	per lb
	24	8+20	28 —	2 0	
	48	16	16 —	4 0	
	60	22+16	38 —	5 0	

40d.					
	18	8	8 at	1s. 6d.	per lb
	24	8+20	28	2	
	48	22+16	38	4	
	60	16	16	5	

40d.					
	18	8+20	28 at	1s. 6d.	per lb
	24	20	20	2	
	48	22	22	4	
	60	22+16	38	5	

40d.					
	18	20+ 8	28 at	1 6	per lb
	24	8+20	28 —	2 0	
	48	16+22	38 —	4 0	
	60	22+16	38 —	5 0	

Note. These seven answers arise from as many various ways of linking the rates of the ingredients together.

2. *A merchant has Canary wine, at 3s. per gallon, Sherry, at 2s. 1d. and Claret at 1s. 5d. per gallon: How much of each sort must he take, to sell it at 2s. 4d. per gallon?

Mean rate 28d.					
	36	3+11	14 at	3 0	per gallon.
	25	8	8	2 1	
	17	8	8	1 5	

* Note, the 2d and 3d questions admit but of one way of linking, and so but of one answer; yet all numbers in the same proportion between themselves, as the numbers which compose the answer, will likewise satisfy the condition of the question.

3. How much barley at 40c. rye at 60c. and wheat at 80c. per bushel, must be mixed together, that the compound may be worth 62½c. per bushel?

Ans. 17½ bushels of barley, 17½ of rye, and 25 of wheat.

4. A goldsmith would mix gold of 19 carats fine, with some of 16, 18, 23 and 24 carats fine, so that the compound may be 21 carats fine: What quantity of each must he take?

Ans. 5oz. of 16 carats fine, 5oz. of 18, 5oz. of 19, 10oz. of 23, and 10oz. of 24 carats fine.

5. It is required to mix several sorts of wine, at 60c. 90c. and $1 15c. per gallon, with water, that the mixture may be worth 75c. per gallon: Of how much of each sort must the composition consist?

Ans. 40galls. of water, 15galls. of wine, at 60c. 15galls. do. at 90c. and 75galls. do. at $1 15c.

CASE II.

When the rates of all the ingredients, the quantity of but one of them, and the mean rate of the whole mixture are given, to find the several quantities of the rest, in proportion to the quantity given.

Rule.

Take the differences between each price, and the mean rate, and place them alternately, as in Case I. Then, as the difference standing against that simple, whose quantity is given, is to that quantity, so is each of the other differences, severally, to the several quantities required.

Examples.

1. A merchant has 40lb of tea, at 6s. per lb, which he would mix with some at 5s. 8d. some at 5s. 2d. and some at 4s. 6d.: How much of each sort must he take, to mix with the 40lb, that he may sell the mixture at 5s. 5d. per lb?

			lb
65	54	7+3	10
	62	3+7	10
	68	3+11	14
	72	11+3	14 stands against the given quantity.

	lb		lb	s. d.	
As 14 : 40 ::	10	:	$28\frac{8}{14}$	at 4 6	per lb
	10	:	$28\frac{8}{14}$	— 5 2	
	14	:	40	— 5 8	

2. A farmer being determined to mix 20 bushels of oats, at 60c. per bushel, with barley, at 75c. rye, at $1, and wheat, at $1 25c. per bushel; I demand the quantity of each, which must be mixed with the 20 bushels of oats, that the whole quantity may be worth 90c. per bushel?

Ans. 70 of barley, 60 of rye, and 30 of wheat, (or 20 of each.)

3. How much gold of 16, 20 and 24 carats fine, and how much alloy, must be mixed with 10oz. of 18 carats fine, that the composition may be 22 carats fine.

Ans. 10oz. of 16 carats fine, 10 of 20, 170 of 24, and 10 of alloy.

ALTERNATION TOTAL.*

CASE III.

When the rates of the several ingredients, the quantity to be compounded, and the mean rate of the whole mixture are given, to find how much of each sort will make up the quantity.

RULE.

Place the differences between the mean rate, and the several prices alternately, as in Case 1; then, as the sum of the quantities, or differences thus determined, is to the given quantity, or whole composition; so is the difference of each rate, to the required quantity of each rate.

EXAMPLES.

1. Suppose I have 4 sorts of currants, at 8d. 12d. 18d. and 22d. per ℔; the worst will not sell, and the best are too dear; I therefore conclude to mix 120℔ and so much of each sort as to sell them at 16d. per ℔; how much of each sort must I take?

$$\begin{array}{rl} & \begin{array}{cc} \text{d.} & \text{℔} \end{array} \\ 16\text{d.} & \left\{\begin{array}{cc} 8 & 6 \\ 12 & 2 \\ 18 & 4 \\ 22 & 8 \end{array}\right. \\ & \text{Sum}=20 \end{array} \quad \text{As } \overset{\text{℔}}{20} : \overset{\text{℔}}{120} :: \left\{\begin{array}{l} 6 : 36 \text{ at } 8\text{d.} \\ 2 : 12 - 12\text{d.} \\ 4 : 24 - 18\text{d.} \\ 8 : 48 - 22\text{d.} \end{array}\right\} \text{per ℔}$$

$$\qquad\qquad\qquad\qquad 120$$

2. A goldsmith has several sorts of gold; viz. of 15, 17, 20 and 22 carats fine, and would melt together, of all these sorts, so much as may make a mass of 40oz. 18 carats fine; how much of each sort is required?

Ans. 16oz. 15 carats fine, 8oz. 17, 4oz. 20, and 12oz. of 22 carats fine.

* To this Case belongs that curious question concerning king Hiero's crown. Hiero, king of Syracuse, gave orders for a crown to be made entirely of pure gold; but suspecting the workmen had debased it, by mixing with it silver or copper, he recommended the discovery of the fraud to the famous Archimedes, and desired to know the exact quantity of alloy in the crown.

Archimedes, in order to detect the imposition, procured two other masses, one of pure gold, and the other of silver, or copper, and each of the same weight with the former; and by putting each separately into a vessel *full* of water, the quantity of water expelled by them, determined their specific bulks; from which, and their given weights, it is easier to determine the quantities of gold and alloy in the crown by this case of Alligation, than by an Algebraic process.

Suppose the weight of each mass to have been 5lb. the weight of the water expelled by the alloy, 23oz. by the gold, 13oz. and by the crown 16oz. that is, that their specifick bulks were as 23, 13, and 16; then, what were the quantities of gold and alloy respectively in the crown?

Here, the rates of the simples are 23 and 13, and of the compound 16, whence,

$16\left\{\begin{array}{l} 13 \quad 7 \text{ of gold} \\ 23 \quad 3 \text{ of alloy} \end{array}\right\}$ And the sum of these is $7+3=10$, which should have been but 5, whence, by the rule,

$$10 : 5 :: \left\{\begin{array}{l} 7 : 3\tfrac{1}{2}\text{lb. of gold} \\ 3 : 1\tfrac{1}{2}\text{lb. of alloy} \end{array}\right\} \text{the Answer.}$$

3. A merchant would mix 4 sorts of wine, of several prices, viz. at 75c. \$1 25c. \$1 50c. and \$1 $62\frac{1}{2}$c. per gallon; of these he would have a mixture of 72 gallons, worth \$1 $37\frac{1}{2}$c. per gallon; what quantity of each sort must he have?

Ans. 8 at 75c. 16 at \$1 25c. 40 at \$1 50c. and 8 at \$1 $62\frac{1}{2}$c. Or, 16 at 75c. 8 at \$1 25c. 8 at \$1 50c. and 40 at \$1 $62\frac{1}{2}$c.

4. How many gallons of water of no value, must be mixed with wine, at 4s. per gallon, so as to fill a vessel of 80 gallons, that may be afforded at 2s. 9d. per gallon?

$$33\left\{\begin{matrix} 0 \\ 48 \end{matrix}\right.\begin{matrix} \text{Gal.} \\ 15 \\ 33 \\ \text{—} \\ \text{Sum } 48 \end{matrix} \qquad \begin{matrix} \text{Gal. Gal.} \\ \text{As } 48 : 80 :: \end{matrix} \quad \begin{matrix} \text{Gal.} \\ \left\{\begin{matrix} 15 : 25 \text{ gallons of water.} \\ 33 : 55 \text{ gallons of wine.} \end{matrix}\right\} \text{Ans.} \end{matrix}$$

CASE IV.*

When more than one of the simples are limited.

RULE.

Find, by Alligation Medial, what will be the rate of a mixture made of the given quantities of the limited simples only; then, consider this as the rate of a limited simple, whose quantity is the sum of the first given limited simples, from which, and the rates of the unlimited simples, by Case II. calculate the quantity.

EXAMPLES.

1. How much wine, at 80c. and at $87\frac{1}{2}$c. per gallon, must be mixed with 8 gallons at 75c. and 12 gallons at 90c. per gallon, that the mixture may be worth $82\frac{1}{2}$c. per gallon?

$$\text{Limited simples}\left\{\begin{matrix} 8 \text{ gallons, at } 75\text{c.} = \$\ 6 \\ 12 \text{ gallons, at } 90 \ = 10\ 80\text{c.} \end{matrix}\right\}$$

$$\begin{matrix} \text{—} & & \text{——} \\ 20 & & 16\ 80 \end{matrix}$$

$$\begin{matrix} & \text{Gal.} & & \$\ \text{c.} & & \text{Gal.} & & \text{c.} \\ \text{As} & 20 & : & 16\ 80 & :: & 1 & : & 84 \text{ per gallon.} \end{matrix}$$

Now, having found the rate of the limited simples, the question may stand thus: How much wine, at 80c. and $87\frac{1}{2}$c. per gallon, must be mixed with 20 gallons at 84c. per gallon, that the mixture may be worth $82\frac{1}{2}$c. per gallon?

$$82\tfrac{1}{2}\left\{\begin{matrix} 80 \\ 84 \\ 87\frac{1}{2} \end{matrix}\right. \quad \begin{matrix} 1\frac{1}{2}+5 \\ 2\frac{1}{2} \\ 2\frac{1}{2} \end{matrix} \left|\begin{matrix} 6\frac{1}{2} \text{ gallons, at } 80\text{c.} \\ 2\frac{1}{2} \text{ ———— } 84 \\ 2\frac{1}{2} \text{ ———— } 87\frac{1}{2} \end{matrix}\right.$$

$$\text{As } 2\tfrac{1}{2} : \left\{\begin{matrix} 6\frac{1}{2} \\ 2\frac{1}{2} \end{matrix}\right\} :: 20 : \left\{\begin{matrix} 52 \text{ gallons, at } 80\text{c. per gallon.} \\ 20 \text{ ———— } 87\frac{1}{2} \text{ ———— } \end{matrix}\right\} \text{Answer.}$$

* The three last Cases need no demonstration, as the 2d and 3d evidently result from the first, and the last from Alligation Medial, and the second Case in Alternate.

Proof.

52 gallons	at 80c.	=	$41	60c.
20	- - $87\frac{1}{2}$	=	17	50
8	- - 75	=	6	
12	- - 90	=	10	80
92	- - $82\frac{1}{2}$	=	75	90

2. How much gold, of 14 and 16 carats fine, must be mixed with 6oz. of 19, and 12 of 22 carats fine, that the composition may be 20 carats fine? Ans. $1\frac{8}{18}$oz. of each sort.

POSITION.

POSITION is a rule, which, by false or supposed numbers, taken at pleasure, discovers the true ones required. It is divided into two parts; *single* and *double*.

SINGLE POSITION.

Single Position teaches to resolve those questions, whose results are proportional to their suppositions: such are those which require the multiplication or division of the number sought by any proposed number; or when it is to be increased or diminished by itself a certain proposed number of times.

Rule.*

1. Take any number, and perform the same operations with it as are described to be performed in the question.
2. Then say, as the sum of the errours is to the given sum, so is the supposed number, to the true one required.

Proof. Add the several parts of the sum together, and if it agrees with the sum, it is right.

Examples.

1. A school master, being asked how many scholars he had, said, If I had as many more as I now have, three quarters as many, half as many, one fourth and one eighth as many, I should then have 435: Of what number did his school consist?

* The operations contained in the question being performed upon the answer or number to be found, will give the result contained in the question. The same operations, performed on any other number, will give a certain result. When the results are proportional to their supposed numbers, it is manifest that one result must be to the result in the question, as the *supposed number* is to the *true* one or answer. In any cases, when the results are not proportional to their suppositions, the answer cannot be found by this rule.

Suppose he had 80	As 290 : 435 :: 80	
As many = 80	80	
$\frac{3}{4}$ as many = 60	———	120
$\frac{1}{2}$ as many = 40	29|0)3480|0(120 Ans.	120
$\frac{1}{4}$ as many = 20	29	90
$\frac{1}{8}$ as many = 10	——	60
——	58	30
290	58	15
	—	——
	0	435 Proof.

2. A person lent his friend a sum of money unknown, to receive interest for the same at 6 per cent. per annum, simple interest, and at the end of 12 years, received for principal and interest $860: What was the sum lent? Ans. $500.

3. A, B and C joined their stocks, and gained $353 12½c. of which A took up a certain sum, B took up four times so much as A, and C, five times so much as B: What share of the gain had each?

Ans. $\begin{cases} \$14\ 12\tfrac{1}{2}c. & \text{A's share.} \\ 56\ 50 & \text{B's share.} \\ 282\ 50 & \text{C's share.} \end{cases}$

4. A, B, C and D spent 35s. at a reckoning, and, being a little dipped, they agreed that A should pay $\frac{2}{3}$, B $\frac{1}{2}$, C $\frac{1}{3}$, and D $\frac{1}{4}$: What did each pay in the above proportion?

Ans. $\begin{cases} & \text{s.} & \text{d.} \\ \text{A,} & 13 & 4 \\ \text{B,} & 10 & 0 \\ \text{C,} & 6 & 8 \\ \text{D,} & 5 & 0 \end{cases}$

5. A certain sum of money is to be divided between 5 men, in such a manner as that A shall have $\frac{1}{4}$, B $\frac{1}{5}$, C $\frac{1}{10}$, D $\frac{1}{20}$, and E the remainder, which is £40: What is the sum?

Suppose £200, then $\frac{1}{4}+\frac{1}{5}+\frac{1}{10}+\frac{1}{20}=120$.

200—120=80. As 80 : 40 :: 200 : 100 Ans.

6. A person, after spending $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had £26$\frac{2}{3}$ left: What had he at first? Ans. £160.

7. A and B, talking of their ages, B said his age was once and an half the age of A; C said his was twice and one tenth the age of both, and that the sum of their ages was 93: What was the age of each? Ans. A's 12, B's 18, and C's 63 years.

8. A vessel has 3 cocks, A, B and C; A can fill it in $\frac{1}{2}$ an hour, B in $\frac{1}{4}$ of an hour, and C in $\frac{1}{3}$ of an hour: In what time will they all fill it together? Ans. $\frac{1}{9}$ hour.

9. A person having about him a certain number of dollars, said that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them would make 57: Pray, how many had he? Ans. 60.

10. A Gentleman bought a chaise, horse and harness, for $500, the horse cost $\frac{1}{4}$ more than the harness, and the chaise $\frac{1}{3}$ more than the horse: What was the price of each?

Ans. $\begin{cases} \text{Harness} & \$127 & 65c. & 9\tfrac{27}{47}m. \\ \text{Horse} & 159 & 57 & 4\tfrac{22}{47} \\ \text{Chaise} & 212 & 76 & 5\tfrac{45}{47} \end{cases}$

11. A and B, having found a purse of money, disputed, who should have it: A said that $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{1}{20}$ of it amounted to £35, and, if B could tell him how much was in it, he should have the whole, otherwise he should have nothing: How much did the purse contain? Ans. £100.

12. A gentleman divided his fortune among his sons; to A he gave \$9, as often as to B \$5, and to C but \$3 as often as to B \$7, yet C's portion came to \$1059: What was the whole estate?
Ans. \$7979 80c.

13. Seven eighths of a certain number exceeds four fifths by 6: What is that number? Ans. 80.

14. What number is that, which, being increased by $\frac{2}{5}$, $\frac{3}{8}$ and $\frac{5}{6}$ of itself, the sum will be $234\frac{3}{4}$? Ans. 90.

DOUBLE POSITION.

Double Position teaches to resolve questions by making two suppositions of false numbers.

Those questions, in which the results are not proportional to their positions, belong to this rule: such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required.

Rule.*

1. Take any two convenient numbers, and proceed with each according to the conditions of the question.

2. Place the result or errours against their positions or suppos-

$$\begin{array}{ccc} \text{Pos.} & & \text{Err.} \\ 30 & & 12 \\ & \times & \\ 20 & & 6 \end{array}$$

ed numbers, thus, and if the errour be too great, mark it with $+$; and if too small with $-$.

3. Multiply them crosswise; that is, the first position by the last errour, and the last position by the first errour.

* The rule is founded on this supposition, that the first errour is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number: When that is not the case, the exact answer to the question cannot be found by this rule.

That the rule is true according to the supposition may be thus demonstrated.

Let A and B be any two numbers produced from a and b by similar operations, it is required to find the number from which N is produced by a like operation.

Put $x =$ number required, and let $N - A = r$, and $N - B = s$. Then according to the supposition on which the rule is founded, $r : s :: x - a : x - b$, whence, by multiplying means and extremes, $rx - rb = sx - sa$; and by transposition, $rx - sx = rb - sa$; and by division, $x = \frac{rb - sa}{r - s} =$ number sought; and if r and s be both negative, the Theorem is the same, and if r or s be negative, x will be equal to $\frac{rb + sa}{r + s}$ which is the rule.

4. If the errours be alike, that is, both too small or both too great, divide the difference of the products by the difference of the errours, and the quotient will be the answer.

5. If the errours be unlike; that is, one too small, and the other too great, divide the sum of the products by the sum of the errours, and the quotient will be the answer.

Note. When the errours are the same in quantity, and unlike in quality, half the sum of the suppositions is the number sought.

EXAMPLES.

1. A lady bought damask for a gown, at 8s. per yard, and lining for it at 3s. per yard; the gown and lining contained 15 yards, and the price of the whole was £3 10s.: How many yards were there of each?

Suppose 6 yards damask, value 48s.
Then she must have 9 yards lining, value 27s.

Sum of their values = 75s.
So that the first errour is 5 too much, or + 5
Again, suppose she had 4 yards of damask, value 32s.
Then she must have 11 yards of lining, value 33s.

Sum of their values = 65s.
So that the second errour is 5 too little or — 5s.

				£	s.	d.
	6	5+				
Then	X		5 yards at 8s. =	2	0	0
	4	5—	10 yards at 3s. =	1	10	0
	20	30		3	10	0 proof.
		20				

Sum of errours = 5+5=10)50

Ans. 5yds. damask, and 15—5=10yds. lining.
Or, 6+4÷2=5 as before.

2. A and B have the same income; A saves $\frac{1}{8}$ of his; but B, by spending £30 per annum more than A, at the end of 8 years finds himself £40 in debt; what is their income, and what does each spend per annum?

Suppose { 80 X 120+ / 160 X 40+ } Ans. Their income is £200 per ann. Also, A spends £175 and B £205 per annum. Then, 80—10=70 A's expense per annum, and 70+30 =100, B's expense per annum. Then 100×8—80×8=160, which should have been 40; therefore, 160—40=120 more than it should be, for the first errour. In like manner proceed for the second errour.

3. A and B laid out equal sums of money, in trade: A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost $225, then A's money was double that of B: What did each lay out? Ans. $600.

4. A labourer was hired for 60 days upon this condition, that for every day he wrought, he should receive 75c.; and for every day he was idle, should forfeit 37½c.; at the expiration of the time he received $18: How many days did he work, and how many was he idle? Ans. He was employed 36 days, and was idle 24.

5. A gentleman has two horses of considerable value, and a carriage worth £100; now if the first horse be harnessed in it, he and the carriage together will be triple the value of the second; but if the second be put in they will be 7 times the value of the first: What is the value of each horse?

Ans. One £20 and the other £40.

6. There is a fish, whose head is 10 feet long; his tail is as long as his head and half the length of his body, and his body as long as the head and tail: What is the whole length of the fish?

Head=10
First, suppose the body 20 X 10— Tail =30
Body=40
2d. suppose it 30 X 5— —

Ans. 80 feet.

7. What number is that, which, being increased by its ½, its ¼, and 5 more, will be doubled? Ans. 20.

8. A farmer, having driven his cattle to market, received for them all $320, being paid at the rate of $24 per ox, $16 per cow, and $6 per calf; there were as many oxen as cows, and 4 times as many calves as cows: How many were there of each sort?

Ans. 5 oxen, 5 cows, and 20 calves.

9. A, B, and C built a ship, which cost them $5000, of which A paid a certain sum, B paid $500 more than A, and C $500 more than both; having finished her, they fixed her for sea, with a cargo worth twice the value of the ship: The outfits and charges of the voyage, amounted to $\frac{1}{8}$ of the ship; upon the return of which, they found their clear gain to be $\frac{2}{3}$ of $\frac{3}{5}$ of the vessel, cargo and expenses: Please to inform me what the ship cost them, severally; what share each had in her, and what, upon the final adjustment of their accompts, they had severally gained?

500 X 1500—
Suppose it cost A
1000 X 500+

Ans. A owned $\frac{7}{40}$ of the ship, which cost him $875, and his share of the gain, was $1093 75c. B owned $\frac{11}{40}$, which cost $1375, and his gain was $1718 75c. C owned $\frac{11}{20}$, which cost $2750, and his gain was $3437 50c.

PERMUTATIONS AND COMBINATIONS.

THE Premutation of Quantities is the shewing how many different ways any given number of things may be changed.

This is also called variation, alternation or changes; and the only thing to be regarded here is the order they stand in; for no two parcels are to have all their quantities placed in the same situation.

The Combination of quantities is the shewing how often a less number of things can be taken out of a greater, and combined together, without considering their places, or the order they stand in.

This is sometimes called election, or choice; and here every parcel must be different from all the rest, and no two are to have precisely the same quantities, or things.

The Composition of Quantities is the taking of a given number of quantities out of as many equal rows of different quantities, one out of every row, and combining them together.

Here no regard is had to their places; and it differs from Combination only as that admits of but one row of things.

Combinations of the same form are those, in which there are the same number of quantities, and the same repetitions; thus, *abcc*, *bbad*, *deef*, &c. are of the same form; but *abbc*, *abbb*, *aacc* are of different forms.

Problem I.

To find the number of permutations, or changes, that can be made of any given number of things all different from each other.

Rule.*

Multiply all the terms of the natural series of numbers, from 1 up to the given number, continually together, and the last product will be the answer required.

Examples.

1. Christ church, in Boston, has 8 bells: How many changes may be rung on them? $1\times2\times3\times4\times5\times6\times7\times8=40320$ Ans.

2. Nine gentlemen met at an inn, and were so pleased with their host, and with each other, that in a frolick, they agreed to tarry so long as they, together with their host, could sit every day in a different position at dinner: Pray how long, had they kept their agreement, would their frolick have lasted?

Ans. $9941\frac{335}{365}$ years.

3. How many changes, or variations, will the alphabet admit of?

Ans. 620448401733239439360000.

* The reason of this rule may be shewn thus, any one thing *a* is capable of one position only, as *a*.

Any two things *a* and *b* are capable of two variations only; as *ab*, *ba*; whose number is expressed by 1×2.

If there be three things *a*, *b* and *c*; then any two of them, leaving out the third, will have 1×2 variations; and consequently when the third is taken in, there will be $1\times2\times3$ variations; and so on, as far as you please.

PROBLEM II.

Any number of different things being given, to find how many changes can be made out of them by taking any given number of quantities at a time.

RULE.*

Take a series of numbers, beginning at the number of things given, and decreasing by 1, as many terms as the number of quantities to be taken at a time; the product of all the terms will be the answer required.

EXAMPLES.

1. How many changes may be rung with 4 bells out of 8?

$$\begin{array}{r} 8 \\ 7 \\ \hline 56 \\ 6 \\ \hline 336 \\ 5 \\ \hline 1680 \end{array} \qquad \text{Or, } 8\times7\times6\times5 \ (=4 \text{ terms}) = 1680 \text{ Ans.}$$

2. How many words can be made with 6 letters of the alphabet, admitting a number of consonants may make a word?

$24\times23\times22\times21\times20\times19$ (6 terms) $=96909120$ Ans.

PROBLEM III.

Any number of things being given, whereof there are several things of one sort, several of another, &c. to find how many changes may be made out of them all.

RULE.†

1. Take the series $1\times2\times3\times4$, &c. up to the number of things given, and find the product of all the terms.

* This Rule, expressed in terms, is as follows; $m\times\overline{m-1}\times\overline{m-2}\times\overline{m-3}$, &c. to n terms; whence m = number of things given, and n = quantities to be taken at a time.

† This Rule is expressed in terms thus; $\dfrac{1\times2\times3\times4\times5, \text{ \&c. to } m.}{1\times2\times3, \text{\&c. to } p\times1\times2\times3, \text{\&c. to } q, \text{\&c.}}$ whence m = number of things given, p = number of things of the first sort, q = number of things of the second sort, &c.

Any 2 quantities, a, b, both different, admit of 2 changes; but if the quantities are the same, or ab becomes aa, there will be only one alteration, which may be expressed by $\frac{1\times2}{1\times2}=1$.

Any 3 quantities, a, b, c, all different from each other, admit of 6 variations; but if the quantities are all alike, or, $a\ b\ c$ become aaa, then the 6 variations will be reduced to 1, which may be expressed by $\frac{1\times2\times3}{1\times2\times3}=1$. Again, if two quantities out of three are alike, or abc become aac; then the 6 variations will be reduced to these 3, aac, caa, aca, which may be expressed by $\frac{1\times2\times3}{1\times2}=3$, and so of any greater number.

2. Take the series 1×2×3×4, &c. up to the number of the given things of the first sort, and the series, 1×2×3×4, &c. up to the number of the given things of the second sort, &c.

3. Divide the product of all the terms of the first series by the joint product of all the terms of the remaining ones, and the quotient will be the answer required.

EXAMPLES.

1. How many variations may be made of the letters in the word *Zaphnathpaaneah*?

1×2×3×4×5×6×7×8×9×10×11×12×13×14×15 (= number of letters in the word) =1307674368000.

1×2×3×4×5 (= number of *a*s) = 120
1×2 (= number of *p*s) = 2
1 (= number of *t*s) = 1
1×2×3 (= number of *h*s) = 6
1×2 (= number of *n*s) = 2

2×6×1×2×120=2880)1307674368000(454053600 Ans.

2. How many different numbers can be made of the following figures, 1223334444? Ans. 12600.

PROBLEM IV.

To find the number of combinations of any given number of things, all different from one another, taken any given number at a time.

RULE.*

1. Take the series 1, 2, 3, 4, &c. up to the number to be taken at a time, and find the product of all the terms.

2. Take a series of as many terms, decreasing by 1, from the given number, out of which the election is to be made, and find the product of all the terms.

3. Divide the last product by the former, and the quotient will be the number sought.

EXAMPLES.

1. How many combinations may be made of 7 letters out of 12?
1×2×3×4×5×6×7 (= the number to be taken at a time)=5040.
12×11×10×9×8×7×6(= same number from 12)=3991680.
5040)3991680(792 Ans.

2. How many combinations can be made of 6 letters out of the 24 letters of the alphabet? Ans. 134596.

* This Rule, expressed algebraically, is $\frac{m}{1} \times \frac{m-1}{2} \times \frac{m-2}{3} \times \frac{m-3}{4}$, &c. to n terms; where m is the number of given quantities, and n those to be taken at a time.

Note. In any given number of quantities, the number of Combinations increases gradually till you come about the even numbers, and then gradually decreases. If the number of quantities be *even*, half the number of places will shew the greatest number of Combinations, that can be made of those quantities; but if odd, then those two numbers which are the middle, and whose sum is equal to the given number of quantities, will shew the greatest number of Combinations.

3. A general was asked by his king what reward he should confer on him for his services; the general only required a penny for every file, of 10 men in a file, which he could make out of a company of 90 men: What did it amount to?

Ans. £23836022841 7s. 11[illegible]d.

4. A farmer bargained with a gentleman for a dozen sheep, (at 2 dollars per head) which were to be picked out of 2 dozen; but being long in choosing them, the gentleman told him that if he would give him a cent for every different dozen which might be chosen out of the two dozen, he should have the whole, to which the farmer readily agreed: Pray what did they cost him?

Ans. $27041 56c.

5. How many locks, whose wards differ, may be unlocked with a key of 6 several wards?

Ans. 63: 6 of which may have one single ward, 15 double wards, 20 triple wards, 15 four wards, 6 five wards, and 1 lock 6 wards.

$$\begin{array}{c} \text{Wards.} \\ \left\{\begin{matrix}1\\2\\3\\4\\5\\6\end{matrix}\right\} \end{array} \text{ in } 6 = \begin{array}{c} \text{Locks.} \\ \left\{\begin{matrix}6\\15\\20\\15\\6\\1\end{matrix}\right\} \end{array} \qquad \begin{array}{c} \text{Wards.} \\ \left\{\begin{matrix}1\\2\\3\\4\\5\end{matrix}\right\} \end{array} \text{ in } 5 = \begin{array}{c} \text{Locks.} \\ \left\{\begin{matrix}5\\10\\10\\5\\1\end{matrix}\right\} \end{array}$$

Problem V.

To find the number of combinations of any given number of things, by taking any given number at a time; in which there are several things of one sort, several of another, &c.

Rule.

Find the number of different forms, which the things, to be taken at a time, will admit of, in the following manner:

1. Place the things so that the greatest indices may be first, and the rest in order.

2. Begin with the first letter and join it to the second, third, fourth, &c. to the last.

3. Join the second letter to the third, fourth, &c. to the last; and so on till they are all done, always rejecting such combinations as have occurred before; and this will give the combinations of all the twos.

4. Join the first letter to every one of the twos; then join the second, third, &c. as before; and it will give the combinations of all the threes.

5. Proceed in the same manner to get the combinations of all the fours, fives, &c. and you will at last get all the several forms of combination, and the number in each form.

6. Having found the number of combinations in each form, add them all together, and the sum will be the number required.

Example.

Let the things proposed be *aaabbc*: It is required to find the number of combinations of every 2, of every 3, and of every 4 of these quantities.

Combinations at large.	Forms.	Combinations in each form.
aa,aa,ab,ab,ac	a^2,b^2	2
aa,ab,ab,ac	*ab,ac,bc*	3
ab,ab,ac		—
bb,bc		5=sum of the twos.
bc		
	a^3	1
aaa,aab,aab,aac	a^2b,a^2c,b^2a,b^2c	4
aab,aab,aac	*abc*	1
abb,abc		—
bbc		6=sum of the threes.
aaab,aaab,aaac	a^3b,a^3c	2
aabb,aabc	a^2b^2	1
abbc	a^2bc,b^2ac	2
		—
		5=sum of the fours.

Ans. 5 combinations of every 2; 6 of every 3, and 5 of every 4 quantities.

Problem VI.

To find the changes of any given number of things, taken a given number at a time; in which there are several given things of one sort, several of another, &c.

Rule.

1. Find all the different forms of combination of all the given things, taken, as many at a time, as in the question, by Problem 5.

2. Find the number of changes in any form, (by Problem 3,) and multiply it by the number of combinations in that form.

3. Do the same for every distinct form, and the sum of all the products will give the whole number of changes required.

Example.

How many changes can be made of every 4 letters out of these 6, *aaabbc*?

No. of forms.	Comb.	Changes.
a^3b,a^3c	2	$\dfrac{1\times2\times3\times4=24}{1\times2\times3\quad=6}=4$
a^2b^2	1	$\dfrac{1\times2\times3\times4=24}{1\times2\times1\times2=4}=6$
a^2bc,b^2ac	2	$\dfrac{1\times2\times3\times4=24}{1\times2\quad=2}=12$

$$\text{Therefore, } \begin{cases} 2\times 4=8 \\ 1\times 6=6 \\ 2\times 12=24 \end{cases}$$

$38 =$ number of changes required.

PROBLEM VII.

To find the compositions of any number, in an equal number of sets, the things being all different.

RULE.

Multiply the number of things in every set continually together, and the product will be the answer required.

EXAMPLES.

1. Suppose there are five companies, each consisting of 9 men; it is required to find how many ways 5 men may be chosen, one out of each company?

Multiply 9 into itself continually, as many times as there are companies. $9\times 9\times 9\times 9\times 9=59049$ different ways, Ans.

2. How many changes are there in throwing 4 dice?

As a die has 6 sides, multiply 6 into itself four times continually, $6\times 6\times 6\times 6=1296$ changes, Ans.

3. Suppose a man undertakes to throw an ace at one throw with 4 dice, what is the probability of his effecting it?

First, $6\times 6\times 6\times 6=1296$ different ways with and without the ace. Then, if we exclude the ace side of the die, there will be 5 sides left; and $5\times 5\times 5\times 5=625$ ways without the ace; therefore there are $1296-625=671$ ways, wherein one or more of them may turn up an ace; and the probability that he will do it, as 671 to 625, Ans.

4. In how many ways may a man, a woman and a child be chosen out of three companies, consisting of 5 men, 7 women and 9 children? Ans. 315.

MISCELLANEOUS MATTERS.

A short method of reducing a Vulgar Fraction, into its equivalent Decimal, by Multiplication.

RULE.

Divide unity or 1 by the denominator, till the remainder is a single figure, 10, 100, &c. if convenient, then multiply the whole quotient, including the remainder after division, by the remainder (which is now the numerator, and the divisor, the denominator) and annex the product to the quotient, then multiply the quotient, thus increased by the last numerator, and annex the product to the

increased quotient; and thus it may be reduced to what exactness you please. But if the numerator of the given fraction exceed 1, you must finally multiply the last product by the said numerator.

EXAMPLES.

1. Reduce $\frac{1}{26}$ to its equivalent decimal.

26)1·00(·03846$\frac{4}{26}$
78 This multiplied by 4 (the numerator) is ·15384$\frac{16}{26}$=$\frac{8}{13}$
—— Which annexed to the quotient ·03846 is ·0384615384$\frac{8}{13}$
220 And ·0384615384$\frac{8}{13}$×8 and annexed to the last product=
208 ·03846153843076923076$\frac{12}{13}$, &c,
——
120
104
——
160
156
——
4

2. Reduce $\frac{5}{246}$.

246)1·000000(·004065$\frac{10}{246}$ and ·0040650$\frac{10}{246}$×10 = ·0040650$\frac{100}{246}$ and this annexed to the quotient is ·00406540650$\frac{100}{246}$, and this multiplied by the given numerator, 5, is ·02032703252$\frac{8}{246}$.

For any number of pounds, avoirdupois, under 28, multiply the decimal ·00892857 by the given number of pounds, which generally gives the decimal true to the sixth place.

A short method of finding the duplicate, triplicate, &c. Ratio of any two numbers, whose difference is small, compared with the two numbers.

FOR THE DUPLICATE RATIO.

RULE.

Assume two numbers, whose difference is small; subtract half their difference from the least, and add it to the greatest, and the two numbers, thus found, will be in the same proportion nearly as the squares of the assumed numbers.

EXAMPLE.

Let the assumed numbers be 10 and 11; Then 11—10=1. 10—·5=9·5 and 11+·5=11·5·

Proof, As $10^2 : 11^2 :: 9·5 : 11·5$ nearly.

FOR A TRIPLICATE RATIO.

RULE.

Subtract the difference of the assumed numbers from the least, and add it to the greatest, and the numbers, thus obtained, will be in the same proportion nearly as the cubes of the assumed numbers.

Let the numbers be 164 and 165: Then 165—164=1. 164—1 =163 and 165+1=166.

Proof, As $164^3 : 165^3 :: 163 : 166$ nearly.

For a quadruplicate proportion subtract, and add once and a half the difference, and so on, for each higher power, increasing the number to be subtracted and added by ·5.

To reduce a Ratio, consisting of large numbers, to its least terms, and very nearly of the same value.

RULE.

1. Divide the greater of the terms by the less, and the least divisor by the remainder, and so on continually, till nothing remain, in the same manner as we get the greatest common measure for reducing a vulgar fraction: This will give a number of ratios, from which we can choose one, that will suit our purpose.

2. Place the first quotient under unit for the first ratio; multiply that by the next quotient, adding nothing to the numerator, and 1 to the product of the denominator, for a new denominator, and it will give a second ratio, nearer than the first: Then, multiply the last ratio by the next quotient, adding the preceding ratio, and so on, continually till you have gone through.

EXAMPLES.

1. Sir Isaac Newton has demonstrated, in his Principia, that the velocity of a comet, moving in a parabola, is to that of a planet, moving in a circular orb, at the same distance from the sun, as $\sqrt{2}$ to 1. Let this be taken for an example.

$\sqrt{2}=1{\cdot}4142$; those motions, then, are as 1·4142 to 1; or as 14142 to 10000?

```
10000)14142(1
      10000
      -----
       4142)10000(2
             8284
             ----
             1716)4142(2
                  3432
                  ----
                   710)1716(2
                       1420
                       ----
                        296)710(2
                            592
                            ---
                            118)296(2
                                236
                                ---
                                 60)118(1
                                     60
                                     --
                                     58)60(1
                                        58
                                        --
                                         2)58(29
                                           58
```

Then $\frac{1}{1}$=first ratio.

$\dfrac{1\times2+0=2}{1\times2+1=3}$=second.

$\dfrac{2\times2+1=5}{3\times2+1=7}$=third.*

$\dfrac{5\times2+2=12}{7\times2+3=17}$=fourth.

$\dfrac{12\times2+5=29}{17\times2+7=41}$=fifth, &c.

* The late Professor Winthrop chose 7 to 5 for a proportion.

2. Geometers have found the proportion of the circumference of a circle to its diameter, to be as 3·1416 to 1: Let this ratio be reduced.

```
10000)31416(3
      30000
      -----
       1416)10000(7
             9912
             ----
               88)1416(16
                  88
                  ---
                  536
                  528
                  ---
                    8)88(11
                      88
```

Then $\frac{1}{3}$=first ratio.

$$\frac{1\times7+0}{3\times7+1}=\frac{7}{22}=\text{second.}$$

$$\frac{7\times16+1}{22\times16+3}=\frac{113}{355}=\text{third}$$

: this is the ratio generally made use of, and is sufficiently exact for very nice calculations.

3. The area of a circle is to its circumscribing square, as ·7854 to 1, very nearly: Let this be reduced.

```
7854)10000(1
     7854
     ----
     2146)7854(3
          6438
          ----
          1416)2146(1
               1416
               ----
                730)1416(1
                     730
                     ---
                     686)730(1
                         686
                         ---
                          44)686(15
                             44
                             ---
                             246
                             220
                             ---
                              26 &c.
```

Then $\frac{1}{1}$=first ratio.

$$\frac{1\times3+0}{1\times3+1}=\frac{3}{4}=\text{second.}$$

$$\frac{3\times1+1}{4\times1+1}=\frac{4}{5}=\text{third.}$$

$$\frac{4\times1+3}{5\times1+4}=\frac{7}{9}=\text{fourth.}$$

$$\frac{7\times1+4}{9\times1+5}=\frac{11}{14}=\text{fifth}$$

: This is very exact, and the proportion generally used.

Therefore, as 14 : 11 :: the square of the diameter of a circle to its area.

To estimate the Distance of Objects on level ground, or at sea, having only the height given.

RULE.

1. To the earth's diameter, (viz. 42056462 feet,) add the height of the eye, and multiply the sum by that height, then the square root of the product is the distance, at which an object on the surface of the earth, or water, can be seen by an eye so elevated.

2. As objects are seen in a straight line, and that line is a tangent to the earth's surface; therefore, *To find the distance of two elevated objects, when the right line joining them touches the earth's surface*

between those objects, (for instance, the line from the eye of the observer to the distance found by the first part of the rule, and from thence to the object;) work for each object separately, and the sum of the square roots of the products is the distance of the two objects from each other.

EXAMPLE.

How far may a mountain be seen on level ground, or at sea, which is a mile high, supposing the eye of the observer elevated 5 feet above the surface?

$$\sqrt{42056462 + 5 \times 5} = 2{\cdot}746 \text{ miles.}$$
$$\sqrt{42056462 + 5280 \times 5280} = 89{\cdot}253 \text{ miles.}$$

Ans. 91·999 miles.

To estimate the Height of Objects on level ground, or at sea, having only the distance given.

RULE.

1. From the given distance, take the distance which the elevation of your eye above the surface will give, found by the last problem.
2. Divide the square of the remainder in feet by 42056462 feet, and the quotient will be the height required.

EXAMPLE.

Being on my return from a foreign voyage, and finding by my reckoning I was about $5\frac{1}{2}$ leagues from Boston light house, it being in the dusk of the evening, with my telescope I descried the lamp of the light house in the horizon, at which time, my eye was elevated 6 feet above the surface of the water: Now, supposing my reckoning to be true, what is the height of the light house above the water?

$5\frac{1}{2}$ leagues$=16{\cdot}5$ miles; then $16{\cdot}5-\sqrt{42056462+6\times6}=13{\cdot}943$ miles$=73619$ feet nearly, and $73619\times73619\div42056462=129$ feet nearly, Ans.

MISCELLANEOUS QUESTIONS.

1. What part of 9d. is $\frac{2}{5}$ of 7d.?

$$\frac{2}{5} \text{ of } \frac{7}{1} = \frac{14}{5}, \text{ and } \frac{9}{1} \div \frac{14}{5} = \frac{1\times14}{9\times5} = \frac{14}{45} \text{ Ans.}$$

2. What number is that from which $\frac{3}{7}$ being taken, the remainder will be $\frac{1}{5}$? Ans. $\frac{22}{35}$.

3. What number is that, to which if $\frac{3}{7}$ of $\frac{12}{5}$ of $\frac{129}{313}$ be added, the total will be 1?

$$\frac{3}{7} \text{ of } \frac{12}{5} \text{ of } \frac{129}{313} = \frac{4644}{10955}, \text{ and } \frac{1}{1} - \frac{4644}{10955} = \frac{\overline{1\times10955}-\overline{1\times4644}}{1\times10955} =$$
$\frac{6311}{10955}$ Ans.

4. What number is that, of which $19\frac{3}{13}$ is $\frac{5}{7}$?

$19\frac{3}{13}=\frac{250}{13}$; then, As $\frac{5}{7} : \frac{250}{13} :: \frac{7}{7} : 26\frac{12}{13}$ Ans.

5. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 60 of them peaches, and 40 cherries: How many trees does the orchard contain?

$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$, and $\frac{12}{12}-\frac{11}{12}=\frac{1}{12}$; therefore, as $\frac{1}{12} : \frac{60+40}{1} :: \frac{12}{12} : 1200$ Ans.

6. A person, who was possessed of $\frac{2}{3}$ of a vessel, sold $\frac{3}{8}$ of his interest for £375: What was the ship worth at that rate?

Ans. £1500.

7. If $\frac{5}{7}$ of $\frac{3}{8}$ of $\frac{4}{5}$ of a ship be worth $\frac{2}{9}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of the cargo, valued at £1000: What did both ship and cargo cost?

£837 12s. $1\frac{25}{39}$d. the cost of the ship; and £1837 12s. $1\frac{25}{39}$d. value of the ship and cargo, Ans.

8. Two ships, A and B, sailed from a certain port at the same time; A sailed north 8 miles an hour, and B east 6 miles an hour: Required by an easy method, to find their distance asunder at every hour's end?

10 miles distant in 1 hour, and $10\times2=20$ miles in 2 hours, &c. Ans.

9. If a body be weighed in each scale of a balance, whose beam is unequally divided, and those different weights of the body be multiplied together, the square root of the product will be the true weight of that body.

Suppose the weight of a bar of silver, in one scale, to be 10oz. and in the other scale 12oz. required the true weight of the bar?

$\sqrt{12\times10}=10{\cdot}95445+$ oz. $=10$ oz. 19 pwt. $2{\cdot}1384+$ gr. Ans.

10. A younger brother received \$3125 92c. which was just $\frac{7}{12}$ of his elder brother's fortune; and $5\frac{3}{8}$ times the elder's money was $1\frac{2}{3}$ the value of their father's estate: Pray, what was their father worth? Ans. \$17281 87c. 2m.

11. A gentleman divided his fortune among his sons, giving A £9 as often as B £5, and to C but £3 as often as to B £7, and yet C's dividend was £1537$\frac{5}{6}$: What did the whole estate amount to?

Ans. £11583 8s. 10d.

12. A gentleman left his son a fortune, $\frac{5}{16}$ of which he spent in 3 months, $\frac{3}{4}$ of $\frac{5}{6}$ of the remainder lasted him 9 months longer, when he had only £537 left: Pray, what did his father bequeath him?

$\frac{16}{16}$=whole legacy, $\frac{16}{16}-\frac{5}{16}=\frac{11}{16}$ left at three months, then $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{11}{16}=\frac{165}{384}$, and $\frac{11}{16}-\frac{165}{384}=\frac{1584}{6144}=\frac{33}{128}=£537$, therefore, as $\frac{33}{128} : \frac{537}{1} :: \frac{1}{1} : £2082$ 18s. $2\frac{2}{11}$d. Ans.

13. A gay young fellow soon got the better of $\frac{2}{7}$ of his fortune; he then gave £1500 for a commission, and his profusion continued till he had but £150 left, which he found to be just $\frac{6}{16}$ of his money after he had purchased his commission: What was his fortune at first? Ans. £3780.

14. A merchant begins the world with \$5000, and finds that by his distillery he clears \$5000 in 6 years: by his navigation \$5000 in $7\frac{1}{2}$ years, and that he spends in gaming \$5000 in 3 years: How long will his estate last?

$$\text{As} \left\{\begin{matrix} 6 \\ 7\frac{1}{2} \\ 3 \end{matrix}\right\} : 5000 :: 1 : \left\{\begin{matrix} 833\frac{1}{3} \\ 666\frac{2}{3} \\ 1666\frac{2}{3} \end{matrix}\right\}$$

As $1666\frac{2}{3} - 833\frac{1}{3} + 666\frac{2}{3} : 1 :: 5000 : 30$ years, Ans.

15. A has £100 of B's money in his hands, for the remittance of which B allows him 9 per cent.: What sum must he remit, to discharge himself of the £100? Ans. $91\frac{81}{109}$.

16. Said Harry to Edmund, I can place four 1s, so that, when added, they shall make precisely 12: Can you do so too?

Ans. $11\frac{1}{1}$.

17. A and B are on opposite sides of a circular field 268 poles about; they begin to go round it, both the same way, at the same instant of time; A goes 22 rods in 2 minutes, and B 34 rods in 3 minutes: How many times will they go round the field, before the swifter overtakes the slower?

$$\begin{matrix} \text{min.} & \text{po.} & & \text{min.} & \text{po.} & \\ 2 : & 22 & \Big\} :: & 1 : & \Big\{ 11 & \text{A goes in a minute.} \\ 3 : & 34 & & & \; 11\frac{1}{3} & \text{B do. do.} \end{matrix}$$

therefore, B gains $11\frac{1}{3} - 11 = \frac{1}{3}$ of a pole of A every minute. And, as $\frac{1}{3}$po. : 1min. :: $\frac{268}{2}$po. (=half round the field): 402min. (=the time in which B will overtake A.) Then,

$$\begin{matrix} & \text{min.} & \text{po.} & \text{min.} & \text{po.} & \\ \text{As } & 1 : & \left\{\begin{matrix} 11 \\ 11\frac{1}{3} \end{matrix}\right\} & :: 402 : & \left\{\begin{matrix} 4422 \\ 4556 \end{matrix}\right. & \begin{matrix} \text{A travels.} \\ \text{B travels.} \end{matrix} \end{matrix}$$

And, $\frac{4422}{268} = 16\frac{1}{2}$ times round the field, A travels;

and $\frac{4556}{268} = 17$ times round the field, B travels.

18. If 15 men can perform a piece of work in 11 days, how many men will accomplish another piece of work, four times as large in a fifth part of the time? Ans. 300 men.

19. If A can do a piece of work alone in 7 days, and B in 12; let them both go about it together: In what time will they finish it?

Days. work. day works. work. work. work. work. day. work. day.

$$\text{As} \left\{\begin{matrix} 7 : 1 :: 1 : \frac{1}{7} \\ 12 : 1 :: 1 : \frac{1}{12} \end{matrix}\right\} \text{Then } \frac{1}{7} + \frac{1}{12} = \frac{19}{84}. \text{ As } \frac{19}{84} : \frac{1}{1} :: \frac{1}{1} : 4\frac{8}{19} \text{ Ans.}$$

20. A and B together can build a boat in 20 days; with the assistance of C they can do it in 12: In what time would C do it by himself?

D. W. D. W. W. W. W. W. D. W. D.

$$\text{As} \left\{\begin{matrix} 20 : 1 :: 1 : \frac{1}{20} \\ 12 : 1 :: 1 : \frac{1}{12} \end{matrix}\right\} \text{Then, } \frac{1}{12} - \frac{1}{20} = \frac{8}{240}. \text{ \& as } 8 : 1 :: 240 : 30 \text{ Ans.}$$

21. A can do a piece of work alone in 13 days, and A and B together in 8 days: In what time can B do it alone?

Ans. $20\frac{4}{5}$ days.

22. A, B, and C can complete a piece of work in 12 days; A can do it alone in 23 days, and B in 37 days: In what time can C do it by himself? Ans. $77\frac{125}{131}$ days.

Another question; Four persons can perform a certain work in the following manner, viz. A, B, and C can do it in 6 days; B, C, and D in 7 days; A, C, and D in 8 days, and A, B, and D in 9 days:

In what time can they all do it together, and in what time can each one do it alone?

The power to do the work is inversely as the time; whence the power of A, B, and C will be $\frac{1}{6}$, of B, C, and D $\frac{1}{7}$, of A, C, and D $\frac{1}{8}$, and of A, B and D $\frac{1}{9}$. Hence $\frac{1}{6}+\frac{1}{7}+\frac{1}{8}+\frac{1}{9}=\frac{1650}{3024}=\frac{275}{504}$, is the power which does the work three times, for each agent is united with others three times.

Then $\frac{504}{275}\times3=\frac{1512}{275}=5\frac{137}{275}$ days, the time in which all together will do the work.

Then $\frac{275}{1512}-\frac{1}{6}=\frac{138}{9072}$=D's power, by taking A, B, and C's from the sum of the whole power to do the work once.

Then $\frac{9072}{138}$ days, = D's time. In the same way, is found $\frac{9072}{354}$d. = A's time. $\frac{9072}{516}$d. = B's time, and $\frac{9072}{642}$d. = C's time, Ans.

23. A cistern, for water, has 2 cocks to supply it; by the first, it may be filled in 45 minutes, and by the second, in 55 minutes; it has likewise a discharging cock, by which it may, when full, be emptied in 30 minutes: Now, if these three cocks be all left open, when the water comes in, in what time will the cistern be filled?

Min.	Cist.	Min.	Cist.
45 :	1 ::	60 :	1·3333
55 :	1 ::	60 :	1·0909
			2·4242
30 :	1 ::	60 :	2·

Gains in an hour ·4242 of a cistern.

	Cist.	Hour.	Cist.	h.	m.	s.	
As	·4242 :	1 ::	1 :	2	21	$26\frac{1}{2}$	Ans.

Or, by vulgar fractions, more accurately, 2h. 21m. $25\frac{5}{7}$s. Ans.

24. A water tub holds 73 gallons; the pipe, which conveys the water to it, usually admits 7 gallons in 5 minutes; and the tap discharges 20 gallons in 17 minutes: Now, supposing these both to be carelessly left open, and the water to be turned on at 4 o'clock in the morning; a servant, at 6, finding the water running, puts in the tap; in what time, after this accident, will the tub be full?

Ans. The tub will be full at 32m. $58\frac{118}{119}$s. after 6.

25. A has a chest of tea, weighing $3\frac{1}{2}$cwt. the prime cost of which is £60: Now, allowing interest at 6 per cent. per annum, how must he rate it per lb to B, so that, by taking his note of hand, payable at 6 months, he may clear $50 by the bargain?

Interest £2 5s. Then as $3\frac{1}{2}$cwt. : £60+£15+£2 5s. :: 1lb : 3s. $11\frac{29}{98}$d. Ans.

26. Suppose the American continental debt to be 18 millions, what annuity, at 6 per cent. per annum, will discharge it in 25 years?

By Table 5, of annuities, page 339, ·07823 is the annuity which £1 will purchase in 25 years, then, ·07823×18000000=

£1408140 Ans.

The annual interest of the debt=1080000

Therefore, there must be a sinking fund of £328140 per ann.

27. The hour and minute hands of a watch are exactly together at 12 o'clock: When are they next together?

The velocities of the two hands of a watch, or clock, are to each other, as 12 to 1; therefore, the difference of velocities is $12-1=11$.

$$\left.\begin{array}{lll} 1 & 5 & 27\frac{3}{11} \\ 2 & 10 & 54\frac{6}{11} \\ 3 & 16 & 21\frac{9}{11} \end{array}\right\} \text{\&c. Ans.}$$

28. A hare starts 12 rods before a hound; but is not perceived by him till she has been up 45 seconds; she scuds away at the rate of 10 miles an hour, and the dog, on view, makes after at the rate of 16 miles an hour: How long will the course hold, and what space will be run over, from the spot where the dog started?

2288 feet = the ground run over by the dog. $97\frac{1}{2}$sec. Ans.

29. In a series of proportional numbers, the first is 4, the third 12, and the product of the second and third is 112·8: What is the difference of the second and fourth? Ans. 18·8.

30. A fellow said that when he counted his nuts, two by two, three by three, four by four, five by five, and six by six, there was still an odd one; but when he told them seven by seven, they came out even: How many had he?

$2\times3\times4\times5\times6=720$, and $\overline{720+1}\div7=103$ even, Ans. 721.

31. There is an island 50 miles in circumference, and three men start together to travel the same way about it: A goes 7 miles per day, B 8, and C 9: When will they all come together again, and how far will each travel?

$\overline{50\times7}+\overline{50\times8}+\overline{50\times9}\div\overline{7+8+9}=50$ days. A 350 miles, B 400, and C 450, Ans.

32. Suppose A leaves Newburyport at 6 o'clock on Monday morning, and travels towards Providence, at the rate of 4 miles per hour without intermission; and that, at 3 in the afternoon, B sets out from Providence for Newburyport, and travels constantly at the rate of 7 miles an hour: Now suppose the distance between the two towns to be 90 miles; whereabout on the road will they meet?

$6+3=9$ hours, and $9\times4=36$ miles, the time and distance A had travelled before B started. Then $90-36=54$ miles remain to be travelled by both; now, as both together lessen the distance $7+4=11$ miles an hour, therefore $\frac{4}{11}$ of $\overline{54}+36=55\frac{7}{11}$ miles from Newburyport; which is near Ames's, at Dedham.

33. If, during ebb tide, a wherry should set out from Haverhill to come down the river, and at the same time, another should set out from Newburyport, to go up the river, allowing the distance to be 18 miles; suppose the current forwards one and retards the other $1\frac{1}{2}$ mile per hour; the boats are equally laden, the rowers equally good, and, in the common way of working in still water, would proceed at the rate of 4 miles per hour: Where, in the river will the two boats meet?

Ans. $12\frac{3}{8}$m. from Haverhill, and $5\frac{5}{8}$m. from Newburyport.

34. A gentleman making his addresses in a lady's family who had five daughters; she told him that their father had made a will, which imported that the first four of the girls' fortunes were, to-

gether, to make \$50000; the last four, \$66000; the three last with the first, \$60000; the three first with the last, 56000; and the two first with the two last, \$64000, which, if he would unravel, and make it appear, what each was to have, as he appeared to have a partiality for Harriet, her third daughter, he should be welcome to her: Pray, what was Miss Harriet's fortune?

$$\left.\begin{array}{l} A+B+C+D \qquad\; =50000 \\ \quad\; B+C+D+E=66000 \\ A \quad\;\; +C+D+E=60000 \\ A+B+C \quad\;\; +E=56000 \\ A+B \quad\;\; +D+E=64000 \\ \hline \qquad\qquad\qquad\;\; 296000 \end{array}\right\}$$

Then, 296000÷4 the number of combinations=74000 the sum of their fortunes.

Then, A+B+C+D+E=74000
And A+B +D+E=64000

Ans. Harriet's fortune=\$10000

35. Three persons purchase a vessel in company, towards the payment whereof A advanced $\frac{2}{5}$, B $\frac{3}{7}$, and C, \$900: What did A and B pay, each, and what part of the vessel had C?

$\frac{6}{35}$ C's part of the vessel. \$2100 A advanced. \$2250 B advanced.

36. A and B cleared, by an adventure at sea, 45 guineas, which was £35 per cent. upon the money advanced, and with which they agreed to purchase a genteel horse and carriage, whereof they were to have the use in proportion to the sums adventured, which was found to be 11 to A, as often as 8 to B: What money did each adventure?

As £35 : 100 :: 45 guineas : £180=the whole adventure.

As 11+8 : 180 :: $\begin{cases} 11 : £104\ 4s.\ 2\frac{10}{19}d.\ A's. \\ 8 : £75\ 15s.\ 9\frac{9}{19}d.\ B's. \end{cases}$

37. A, B and C are to share £100 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$, respectively; but C dying, it is required to divide the whole sum properly, between the other two?

£	s.	d.		
57	2	$10\frac{2}{7}$	A's share in all,	Ans.
42	17	$1\frac{5}{7}$	B's share in all,	

Proof 100

38. A, B and C have among them 135 guineas; A's+B's are to B's+C's, as 5 to 7, and C's—B's to C's+B's as 1 to 7: How many had each?

A+B. B+C.

Suppose A's+B's=50; then, as 5 : 7 :: 50 : 70; as 7 : 1 :: 70 : 10=C's—B's; then 70—10=60, and 60÷2=30=B's; 50—30=20=A's, and 30+10=40=C's, by the supposition: Now 20+30+40=90, which should have been 135, therefore,

As 90 : 135 :: $\begin{cases} 20 : 30=A's. \\ 30 : 45=B's. \\ 40 : 60=C's. \end{cases}$

Sum=135 proof.

39. There are three horses, belonging to different men, employed in a team to draw a load of salt from Newburyport to Boston,

for £2 10s. : A and B are supposed to do $\frac{3}{11}$ of the work; A and C $\frac{5}{13}$ and B and C $\frac{4}{14}$ of it; they are to be paid proportionally: Can you divide it as it should be?

Ans. A's$=19\frac{332}{472}$s.
B's$=\ 9\frac{102}{472}$s.
C's$=21\frac{38}{472}$s.

Proof 50s.=the sum.

40. I would put 20 hogsheads of London beer into 10 wine pipes, and desire to know what the cask must contain, which will receive the difference, 231 solid inches being the wine gallon, and 282 that of beer.

Beer hhd. = 54 gall. and $54\times282\times20=304560$ solid inches.
Wine pipe $=126$ gall. and $126\times231\times10=291060$ solid inches, and

$$\frac{304560-291060}{282}=47\tfrac{41}{47} \text{ beer gallons, Ans.}$$

41. Being about to plant 5292 trees equally distant in rows, the length of the grove is to be three times the breadth: How many of the shorter rows will there be?

Ans. viz. $\frac{1}{3}$ of the trees are to form an exact square, the side whereof being 42, shews how many come into a short row.

42. A general, disposing his army into a square battalion, found he had 231 over and above, but increasing each side with one soldier, he wanted 44 to fill up the square: How many men did his army consist of?

$231+44=275$, and $\overline{275-1}\div2=137$, then $\overline{137\times137}+231=19000$ Ans. Proof, $138\times138=19044$.

43. I want the length of a shoar, the bottom of which, being set 9 feet from the perpendicular side of a house, will support a weak place in the wall, $22\frac{1}{2}$ feet from the ground?

Ans. 24 feet, $2\frac{3}{4}$ inches.

44. A line 35 yards long will exactly reach from the top of a fort, standing on the brink of a river, known to be 27 yards broad, to the opposite bank: What is the height of the wall?

Ans. 22 yards, $9\frac{3}{4}$ inches, nearly.

45. Suppose a light-house built on the top of a rock; the distance between the place of observation and that part of the rock level with the eye 620 yards; the distance from the top of the rock to the place of observation, 846 yards, and from the top of the light-house 900 yards: the height of the light-house is required?

$$\sqrt{\overline{900\times900}-\overline{620\times620}}-\sqrt{\overline{846\times846}-\overline{620\times620}}=76\cdot77\text{yds. Ans.}$$

46. *The sum and difference of the squares of two numbers given, to find those numbers.*

RULE.

From the sum take the difference, and half the remainder is the square of the less, which, taken from the sum of the squares, will give the square of the greater.

A and B have between them a number of guineas, which are to be so divided, that the sum of their squares may be 208, and the difference of their squares 80; supposing A's the greater number, how many has he more than B?

$\overline{208-80}\div2=64$ the square of B's, and $208-64=144$ the square of A's; therefore $\sqrt{144}-\sqrt{64}=4$, Ans.

47. *Having the sum of two numbers, and the sum of their squares given, to find those numbers.*

RULE.

From the square of their sum take the sum of their squares: then from the sum of their squares take this remainder, and the square root of the difference will be the difference of the two numbers. To half their sum add their difference, and the sum will be the greater. From half the sum take half their difference, and the remainder will be the less.

A and B have 50 guineas between them, which are to be so divided, as that the sum of the squares of the two numbers shall be 1300: How many had each, supposing A to have the greater number?

$\overline{50\times50}-1300=1200$; then, $\sqrt{1300-1200}=10$ difference. Now $\overline{50\div2}+\overline{10\div2}=30$=A's. And $\overline{50\div2}-\overline{10\div2}=20$=B's, Ans.

48. *Having the difference of two numbers, and the sum of their squares given, to find those numbers.*

RULE.

From the sum of their squares take the square of their difference: to the sum of the squares add the remainder, and the square root of this sum will be the sum of the required numbers; then, with the half sum and half difference proceed as in the last question.

A number of guineas are to be divided between A and B, in such a manner that A may have 50 more than B, and that the sum of the squares of the respective shares may be 12500: What number had each?

$12500-\overline{50\times50}=10000$, and $\sqrt{12500+10000}=150$=sum of their shares. Then, $\overline{150\div2}+\overline{50\div2}=100$ A's; and $\overline{150\div2}-\overline{50\div2}=$ 50 B's, Ans.

49. *Having the sum of the squares of two numbers, and the square of their half sum given, to find those numbers.*

RULE.

From the sum of the squares take twice the square of the half sum, and the square root of half the remainder will be their half difference, with which and the half sum proceed as before directed.

Let the sum of the squares of two numbers be 3161, and the square of their half sum 1560·25: Required those numbers?

$3161-\overline{1560{\cdot}25\times2}=40{\cdot}5$ $40{\cdot}5\div2=20{\cdot}25$ and $\sqrt{20{\cdot}25}=4{\cdot}5=\frac{1}{2}$ difference, and $\sqrt{1560{\cdot}25}=39{\cdot}5=\frac{1}{2}$ sum; then, $39{\cdot}5+4{\cdot}5=44$ the greater, and $39{\cdot}5-4{\cdot}5=35$ the less, Ans.

50.—1. *If the quantity of matter, (or weights) of any two bodies, put in motion, be equal, the force by which they are moved will be in proportion to their velocities, or swiftness of motion.*

2. *If the velocities of these bodies be equal, their forces will be directly as the quantities of matter contained in them, that is, as their weights.*

3. *If both the quantities of matter, and the velocities be unequal, the forces with which the bodies are moved, will be in a proportion compounded of their quantities of matter and velocities.*

Suppose the battering ram of Vespasian weighed 60000lb; that it was moved at the rate of 24 feet in one second, and that this was sufficient to demolish the walls of Jerusalem: With what velocity must a cannon ball, which weighs 42lb be moved, to do the same execution?

The velocity of the ram being 24, and the weight of the ball 42, compounded, will make a fraction$=\frac{24}{42}=\frac{4}{7}$, and $\frac{4}{7}\times60000=34285\frac{5}{7}$ feet in a second, Ans.

51. A body weighing 30lb is impelled by such a force as to send it 20 rods in a second: With what velocity would a body weighing 12lb move, if it were impelled by the same force?

$\frac{30\times20}{12}=50$ rods in a second, Ans.

OF GRAVITY.

52. *The gravity of bodies above the surface of the earth decreases in a duplicate ratio (or as the squares of their distances) in semidiameters of the earth, from the earth's centre.*

Supposing a body to weigh 400lb at 2000 miles above the earth's surface: What would it weigh at the surface, estimating the earth's semidiameter at 4000 miles?

From the centre to the given height being $1\frac{1}{2}$ semidiameters; multiply the square of $1\frac{1}{2}$ by the weight, and the product will be the answer. $1{\cdot}5\times1{\cdot}5\times400=900$lb Ans.

53. If a body weigh 900lb at the surface of the earth, what will it weigh at 2000 miles above the surface?

This being the reverse of the last, therefore, $1+{\cdot}5=1{\cdot}5$ and $900\div\overline{1{\cdot}5\times1{\cdot}5}=400$lb Ans.

54. A certain body on the surface of the earth, weighs 180lb: How high must it be carried to weigh but 20lb?

$\sqrt{180\div20}=3$, Ans. 3 semidiameters from the earth's centre, that is 8000 above its surface.

55. To what height must a ball be raised to lose half its weight? As 1 : $\overline{3982\cdot06\times3982\cdot06}$ :: 2 : 31713603·6872, and $\sqrt{31713603\cdot6872}$ =5631·48 : and 5631·48—3982·06=1649·42 miles, Ans.

56. *At what distance from the earth would a balloon be suspended between the earth and moon?*

RULE.

As the sum of the square roots of their quantities of matter is to the distance of their centres, so is the square root of the quantity of matter in the earth, to the distance from the earth's centre.

The proportional quantity of matter in the earth being to that in the moon as 41·24 to 1 : and the distance of their centres 240000+3982·06+1090: therefore, as $\overline{\sqrt{41\cdot24}+\sqrt{1}}$: $\overline{240000+3982\cdot06+1090}$ :: $\sqrt{41\cdot24}$: 212051·49. And 212051·49—3982·06=208069·43 miles from the earth's surface, Ans.

57.—1. *If the diameters of two globes be equal, and their densities different, the weight of a body on their surfaces will be as their densities.*

2. *If their densities be equal, and diameters different, the weight will be as their diameters.*

3. *If their diameters and densities be both different, the weight will be as the product of their diameters and densities.*

If a stone weigh 100lb at the surface of the earth, required its weight at the surfaces of the sun and the several planets, whose densities are known respectively?

	Sun.	Jupiter.	Saturn.	Earth.	Moon.
Their densities	100	78·5	36	392·5	464
Diameters in Eng. miles.	883217·58.	89170·81.	79042·35.	7964·12	2180

As $\overline{7964\cdot12\times392\cdot5}$: 100 :: $\left\{\begin{array}{l}\overline{883217\cdot58\times100} : 2825\cdot46\text{lb. at the Sun.}\\ \overline{89170\cdot81\times78\cdot5} : 220\cdot41\text{lb. at Jupiter.}\\ \overline{79042\cdot35\times36} : 91\cdot06\text{lb. at Saturn.}\\ \overline{2180\times464} : 32\cdot35\text{lb. at the Moon.}\end{array}\right.$

58. If the attraction of the moon raise a tide on the earth 5 feet, what will be the height of a tide raised by the earth on the surface of the moon under similar circumstances?

The attraction of one of those bodies on the other's surface is directly as its quantity of matter, and inversely as its diameter; therefore, as $\overline{2180\times2180\times2180\times464}$: 5 :: $\overline{7964\times7964\times7964\times392\cdot5}$: 206·22 directly. And as 2180 : 206·22 :: 7964 : 56·448 inversely, Ans.

OF THE FALL OF BODIES.

59. Heavy bodies near the surface of the earth, fall one foot the first quarter of a second; three feet the second quarter; five feet in the third, and seven feet in the fourth quarter; that is, 16 feet in the first second.*

* The exact velocity in *vacuo* is 16·1 in the second; but in the air it will be scarcely 16 feet.

The velocities, acquired by bodies in falling, are in proportion to the squares of the times in which they fall; for instance, Let go three bullets together; stop the first at one second, and it will have fallen 16 feet. Stop the next at the end of the second second, and it will have fallen ($2\times2=4$) four times 16, or 64 feet; and stop the last at the end of the third second, and the distance fallen will be ($3\times3=9$) nine times 16, or 144 feet, and so on.

Or, which is the same, the space fallen through (in feet) is always equal to the square of the time in 4ths of a second.

Or, by multiplying 16 feet by so many of the odd numbers, beginning at unity, as there are seconds in any given time; viz. by 1 for the first second, by 3 for the second, by 5 for the third, and so on, these several products will give the spaces fallen through, in each of the several seconds, and their sum will be the whole distance fallen.

The velocity given, to find the space fallen through.

RULE.

1. The square root of the feet, in the space fallen through, will ever be equal to one eighth of the velocity acquired at the end of the fall; therefore,

2. Divide the velocity by 8, and the square of the quotient will be the distance fallen through, to acquire that velocity.

Suppose the velocity of a cannon ball to be about $\frac{1}{8}$ of a mile, or 660 feet per second: From what height must a body fall, to acquire the same velocity per second?

$660\div8=82\cdot5$ and $82\cdot5\times82\cdot5=6806\frac{1}{4}$ feet,$=1\frac{37}{128}$ mile, Ans.

60. *The time given, to find the space fallen through.*

RULE.

1. The square root of the feet, in the space fallen through, will ever be equal to four times the number of seconds the body has been falling; therefore,

2. Multiply the time by 4, and the sqnare of the products will be the space fallen through in the given time.

How many feet will a body fall in 5 seconds?

$5\times4=20$, and $20\times20=400$ feet, Ans.

61. A bullet is dropped from the top of a building, and found to reach the ground in $1\frac{3}{4}$ seconds: Required its height?

$1\cdot75\times4=7$, and $7\times7=49$ feet, Ans. Or, $1\frac{3}{4}=7$qrs. and $7\times7=49$. Or, $1\cdot75\times1\cdot75\times16=49$ feet, Ans.

62. What is the difference between the depths of two wells, into each of which should a stone be dropped in the same instant, one would reach the bottom in 5 seconds, and the other in 3?

$5\times4=20$, and $20\times20=400$ feet.
$3\times4=12$, and $12\times12=144$ feet.

Ans. 256 feet.

63. Ascending bodies are retarded in the same ratio that descending bodies are accelerated; therefore, if a ball, discharged from a gun, return to the earth in 12 seconds: How high did it ascend?

The ball being half of the time, or 6 seconds, in its ascent, therefore, $6\times4=24$, and $24\times24=576$ feet, Ans.

64. *The velocity per second given, to find the time.*

RULE.

1. Four times the number of seconds, in which a body has been falling, is equal to one eighth of the velocity, in feet, per second, acquired at the end of the fall; therefore,

2. Divide the given velocity by 8, and one fourth part of the quotient will be the answer.

How long must a bullet be falling, to acquire a velocity of 160 feet per second? $160\div8=20$, and $20\div4=5$ seconds, Ans.

65. *The space through which a body has fallen, given, to find the time it has been falling.*

RULE.

1. Four times the number of seconds, in which the body has been falling, will ever be equal to the square root of the space, in feet, through which it has fallen; therefore,

2. Divide the square root of the space fallen through by 4, and the quotient will be the time, in which it was falling.

In how many seconds will a bullet fall through a space of 10125 feet? $\sqrt{10125}=100{\cdot}6$, and $100\ 6\div4=25{\cdot}15$ seconds$=25''\ 9'''$ Ans.

66. In what time will a musket ball, dropped from the top of a steeple, 484 feet high, come to the ground?

$\sqrt{484}=22$, and $22\div4=5\frac{1}{2}$ seconds, Ans.

67. *To find the velocity, per second, with which a heavy body will begin to descend, at any distance from the earth's surface.*

RULE.

As the square of the earth's semidiameter is to 16 feet, so is the square of any other distance from the earth's centre, inversely, to the velocity with which it begins to descend per second.

With what velocity, per second, will an iron ball begin to descend if raised 3000 miles above the earth's surface?

As $4000\times4000 : 16 :: \overline{4000+3000}\times\overline{4000+3000} : 5.22449$ feet, Ans.

68. How high must a ball be raised above the earth's surface, to begin to descend with a velocity of 5·22449 feet per second?

As $16 : 4000\times4000 :: 5{\cdot}22449 : 49000000$, and $\sqrt{49000000}=7000$.
Wherefore, $7000-4000=3000$ miles, Ans.

69. *To find the mean velocity of a falling body.*

RULE.

Divide the space fallen through by the number of seconds it was falling, and the quotient will be the mean velocity.

A musket ball dropped from the top of a steeple 484 feet high in $5\frac{1}{2}$ seconds: Required its mean velocity?

$484\div5{\cdot}5=88$ feet per second, Ans.

70. *To find the velocity acquired by a falling body, per second, (or by a stream of water, having the perpendicular descent given) at the end of any given period of time.*

RULE.

1. The velocity acquired at the end of any period is equal to twice the mean velocity, with which it passed during that period.

Or, 2. Multiply the perpendicular space fallen through by 64, and the square root of the product is the velocity required.

If a ball fall through a space of 484 feet in $5\frac{1}{2}$ seconds, with what velocity will it strike?

By the former part of the rule.

$484 \div 5{\cdot}5 = 88$, and

$88 \times 2 = 176$, Ans.

By the latter part, without regarding the time.

$\sqrt{484 \times 64} = 176$, Ans.

71. There is a sluice, or flume, one end of which is $2\frac{1}{2}$ feet lower than the other: What is the velocity of the stream per second?

$2{\cdot}5 \times 64 = 160$, and $\sqrt{160} = 12{\cdot}649$ feet, Ans.

72. *The velocity, with which a falling body strikes, given, to find the space fallen through.*

RULE.

Divide the square of the velocity by 64, and the quotient will be the height required.

If a ball strike the ground with a velocity of 56 feet per second, from what height did it fall?

$\overline{56 \times 56} \div 64 = 49$ feet, Ans.

73. The mean velocity of a fluid, or stream, is 12·649 feet per second: What is the perpendicular fall of the stream?

$\overline{12{\cdot}649 \times 12{\cdot}649} \div 64 = 2\frac{1}{2}$ feet, Ans.

74. *The weight of a body, and the space fallen through, given, to find the force with which it will strike.*

RULE.

The momentum, or force, with which a falling body strikes, is equal to its weight multiplied by its velocity; therefore, find the velocity, by Problem 70, and multiply it by the weight, which will produce the force required.

If the rammer, used for driving the piles of Charlestown bridge, weighed $2\frac{1}{4}$ tons, or 4500℔ and fell through a space of 10 feet, with what force did it strike the pile?

$\sqrt{10 \times 64} = 25{\cdot}3 =$ velocity, and $25{\cdot}3 \times 4500 = 113850$℔ momentum, Answer.

75. *The weight and momentum, or striking force, given, to find the space fallen through.*

RULE.

Divide the momentum by the weight, and the quotient will be the velocity; then divide the square of the velocity by 64, and the quotient will be the space fallen through.

If the aforementioned rammer weighed 4500lb and struck with a force of 113850lb: From what height did it fall?

$113850 \div 4500 = 25 \cdot 3$, and $\overline{25 \cdot 3 \times 25 \cdot 3} \div 64 = 10$ feet, Ans.

76. If it were required to know with what quantity of motion, momentum or force, a fluid, moving with a given velocity, strikes upon a fixed obstacle,

RULE.

By Problem 72 find the fall, which will produce the given velocity; multiply that height by 62·5lb Avoird. for clean river water, by 63lb for dirty water, and by 64 for sea water.

Suppose a stream of clear water to move at the rate of 5 feet per second, and to meet with a fixed obstacle (or bulk head) 15 feet wide and 4 feet high: What is the momentary, instantaneous pressure of the stream?

$\overline{5 \times 5} \div 64 = \frac{25}{64}$ and $25 \div 64 = \cdot 39$ of a foot, for the perpendicular fall of the water Now $62 \cdot 5 \times \cdot 39 = 24 \cdot 375$lb the pressure upon each square foot, which, multiplied by 60, (the number of square feet in the obstacle) gives 1462·5lb going with the given velocity of 5 feet per second; therefore, $1462 \cdot 5 \times 5 = 7312 \cdot 5$lb Ans.*

77. The velocity of water, spouting through a sluice, or aperture in a reservoir, or bulk head, is the same that a body would acquire by falling through a perpendicular space equal to that between the top of the water in the reservoir and the aperture.

What is the velocity of water issuing from a head of 5 feet deep?

By Problem 70th $64 \times 5 = 320$, and $\sqrt{320} = 18$ feet nearly, Ans.

78. If the velocity of a stream issuing through the bulk head of a mill, be 16 feet per second, what head of water is there.

$\overline{16 \times 16} \div 64 = 4$ feet, Ans.

79. The quantity of water discharged from a hole in a vessel, is as the square root of the height of water above the aperture.

A miller has a head of water 4 feet above the sluice: How high must the water be raised above the opening, so that half as much again water may be discharged from the sluice in the same time?

$\sqrt{4} = 2$, and half as much again as 2, is $2 + 1 = 3$, for the square root of the required depth; therefore, $3 \times 3 = 9$ feet high, Ans.

OF PENDULUMS,

80. The time of a vibration, in a cycloid, is to the time of a heavy body's descent through half its length, as the circumference of a circle to its diameter, that is, as 3·1416 to 1: therefore, (as a body descends freely, by gravity, through about 193·5 inches in the first second) to find the length of a pendulum vibrating seconds.

RULE.

As $3 \cdot 1416 \times 3 \cdot 1416 : 1 \times 1 :: 193 \cdot 5 : 19 \cdot 6$ inches, the half length, and $19 \cdot 6 \times 2 = 39 \cdot 2$ inches, the length.

* Water being a yielding substance, loses two thirds of its power in producing effects.

81. *To find the length of a pendulum, that will swing any given time.*

RULE.

Multiply the square of the seconds in any given time by 39·2 and the product will be the length required, in inches.

Required the lengths of several pendulums, which will respectively swing $\frac{1}{4}$ seconds, $\frac{1}{2}$ seconds, seconds, minutes and hours?

$\cdot25\times\cdot25\times39\cdot2=2\cdot45$ inches for $\frac{1}{4}$ seconds. $\cdot5\times\cdot5\times39\cdot2=9\cdot8$ inches for $\frac{1}{2}$ seconds. $1\times1\times39\cdot2=39\cdot2$ inches for seconds, as above; $60\times60\times39\cdot2=$the inches in 2 miles and 1200 feet, for minutes; and 1 hour=3600 seconds, therefore $3600\times3600\times39\cdot2$ =the inches in 8018 miles and 96 feet, for hours, Ans.

82. What is the difference between the length of a pendulum, which vibrates half seconds and one which swings three seconds?

$\overline{3\times3\times39\cdot2}-\overline{\cdot5\times\cdot5\times39\cdot2}=343$ inches$=28\frac{7}{12}$ feet, Ans.

83. *To find the time which a pendulum of any given length will swing.*

RULE.

Divide the given length by 39·2, and the quotient will be the square of the time in seconds.

Or, as 6·261 (the square root of 39·2) is to the square root of the given length, so is 1 second to the time of 1 oscillation: that is, divide the square root of the given length by 6·261, and the quotient will be the time of one vibration of that pendulum.

How often will a pendulum of 9·8 inches vibrate in a second?

By the former part of the rule, $9\cdot8\div39\cdot2=\cdot25$ of a second, and $\sqrt{\cdot25}=.5$ of a second, the time of one vibration, that is, it vibrates half seconds, or $60\div\cdot5=120$ times in a minute.

By the latter part. $\sqrt{9\cdot8}=3\cdot13$, and $\sqrt{39\cdot2}=6\cdot261$, therefore, $3\cdot13\div6\cdot261=5$ of a second.

84. I observed, that while a stone was falling from a precipice, a string, (with a bullet at the end) which measured 25 inches, (to the middle of the ball,) had made 5 vibrations: What was the height of the precipice?

$25\div39\cdot2=\cdot6377+$, and $\sqrt{\cdot6377}=\cdot7986-$of a second, the time of one vibration, and $\cdot7986\times5=4$ seconds, nearly, the time of the stone's descent; then $4\times4=16$, and $16\times16=256$ feet, Ans.

85. *To find the true depth of a well, by dropping a stone into it, also the time of the stone's descent and of the sound's ascent.*

RULE.

1. Take a line of any length, and by the last Problem find the time from the dropping of the stone till you hear it strike the bottom.

2. Multiply 73088 ($=16\times4\times1142$; 1142 feet being the distance, which sound moves in a second) by the number of seconds till you hear the stone strike the bottom.

3. To this product add 1304164 (=the square of 1142) and from the square root of the sum take 1142.

4. Divide the square of the remainder by 64 ($=16\times4$) and the quotient will be the depth of the well in feet.

5. Divide the depth by 1142, and the quotient will be the time of the sound's ascent, which, being taken from the whole time, will leave the time of the stone's descent in seconds.

Suppose I drop a stone into a well, and a string with a plummet, which measured to the middle of the ball, 25 inches, made 5 vibrations before I heard the stone strike the bottom: Required the depth, time of the stone's descent, and of the sound's ascent;

$25 \div 39{\cdot}2 = {\cdot}6377$, and $\sqrt{{\cdot}6377} = {\cdot}7986$, and ${\cdot}7986 \times 5 = 4$ seconds to the hearing of it strike; then, $\sqrt{73088 \times 4 + 1304164} - 1142 = 121{\cdot}53$; and $121{\cdot}53 \times 121{\cdot}53 \div 64 = 230{\cdot}77$ feet, the depth, and $23077 \div 1142 = {\cdot}2$ of a second, the time of the sound's ascent, and $4 - {\cdot}2 = 3{\cdot}8$ seconds, the time of the stone's descent.

OF THE LEVER OR STEELYARD.

86. It is a principle in mechanicks, that the power is to the weight, as the velocity of the weight, to the velocity of the power. Therefore, to find what weight may be raised or balanced by any given power, say;

As the distance between the body to be raised or balanced, and the fulcrum or prop, is to the distance between the prop and the point where the power is applied; so is the power to the weight which it will balance.

If a man, weighing 160℔, rest on the end of a lever 10 feet long, what weight will he balance on the other end, supposing the prop one foot from the weight?

The distance between the weight and prop being 1 foot, the distance from the prop to the power is $10 - 1 = 9$ feet; therefore, as 1 foot : 9 feet :: 160℔ : 1440℔, Ans.

87. If a weight of 1440℔ were to be raised with a lever 10 feet long, and the prop fixed one foot from the weight, what power or weight, applied to the other end of the lever would balance it?

As 9 : 1 :: 1440 : 160℔, Ans.

88. If a weight of 1440℔ be placed 1 foot from the prop, at what distance from the prop must a power of 160℔ be applied, to balance it? As 160 : 1440 :: 1 : 9 feet, Ans.

89. At what distance from a weight of 1440℔, must a prop be placed, so as that a power of 160℔, applied 9 feet from the prop may balance it? As 1440 : 160 :: 9 : 1 foot, Ans.

90. In giving directions for making a chaise, the length of the shafts between the axletree and backband, being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should be fixed. The chaise maker advised to place it 30 inches before the axletree; others supposed 20 inches would be a sufficient incumbrance for the horse: Now supposing two passengers to weigh 3cwt. and the body of the chaise $\frac{3}{4}$cwt. more: What will the beast in both these cases bear, more than his harness?

Weight of the chaise and passengers $3\frac{3}{4}$cwt. = 420℔, and 9 feet = 108 inches.

$$\text{Then, as } \overset{\text{In.}}{108} : \overset{℔}{420} :: \begin{cases} \overset{\text{In.}}{30} : \overset{℔}{116\frac{2}{3}} \\ 20 : 77\frac{7}{9} \end{cases} \text{Ans.}$$

OF THE WHEEL AND AXLE.

91. The proportion for the wheel and axle (in which the power is applied to the circumference of the wheel, and the weight is raised by a rope, which coils about the axle as the wheel turns round) is, as the diameter of the axle is to the diameter of the wheel, so is the power applied to the wheel, to the weight suspended by the axle.

A mechanick would make a windlass in such a manner, as that 1lb applied to the wheel, should be equal to 10lb suspended from the axle; now, supposing the axle to be six inches diameter, required the diameter of the wheel?

lb. in. lb. in.
As 10 : 6 :: 1 : 60 inversely, the diameter required.

92. Suppose the diameter of the wheel to be 60 inches: Required the diameter of the axle, so as that 1lb on the wheel may balance 10lb on the axle?

lb. in. lb. in.
Inversely, as 1 : 60 :: 10 : 6 diameter required.

93. Suppose the diameter of the axle 6 inches, and that of the wheel 60 inches, what power at the wheel will balance 10lb at the axle?

in. lb. in. lb.
Inversely, 6 : 10 :: 60 : 1 Ans.

94. Suppose the diameter of the wheel 60 inches, and that of the axle 6 inches; what weight at the axle will balance 1lb at the wheel?

in. lb. in. lb.
Inversely, as 60 : 1 :: 6 : 10 Ans.

OF THE SCREW.

95. The power is to the weight, which is to be raised, as the distance between two threads of the screw, is to the circumference of a circle described by the power applied at the end of the lever.

Rule.

Find the circumference of the circle described by the end of the lever; then, as that circumference is to the distance between the spiral threads of the screw: so is the weight to be raised, to the power which will raise it, abating the friction, which is not proportional to the quantity of surface; but to the weight of the incumbent part; and, at a medium, $\frac{1}{3}$ part of the effect of the machine is destroyed by it, sometimes more and sometimes less.

There is a screw, whose threads are an inch asunder; the lever by which it is turned 30 inches long, and the weight to be raised a ton, or 2240lb: What power or force must be applied to the end of the lever, sufficient to turn the screw—that is, to raise the weight.

The lever being the semidiameter of the circle, the diameter is 60 inches; then, $3{\cdot}1416\times60=188{\cdot}496$ inches, the circumference:

in. in. lb. lb.
Therefore, as 188·496 : 1 :: 2240 : 11·88, Ans.

96. Let the lever be 30 inches, (the circumference of which is found to be 188·496) the threads 1 inch asunder, and the power 11·88lb: Required the weight to be raised?

in. in. lb. lb.
As 1 : 188·496 :: 11·88 : 2240 nearly, Ans.

97. Let the weight be 2240℔, the power 11·88℔, and the lever 30 inches: Required the distance between the threads?

lb. lb. in. in.
As 2240 : 11·88 :: 288·496 : 1 nearly, Ans.

98. Let the power be 11·88℔, the weight 2240℔, and the threads an inch asunder, to find the length of the lever.

lb. lb. in. in.
As 11·88 : 2240 :: 1 : 188·5; then, as 355 : 113 :: 188·5 : 60 inches nearly, the diameter, and 60÷2=30 inches, Ans.

99. Suppose one of those meteors, called fire balls, to move parallel to the earth's surface, and 50 miles from it, at the rate of 20 miles per second: In what time would it move round the earth?

The Earth's diameter is 7964 English miles; then, $7964+\overline{50\times2}=$ $\underline{8064}$=the diameter of the circle described by the ball. Then, $\overline{8064\times3\ 1416}$=25333·8624 miles, its circumference, and 25333·8624 ÷20=1266·69312 seconds=21′ 6″ 41‴ 35⁗ 13′′′′′ 55′′′′′′ 12′′′′′′′, Ans.

100. Sound, uninterrupted, moves about 1142 feet in a second: How long, then, after firing a cannon at Newburyport, before it will be heard at Ipswich, estimating the distance at 10 miles in a right line?

10 miles=52800 feet, and 52800÷1142=$46\frac{134}{571}$ seconds, Ans.

101. In a thunder storm I observed by my clock that it was 6 seconds between the lightning and thunder: at what distance was the explosion? 1142×6=6852feet=$1\frac{131}{440}$ miles, Ans.

102. Tubes may be made of gold, weighing not more than at the rate of $\frac{1}{1625}$ of a grain per foot: What would be the weight of such a tube, which would extend across the Atlantick, from Boston to London, estimating the distance at 1000 leagues?

1000×3=3000 miles, and 3000×5280=15840000 feet, and 15840000×$\frac{1}{1625}$=9747$\frac{9}{13}$gr. or rather, 1℔ 8oz. 6pwt. 3$\frac{9}{13}$gr. Ans.

103. The mean distances of the Planets from the Sun, in English miles, are as follow: viz. Mercury 36686617·5; Venus 68552135·83; Earth 94772980; Mars 144404783·33; Jupiter 492912533·33; Saturn 903957657·5: Now, as a cannon ball, at its first discharge, flies about a mile in 8 seconds, and sound 1142 feet in a second: In what time, at the above rate, would a bullet pass from the Earth to the Sun? and sound move from the Sun to Saturn?

94772980×8″=758183840=24years, 15days, 6 hours, 27 minutes, 20 seconds, for the passage of the Ball. And 903957657·5×5280= 4772396431600 feet, and 4772896431600÷1142=132 years, 192 days, 21h. 42m. 21$\frac{482}{571}$s. sound passing from the Sun to Saturn, Ans.

104. Light passes from the Sun to the Earth in 8·2 minutes: In what time would it pass from the sun to the *Georgium Sidus*, it being 1803930416·66 English miles?

As 94772980 : 8·2 :: 1803930416·66 : 2h. 36m. 4″ 50‴, Ans.

105. The Sun's diameter is 883217·58 English miles; Jupiter's is 89170·81; Saturn's 79042·35; Georgium 35109; Mercury's 3222·48; Venus' 7637·85; Earth's 7961·12; Mars' 4189·69;

and the Moon's 2180: Required the comparative magnitude between each of those bodies and the Earth?

$$\left.\begin{array}{l}\overline{883217{\cdot}58\times883217{\cdot}58\times883217{\cdot}58}\\ \overline{89170{\cdot}81\times89170{\cdot}81\times89170{\cdot}81}\\ \overline{97042{\cdot}35\times97042{\cdot}35\times97042{\cdot}35}\\ \overline{35109\times35109\times35109}\end{array}\right| \div\overline{7964{\cdot}12\times7964{\cdot}12\times7964{\cdot}12}=\left|\begin{array}{r}1363724\\ 1402{\cdot}65\\ 982\\ 99{\cdot}57\end{array}\right| \text{greater.}$$

$$\overline{7964{\cdot}12\times7964{\cdot}12\times7964{\cdot}12}\div\left\{\begin{array}{l}3222{\cdot}48\times3222{\cdot}48\times3222{\cdot}48=93{\cdot}12\\ 7687{\cdot}85\times7687{\cdot}85\times7687{\cdot}85=1{\cdot}11\\ 4189{\cdot}69\times4189{\cdot}69\times4189{\cdot}69=6{\cdot}86\\ 2180\times2180\times2180=48{\cdot}74\end{array}\right\}\text{less.}$$

N. B. The above diameters and mean distances in English miles answer to the same in geographical miles, as they were deduced from observations on the transits of Venus over the Sun in 1761 and 1769.

106. Suppose the density of the Moon 464, and that of the Earth 392·5: Required the proportion between the quantity of matter in the Earth and in that of the Moon, allowing the Earth's diameter to be 7964·12, and the Moon's 2180 miles, and supposing the Earth a complete sphere, which, however, it is not?

There is $\frac{7964{\cdot}12\times7964{\cdot}12\times7964{\cdot}12\times392{\cdot}5}{2180\times2180\times2180\times464}=41{\cdot}24$ times the quantity of matter in the Earth that there is in the Moon; or, the Earth's weight is so many times that of the Moon.

107. The mean diameter of the Earth's orbit, (or annual path round the Sun) supposing it a circle, is in English miles 190437141·7: Required its mean motion, (or the space through which it moves in its orbit,) per minute?

190437141·7×3·1416=598277324·36 miles in circumference; then,

Days.
As 365·25 : 598277324·36 :: 1′ : 1137·49 miles, Ans.

N. B. The Earth's diurnal motion round its axis is 17¼ miles per minute, at the equator.

OF THE SPECIFICK GRAVITIES OF BODIES.

The specifick gravities of bodies are as their densities, or weights, bulk for bulk; thus, a body is said to have two or three times the specifick gravity of another, when it contains two or three times as much matter in the same space.

A body, immersed in a fluid, will sink, if it be heavier than its bulk of the fluid. If it be suspended therein, it will lose so much of what it weighed in the air, as its bulk of the fluid weighs. Hence, all bodies of equal bulk, which will sink in fluids, lose equal weights when suspended therein, and unequal bodies lose in proportion to their bulks.

The *hydrostatick balance* differs very little from a common balance that is nicely made; only it has a hook at the bottom of each scale, on which small weights may be hung by horse hairs, so that a body suspended by the hair, may be immersed in water without wetting the scales.

How to find the Specifick Gravities of Bodies.

If the body, thus suspended under the scale, at one end of the balance, be first counterpoised in air by weights in the opposite scale, and then immersed in water, the equilibrium will be immediately destroyed; then, if as much weight be put into the scale, to which the body is suspended, as will restore the equilibrium, (without altering the weights in the opposite scale) that weight, which restores the equilibrium, will be equal to a quantity of water as big as the immersed body; and if the weight of the body in air be divided by what it loses in water, the quotient will shew how much that body is heavier than its bulk of water. Thus, if a guinea suspended in air, be counterbalanced by 129 grains in the opposite scale, and then, upon being immersed in water, it becomes so much lighter as to require $7\frac{1}{4}$ grains to be put into the scale over it, to restore the equilibrium, it shews that a quantity of water, of equal bulk with the guinea, weighs 7·25 grains; by which divide 129 (the weight of the guinea in air) and the quotient will be 17·793; which shews that the guinea is 17·793 times as heavy as its bulk of water.

Thus may any piece of gold be tried, by weighing it first in air, and then in water; and if, upon dividing the weight in air by the loss in water, the quotient comes out 17·793, the gold is good: If the quotient be 18, or between 18 and 19, the gold is very fine: but if it be less than 17, the gold is too much alloyed by being mixed with some other metal.

If silver be tried in this manner and found to be 11 times as heavy as water, it is very fine: If it be $10\frac{1}{2}$ times as heavy, it is standard; but if it be of any less weight compared with water, it is mixed with some lighter metal, such as tin, &c.

If a piece of brass, glass, lead, or silver, be immersed and suspended in different sorts of fluids, the different losses of weight therein will shew how much heavier it is than its bulk of the fluid; that fluid being lightest, in which the immersed body loses least of its aerial weight.

Common clear water, for common uses, is generally made a standard for comparing bodies by, whose gravity may be represented by unity, or 1, or, in case great accuracy be required, by 1·000, where 3 cyphers are annexed to give room to express the ratios of other gravities in larger numbers in the table. In doing this there is a twofold advantage; the first is, that, by this mean the specifick gravities of bodies may be expressed to a much greater degree of accuracy. The second is, that the numbers of the Table, considered as whole numbers, do also express the ounces Avoirdupois contained in a cubick foot of every sort of matter therein specified; because a cubick foot of common water, is found by experiment to weigh very nearly 1000 ounces Avoirdupois, or $62\frac{1}{2}$ pounds.

A TABLE

OF THE SPECIFICK GRAVITIES OF SEVERAL SOLID AND FLUID BODIES; WHERE THE SECOND COLUMN CONTAINS THEIR ABSOLUTE WEIGHT, AND THE THIRD THEIR RELATIVE WEIGHT, IN AVOIRDUPOIS OUNCES.

A Cubick Foot of	Abso. wt.	Rela. wt.	A Cubick Foot of	Abso. wt.	Rela. wt.
Platina rendered malleable and hammered	20170	20·170	Brick	2000	2·000
			Liver Sulphur	2000	2·000
Very fine Gold	19637	19·637	Nitre	1900	1·900
Standard Gold	18888	18·888	Alabaster	1875	1·875
Guinea Gold	17793	17·793	Dry Ivory	1825	1·825
Moidore Gold	17140	17·140	Brimstone	1800	1·800
Quicksilver	13600	13·600	Solid subs. of Gun Pow.	1745	1·745
Lead	11325	11·325	Alum	1714	1·714
Fine Silver	11087	11·087	Ebony	1117	1·117
Standard Silver	10535	10·535	Human Blood	1054	1·054
Rose Copper	9000	9·000	Amber	1030	1·030
Copper	8843	8·843	Cow's Milk	1030	1·030
Plate Brass	8000	8·000	Sea Water	1030	1·030
Steel	7852	7·852	Pure Water	1000	1·000
Cast Brass	7850	7·850	Red Wine	993	0·993
Iron	7645	7·645	Oil of Amber	978	0·978
Block Tin	7321	7·321	Proof Spirits	925	0·925
Cast Iron	7135	7·135	Dry Oak	925	0·925
Lead Ore	6800	6·800	Olive Oil	913	0·913
Copper Ore	3775	3·775	Loose Gun Powder	872	0·872
Diamond	3400	3·400	Spirits of Turpentine	864	0·864
Crystal Glass	3150	3·150	Alcohol or Pure Spirit	850	0·850
White Marble	2707	2·707	Elm and Ash	800	0·800
Black Marble	2704	2·704	Oil of Turpentine	772	0·772
Rock Crystal	2658	2·658	Dry Crab Tree	765	0·765
Green Glass	2620	2·620	Æther	732	0·732
Clear Glass	2600	2·600	White Pine	569	0·569
Stone: Flint	2582	2·582	Sassafras Wood	482	0·482
Stone: Paving	2570	2·570	Cork	240	0·240
Stone: Cornelian	2568	2·568	Common Air	$1\frac{25}{100}$	0·00125
Stone: Free	2352	2·352	Inflammable Air	$0,\frac{12}{100}$	0·00012

The use of the Table of Specifick Gravities will best appear by several Examples.

How to discover the quantity of adulteration in metals.

Suppose a body be compounded of gold and silver, and it be required to find the quantity of each metal in the compound.

First, find the Specifick gravity of the compound, by weighing it in air and in water, and dividing its aerial weight by what it loses thereof in water, and the quotient will shew its specifick gravity, or how many times heavier it is than its bulk of water. Then, subtract the specifick gravity of silver (found in the Table) from that of the compound, and the specifick gravity of the compound from that of the gold: the first remainder will shew the bulk of gold, and the latter, the bulk of silver in the whole compound; and if these remainders be multiplied by the respective specifick gravities, the products will shew the proportional weights of each metal in the body.

Suppose the specifick gravity of the compounded body be 14; that of standard silver (by the Table) is 10·535, and that of standard gold 18·888; therefore, 10·535 from 14, remains 3·465, the proportional *bulk* of the gold in the compound; and 14 from 18·888, remains 4·888, the proportional *bulk* of silver in the compound: then, 18·888, the specifick gravity of gold, multiplied by the first remainder 3·465, produces 65·447 for the proportional *weight* of gold; and 10·535, the specifick gravity of silver, multiplied by the last remainder, produces 51·495 for the proportional weight of silver in the whole body: So that for every 65·447 ounces or pounds of gold, there are 51·495 ounces or pounds of silver in the body.

Hence it is easy to know whether any suspected metal be genuine, or alloyed or counterfeit, by finding how much heavier it is than its bulk of water, and comparing the same with the Table; if they agree, the metal is good; if they differ, it is alloyed or counterfeited.

How to try Spiritous Liquors.

A cubick inch of good brandy, rum, or other proof spirits, weighs 234 grains; therefore if a true inch cube of any metal weighs 234 grains less in spirits than in air, it shews the spirits are proof: If it lose less of its aerial weight in spirits, they are above proof; if it lose more, they are under proof; for, the better the spirits are, the lighter they are, and the worse, the heavier.

Or, let any solid, of sufficient specifick gravity, be weighed first in air, then in water, and then in another liquid; from its weight in the air take its weight in water, and the remainder is the weight of its bulk of water. From its weight in air take its weight in the other liquid, and the remainder is the weight of the same quantity of that liquid. Divide the weight of this quantity of liquid by the weight of the same quantity of water, and the quotient will be the specifick gravity of the liquid.

All bodies expand with heat and contract with cold; but some more, and some less than others: therefore the specifick gravities of bodies are not precisely the same in summer as in winter.

The four following Problems, relating to spiritous liquors, are wrought by Alligation.

108. What proportion of rectified spirits of wine must be mixed with water, to make proof spirit, the specifick gravity of the rectified spirits being 850, that of proof spirit 925, and of water 1000?

$$925 \left\{ \begin{matrix} 1000 \\ 850 \end{matrix} \right\} \begin{matrix} 75 \\ 75 \end{matrix} \Big\} \text{ Or equal measures.}$$

109. What proportional *weight* of rectified spirits of wine and water must be mixed, to make proof spirit, the specifick gravities as before?

$$\text{Ans. } \frac{1000}{850} = \frac{20}{17}, \text{ or as 20 to 17.}$$

110. What is the specifick gravity of best French brandy, consisting of 5 parts, measure, of rectified spirits of wine, and 3 parts water?

$$\begin{array}{r} 850\times5=4250 \\ 1000\times3=3000 \\ \hline 5+3=\ 8\ )\ 7250 \\ \hline 906{\cdot}25 \end{array}$$

906·25=specifick gravity.

111. A retailer has 30 gallons of rum, whose specifick gravity is 900: How much water must he add to reduce it to standard proof?

$925 \left\{ \begin{array}{l} 1000 \searrow 25 \\ 900 \nearrow 75 \end{array} \right\}$ g. rum. g. wat. g. rum. g. wat. As 75 : 25 :: 30 : 10 to be added.

112. The cubick inch of common glass weighs about 1·36oz. Troy: ditto of salt water ·5427oz. ditto of brandy ·48927oz. Suppose then, a seaman has a gallon of brandy in a bottle, which weighs 4½lb Troy, out of water, and to conceal it, throws it overboard into salt water: Pray, will it sink or swim, and by how much is it heavier or lighter than the same bulk of salt water?

$4\frac{1}{2}$lb=54oz.=weight of bottle $\frac{54}{1{\cdot}36}$=39·7059 cub. in. in the bottle.
Add 231· =do. in the brandy.

270.7059=ditto in both.

Then, 270 7059×·5427=146·912oz.=weight of salt water occupied by the bottle and brandy. And ·48927 (=weight of a cubick inch of brandy) ×231=113·02+oz. and 113·02+54=167·02oz. =weight of the bottle and brandy. From this take the weight of the salt water, viz. 146·192oz. Ans. Supposing the bottle full, it is 20·11oz. heavier than the same bulk of salt water, and therefore will sink.

Given the weight to be raised by a balloon, to find its diameter.

RULE.

1. As the specific difference between common and inflammable air, is to one cubick foot: so is any weight to be raised, to the cubick feet contained in the balloon.

2. Divide the cubick feet by ·5236, and the cube root of the quotient will be the diameter required, to balance it with common air; but, to raise it, the diameter must be somewhat greater, or the weight somewhat less.

113. I would construct a spherical balloon, of sufficient capacity to ascend with 4 persons, weighing, one with another, 160lb, and the balloon and a bag of sand weighing 60lb: Required the diameter of the balloon?

By the Table of Specifick Gravities, page 388, I find a cubick foot of common air weighs 1·25 ounces Avoirdupois, and a cubick foot of inflammable air ·12 of an ounce Avoirdupois; therefore,

$1{\cdot}25-{\cdot}12=1{\cdot}13$oz. difference. And $160\times4+60=700$ ℔ $=11200$ oz.

As oz. $1{\cdot}13$: cub. foot 1 :: oz. 11200 : cub. feet $9911{\cdot}5044$. And $\sqrt[3]{\dfrac{9911{\cdot}5044}{{\cdot}5236}}=26.65$ feet, [Ans.

Given the diameter of a balloon, to find what weight it is capable of raising.

RULE.

1. Multiply the cube of the diameter by ·5236, and the product will be the content in cubic feet.

2. As one cubick foot is to the specifick difference between common and inflammable air; so is the content of the balloon to the weight it will raise.

114. The diameter of a balloon is 26·65 feet: What weight is it capable of raising?

$26{\cdot}65\times26{\cdot}65\times26{\cdot}65\times{\cdot}5236=9911{\cdot}4+$ cubick feet. And

As cub. foot 1 : oz. 1·13 :: cub. feet 9911·4+ : oz. 11199·882=700℔ nearly.

If the magnitude of any body be multiplied by its specifick gravity, the product will be its absolute weight.

115. What weight of lead will cover a house, the area of whose roof is 6000 feet, and the thickness of the lead $\frac{1}{120}$ of a foot? $6000\times\frac{1}{120}=50$ cub. feet, and its specifick gravity $11325\times50=$ 566250 ounces = 15 tons 15 cwt. 3 qrs. 26 lbs. 10 oz. Ans.

To find the magnitude of any thing when the weight is known.

Divide the weight by the specifick gravity in the Table, and the quotient will be the magnitude sought.

116. What is the magnitude of several fragments of clear glass, whose weight is 13 ounces?

$13\div2600={\cdot}005$ of a cubick foot, and $\cdot005\times1728=8.640$ cubick inches, Ans.

Having the magnitude and weight of any body given, to find its specifick gravity.

Divide the weight by the magnitude, and the quotient will be the specifick gravity.

117. Suppose a piece of marble contains 8 cubick feet, and weighs 1353½℔ or 21656 ounces: What is the specifick gravity?

$21656\div8=2707$ the specifick gravity required, as by the Table.

To find the quantity of pressure against the sluice or bank, which pens water.

Multiply the area of the sluice, under water, by the depth of the centre of gravity, (which is equal to half the depth of the water) in feet, and that product again by 62½ (the number of pounds Avoir-

dupois in a cubick foot of fresh water) or by 64·4lb (the Avoirdupois weight of a cubick foot of salt water) and the product will be the number of pounds required.

118. Suppose the length of a sluice or floom be 30 feet, the width at bottom 4 feet, and the depth of the water 4 feet; what is the pressure against the side of the sluice ?

30×4=120 feet the area of the bottom, and 120×2 (the depth of the centre of gravity) gives 240 cubick feet, and 240×62·5=15000lb=6T. 13cwt. 3qrs. 20lb Ans.

The perpendicular pressure of fluids on the bottoms of vessels is estimated by the area of the bottom multiplied by the altitude of the fluid.

119. Suppose a vessel 3 feet wide, 5 feet long, and 4 feet high, what is the pressure on the bottom, it being filled with water to the brim?

3×5=15 square feet, the area of the bottom, and 15×4=60 cubick feet, and 60×62·5=3750lb=33 cwt. 1 qr. 26lb.

THE USE OF THE BAROMETER.

The Barometer is so formed, that a column of quicksilver is supported within it to such a height as to counterbalance the weight of a column of air, of an equal diameter, extending from the barometer to the top of the atmosphere.

120. At the surface of the earth, the height of this column of quicksilver is, at an average, almost 30 inches; when the barometer is at that height; what is the pressure of atmosphere on a square foot, and on the surface of a man's body, estimated at 14 square feet?

As the cubick foot of quicksilver is 13600 ounces, Avoirdupois, and as the height in the barometer, is 2·5 feet, therefore 13600×2·5 =34000 ounces, =2125 pounds on a square foot; and 2125×14= 29750 pounds on a man's body.

121. If the mercury in a barometer, at the bottom of a tower, be observed to stand at 30 inches, and, on being carried to the top of it, be observed at 29·9 inches: What is the height of the tower?

Divide 13600, the specifick gravity of quicksilver, by 1·25, the specifick gravity of air, and the quotient will be the height of the tower, in tenths of an inch.

$$\frac{13600}{1{\cdot}25}=10880 \text{ tenths, and } \frac{10880}{10}=1088 \text{ inch.}=90\tfrac{2}{3} \text{ feet Ans.}$$

The number of feet, in height, of the atmosphere, corresponding with $\frac{1}{10}$ of an inch on the barometer is variable, depending on the temperature and density of the atmosphere.

The variation, depending on the temperature, is shewn in the following Table, calculated for every 5 degrees, from 32 to 80, Fahrenheit's Thermometer, from whence it may be easily calculated, for the intermediate degress by allowing $\frac{21}{100}$ of a foot for each degree.

TABLE.

Thermo.	Feet.
32°	86·86
35	87·49
40	88·54
45	89·60
50	90·66
55	91·72
60	92·77
65	93·82
70	94·88
75	95·93
80	96·99

The altitude, thus found; will be to the altitude corrected for the density of the air, inversely, as the mean height of the barometer, at the two stations, is to 30 inches; therefore,

RULE.—Multiply the mean height corresponding to the mean temperature of the two barometers (found in the Table) by the *tenths* of an inch in the difference of the two barometers, and this product by 30; divide this last product by the mean height of the two barometers, and the quotient will be the answer, or height required, with the errour of a few feet only, if the height be less than a mile.*

122. At the first station, suppose the barometer to stand at 29, and the thermometer at 60; at the second station, the barometer at 28, and the thermometer at 40: What is the height of the 2d station or the distance between the two places of observation?

Barometer.

$$\text{Add}\begin{cases}\text{First station} = 29\\ \text{Second station} = 28\end{cases}$$

$\frac{1}{2})57$

$\frac{1}{2}$ sum $=28{\cdot}5=$ mean height of the two barometers.

29
28

Difference $= 1 = 10$ tenths of an inch.

Thermometer.

First station $=60$
Second station $=40$

$\frac{1}{2})100$

$50=$ mean height of the two thermometers, against which, in the Table you will find 90·66, the mean temperature of the two barometers. Now, according to the rule $90{\cdot}66\times10\times30\div28{\cdot}5=954{\cdot}3$ feet, the Answer, nearly.

* Let $h=$ mean height of the barometer at its two stations, (or of two barometers, one at each station) in inches; $d=$ difference of the two barometers in *tenths* of an inch; and $n=$ number from the Table answering to the mean temperature of the two thermometers accompanying the barometer, then $\frac{30dn}{h}=$ the altitude required nearly.

The Act of Congress of April 29, 1816, regulating the currency within the United States of the gold coins of Great Britain, France, &c. enacted,

That, of the gold coins of Great Britain and Portugal,

	27 grs.=100 cents, or 1 pwt.=88⅞ cents;	
Of France,	27½grs.= do.	do. =87¼ do.
Spain,	28½grs.= do.	do. =84 do.

Crowns of France, weighing 449grs.=110cents, or 1oz.=117c.

Five franc pieces, weighing 386grs.=93·3 cts. or 1oz.=116 cts.

The Spanish dollar, weighing not less than 415grs.=100 cents.

The Federal dollar is to be of the weight and purity of the Spanish dollar; but, it is to weigh 416grs. and to contain 371¼grs. of pure silver.

The pound Sterling of England is $4·44 in the United States. The dollar is reckoned, therefore, at four shillings and six pence sterling; but in 1820, four shillings and six pence of English silver coin contained only 363¼grs. of pure silver, being 8grs. less than is contained in the Federal dollar.

One pound Troy weight of Standard Gold in England, contains 5280grs. of pure gold, and is coined into £46 14s. 6d. or 11214 pence. Hence a pound sterling contains 113·0016grs. of pure gold.

The Eagle contains 247·5grs. of pure gold, and hence the pound sterling is worth in gold $4 56·572, and hence the dollar is worth in English gold 4s. 4·5656d.

One pound Troy of Standard silver in England contains 5328grs. of pure silver, and is coined into 66 shillings or 792 pence. As the dollar contains 371¼grs. of pure silver, the dollar is worth in English silver 4s. 7·1858d.

In a pound sterling there is 1614·545grs. of pure silver, which is equal in silver to $4·34 3943.

Taking the mean of the values of the dollar and the pound sterling in gold and silver,

The value of the dollar is 4s 5·8757d. sterling.

And the value of the pound sterling is $4·45·7331.

This mean value of the dollar and the pound sterling is very near the values at which they are commonly estimated.

[See "Report of the Secretary of State upon Weights and Measures," appendix C. to Congress, Feb. 1821.]

The standard price of gold in England, is £3 17s. 10½d. an oz. and of silver 5s. 2d. an oz. The standard weight of the English guinea is 5pwt. 9⅜grs.; but it usually weighs 5pwt. 8grs.

The standard coin of France is to contain one tenth of alloy, and the standard value of gold to silver is 15 to 1.

The following Tables are calculated according to the value of Foreign Gold established by act of Congress, April 29, 1816.

TABLE I.

Value of English and Portuguese Gold, in dollars, cents, and mills, throughout the United States.

Gr.	Cts.	m.	Pwts.	$	cts.
1	3	7	1	0	88⅞
2	7	4	2	1	77¾
3	11	1	3	2	66⅔
4	14	8	4	3	55
5	18½	0	5	4	44
6	22	2	6	5	33⅓
7	25	9	7	6	32
8	29	6	8	7	11
9	33⅓	0	9	8	0
10	37	0	10	8	89
11	40	7	11	9	77¾
12	44	4	12	10	66
13	48	1	13	11	55½
14	51	8	14	12	44
15	55	5	15	13	33⅓
16	59¼	0	16	14	22
17	63	0	17	15	11
18	66⅔	0	18	16	0
19	70	4	19	16	89
20	74	0	1oz.	17	77¾
21	77¾	0			
22	81⅓	0			
23	85	2			

Note. 88⅞ cents, the value of 1 penny-weight of Eng. and Portugs. Gold.

TABLE II.

Value of French Gold, in dolls. cents, and mills, in the United States.

Gr.	Cts.	m.	Pwt.	$	Cts.
1	3	6 4/11	1	0	87¼
2	7	3	2	1	74½
3	10	9	3	2	61¾
4	14	5 5/11	4	3	49
5	18	2	5	4	36¼
6	21	8	6	5	23½
7	25	4 6/11	7	6	10¾
8	29	1	8	6	98
9	32	7 3/11	9	7	85¼
10	36	3 7/11	10	8	72½
11	40	0	11	9	59¾
12	43	6 4/11	12	10	47
13	47	3	13	11	34¼
14	50	9	14	12	21½
15	54	5½	15	13	08¾
16	58	2	16	13	96
17	61	8	17	14	83¼
18	65	4 6/11	18	15	70½
19	69	1	19	16	57¾
20	72	7	1oz.	17	45
21	76	3 7/11			
22	80	0			
23	83	6 4/11			

Note. 87¼ cents is the value of 1 pwt. of Fre Gold.

TABLE III.

Value of Spanish Gold, in dolls. cents, and mills, in the United States.

Gr.	Cts.	m.	Pwt.	$	Cts.
1	3	5	1	0	84
2	7	0	2	1	68
3	10	5¼	3	2	52
4	14	0⅕	4	3	36
5	17	5½	5	4	20
6	21	0	6	5	04
7	24	5⅝	7	5	88
8	28	0⅔	8	6	72
9	31	5¾	9	7	56
10	35	1	10	8	40
11	38	6	11	9	24
12	42	1	12	10	08
13	45	6	13	10	92
14	49	1	14	11	76
15	52	6½	15	12	60
16	56	1⅓	16	13	44
17	59	6⅓	17	14	28
18	63	1½	18	15	12
19	66	6½	19	15	96
20	70	2	1oz.	16	80
21	73	7			
22	77	2			
23	80	7			

Note. 1 pwt. is by law=84 cents.

Federal Money.	N. Hampshire, Massachusetts, R. Island, Con. & Virginia.	New York and North Carolina.	New Jersey, Pennsylvania, Delaware, and Maryland.	South Carolina and Georgia.	Canada and Nova-scotia.	French.
$ c.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	Liv. Tour. Sou.
0·0 1	18/25	24/25	9/10	14/25	3/5	1 1/20
0·0 2	1 11/25	1 23/25	1 8/10	1 3/25	1 1/5	2 1/10
0·0 3	2 4/25	2 22/25	2 7/10	1 17/25	1 4/5	3 3/20
0·0 4	2 22/25	3 21/25	3 6/10	2 6/25	2 2/5	4 1/5
0·0 5	3 3/5	4 4/5	4 5/10	2 4/5	3	5 1/4
0·0 6	4 3/25	5 19/25	5 4/10	3 9/25	3 3/5	6 3/10
0·0 7	5 1/25	6 18/25	6 3/10	3 23/25	4 1/5	7 7/20
0·0 8	5 19/25	7 17/25	7 2/10	4 12/25	4 4/5	8 2/5
0·0 9	6 12/25	[illegible] 16/25	8 1/10	5 1/25	5 2/5	9 9/20
0·1 0	7 1/5	9 3/5	9	5 3/5	6	10 1/2
[illegible]	1 2 2/5	1 7 1/5	1 6	11 1/5	1 0	1 1
0·3 0	1 9 3/5	2 4 4/5	2 3	1 4 4/5	1 6	1 11 1/2
0·4 0	2 4 4/5	3 2 2/5	3 0	1 10 2/5	2 0	2 2
0·5 0	3 0	4 0	3 9	2 4	2 6	2 12 1/2
0·6 0	3 7 1/5	4 9 3/5	4 6	2 9 3/5	3 0	3 3
0·7 0	4 2 2/5	5 7 1/5	5 3	3 3 1/5	3 6	3 13 1/2
0·8 0	4 9 3/5	6 4 4/5	6 0	3 8 4/5	4 0	4 4
0·9 0	5 4 4/5	7 2 2/5	6 9	4 2 2/5	4 6	4 14 1/2
1·0 0	6 0	8 0	7 6	4 8	5 0	5 5
2·0 0	12 0	16 0	15 0	9 4	10 0	10 10
3·0 0	18 0	1 4 0	1 2 6	14 0	15 0	15 15
4·0 0	1 4 0	1 12 0	1 10 0	18 8	1 0 0	21 0
5·0 0	1 10 0	2 0 0	1 17 6	1 3 4	1 5 0	26 5
6·0 0	1 16 0	2 8 0	2 5 0	1 8 0	1 10 0	31 10
[illegible]	2 2 0	2 16 0	2 12 6	1 12 8	1 15 0	36 15
8·0 0	2 8 0	3 4 0	3 0 0	1 17 4	2 0 0	42 0
9·0 0	2 14 0	3 12 0	3 7 6	2 2 0	2 5 0	47 5
10·0 0	3 0 0	4 0 0	3 15 0	2 6 8	2 10 0	52 10
20·0 0	6 0 0	8 0 0	7 10 0	4 13 4	5 0 0	105 0
30·0 0	9 0 0	12 0 0	11 5 0	7 0 0	7 10 0	157 10
40·0 0	12 0 0	16 0 0	15 0 0	9 6 8	10 0 0	210 0
50·0 0	15 0 0	20 0 0	18 15 0	11 13 4	12 10 0	262 10
60·0 0	18 0 0	24 0 0	22 10 0	14 0 0	15 0 0	315 0
70·0 0	21 0 0	28 0 0	26 5 0	16 6 8	17 10 0	367 10
80·0 0	24 0 0	32 0 0	30 0 0	18 13 4	20 0 0	420 0
90·0 0	27 0 0	36 0 0	33 15 0	21 0 0	22 10 0	472 10
100·0 0	30 0 0	40 0 0	37 10 0	23 6 8	25 0 0	525 0
200·0 0	60 0 0	80 0 0	75 0 0	46 13 4	50 0 0	1050 0
300·0 0	90 0 0	120 0 0	112 10 0	70 0 0	75 0 0	1575 0
400·0 0	120 0 0	160 0 0	150 0 0	93 6 8	100 0 0	2100 0
500·0 0	150 0 0	200 0 0	187 10 0	116 13 4	125 0 0	2625 0
600·0 0	180 0 0	240 0 0	225 0 0	140 0 0	150 0 0	3150 0
700·0 0	210 0 0	280 0 0	262 10 0	163 6 8	175 0 0	3675 0
800·0 0	240 0 0	320 0 0	300 0 0	186 13 4	200 0 0	4200 0
900·0 0	270 0 0	360 0 0	337 10 0	210 0 0	225 0 0	4725 0
1000·0 0	300 0 0	400 0 0	375 0 0	233 6 8	250 0 0	5250 0

N. Ham. Mass. R. Island, Con. and Virginia.	Federal Coin.	New York and North Carolina.	N. Jersey, Pennsylvania, Delaware, & Maryland.	S. Carolina and Georgia.	English Money.	French Money.
£ s. d.	$ Cts.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	Liv. Tour. } Sou.
1	0·01$\frac{7}{18}$	$1\frac{1}{3}$	$1\frac{1}{4}$	$\frac{7}{9}$	$\frac{3}{4}$	$1\frac{11}{24}$
2	0 02$\frac{7}{9}$	$2\frac{2}{3}$	$2\frac{1}{2}$	$1\frac{5}{9}$	$1\frac{1}{2}$	$2\frac{11}{12}$
3	0·04$\frac{1}{6}$	4	$3\frac{3}{4}$	$2\frac{3}{9}$	$2\frac{1}{4}$	4 $\frac{3}{8}$
4	0·05$\frac{5}{9}$	$5\frac{1}{3}$	5	$3\frac{1}{9}$	3	5 $\frac{5}{6}$
5	0·06$\frac{17}{18}$	$6\frac{2}{3}$	$6\frac{1}{4}$	$3\frac{8}{9}$	$3\frac{3}{4}$	$7\frac{7}{24}$
6	0·08$\frac{1}{3}$	8	$7\frac{1}{2}$	$4\frac{6}{9}$	$4\frac{1}{2}$	8 $\frac{3}{4}$
7	0 09$\frac{13}{18}$	$9\frac{1}{3}$	$8\frac{3}{4}$	$5\frac{4}{9}$	$5\frac{1}{4}$	$10\frac{5}{24}$
8	0·11$\frac{1}{9}$	$10\frac{2}{3}$	10	$6\frac{2}{9}$	6	11 $\frac{2}{3}$
9	0·12$\frac{1}{2}$	1 0	$11\frac{1}{4}$	7	$6\frac{3}{4}$	13 $\frac{1}{8}$
10	0·13$\frac{8}{9}$	1 $1\frac{1}{3}$	1 $0\frac{1}{2}$	$7\frac{7}{9}$	$7\frac{1}{2}$	$14\frac{7}{12}$
11	0·15$\frac{5}{18}$	1 $2\frac{2}{3}$	1 $1\frac{3}{4}$	$8\frac{5}{9}$	$8\frac{1}{4}$	$16\frac{1}{24}$
1 0	0·16$\frac{2}{3}$	1 4	1 3	$9\frac{3}{9}$	9	17 $\frac{1}{2}$
2 0	0·33$\frac{1}{3}$	2 8	2 6	1 $6\frac{6}{9}$	1 6	1 15
3 0	0·50	4 0	3 9	2 4	2 3	2 12 $\frac{1}{2}$
4 0	0·66$\frac{2}{3}$	5 4	5 0	3 $1\frac{3}{9}$	3 0	3 10
5 0	0·83$\frac{1}{3}$	6 8	6 3	3 $10\frac{6}{9}$	3 9	4 7 $\frac{1}{2}$
6 0	1·00	8 0	7 6	4 8	4 6	5 5
7 0	1·16$\frac{1}{3}$	9 4	8 9	5 $5\frac{3}{9}$	5 3	6 2 $\frac{1}{2}$
8 0	1·33$\frac{1}{3}$	10 8	10 0	6 $2\frac{6}{9}$	6 0	7 0
9 0	1·50	12 0	11 3	7 0	6 9	7 17 $\frac{1}{2}$
10 0	1·66$\frac{2}{3}$	13 4	12 6	7 $9\frac{3}{9}$	7 6	8 15
1 0 0	3·33$\frac{1}{3}$	1 6 8	1 5 0	15 $6\frac{6}{9}$	15 0	17 10
2 0 0	6·66$\frac{2}{3}$	2 13 4	2 10 0	1 11 $1\frac{3}{9}$	1 10 0	35 0
3 0 0	10·00	4 0 0	3 15 0	2 6 8	2 5 0	52 10
4 0 0	13·33$\frac{1}{3}$	5 6 8	5 0 0	3 2 $2\frac{6}{9}$	3 0 0	70 0
5 0 0	16·66$\frac{2}{3}$	6 13 4	6 5 0	3 17 $9\frac{3}{9}$	3 15 0	87 10
6 0 0	20·00	8 0 0	7 10 0	4 13 4	4 10 0	105 0
7 0 0	23·33$\frac{1}{3}$	9 6 8	8 15 0	5 8 $10\frac{6}{9}$	5 5 0	122 10
8 0 0	26·66$\frac{2}{3}$	10 13 4	10 0 0	6 4 $5\frac{3}{9}$	6 0 0	140 0
9 0 0	30·00	12 0 0	11 5 0	7 0 0	6 15 0	157 15
10 0 0	33·33$\frac{1}{3}$	13 6 8	12 10 0	7 15 $6\frac{6}{9}$	7 10 0	175 0
20 0 0	66·66$\frac{2}{3}$	26 13 4	25 0 0	15 11 $1\frac{3}{9}$	15 0 0	350 0
30 0 0	100 00	40 0 0	37 10 0	23 6 8	22 10 0	525 0
40 0 0	133·33$\frac{1}{3}$	53 6 8	50 0 0	31 2 $2\frac{6}{9}$	30 0 0	700 0
50 0 0	166·66$\frac{2}{3}$	66 13 4	62 10 0	38 17 $9\frac{3}{9}$	37 10 0	875 0
60 0 0	200 00	80 0 0	75 0 0	46 13 4	45 0 0	1050 0
70 0 0	233·33$\frac{1}{3}$	93 6 8	87 10 0	54 8 $10\frac{6}{9}$	52 10 0	1225 0
80 0 0	266 66$\frac{2}{3}$	106 13 4	100 0 0	62 4 $5\frac{3}{9}$	60 0 0	1400 0
90 0 0	300·00	120 0 0	112 10 0	70 0 0	67 10 0	1575 0
100 0 0	333·33$\frac{1}{3}$	133 6 8	125 0 0	77 15 $6\frac{6}{9}$	75 0 0	1750 0
200 0 0	666 66$\frac{2}{3}$	266 13 4	250 0 0	155 11 $1\frac{3}{9}$	150 0 0	3500 0
300 0 0	1000·00	400 0 0	375 0 0	233 6 8	225 0 0	5250 0
400 0 0	1333·33	533 6 8	500 0 0	311 2 $2\frac{6}{9}$	300 0 0	7000 0
500 0 0	1666·66	666 13 4	625 0 0	388 17 $9\frac{3}{9}$	375 0 0	8750 0

TABLE

OF THE VALUE OF SEVERAL PIECES OF COIN, IN THE FEDERAL COIN, AND THE SEVERAL CURRENCIES OF THE UNITED STATES.

		Federal Coin.		N. Hamp. Mass. R. Island, Con. and Virginia.			New York and North Carolina.			N. Jersey, Pennsylvania, Delaware and Maryland.			S. Carolina and Georgia.		
		Cents.		£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
$\frac{1}{16}$ of a Dollar		0·06	$\frac{1}{4}$			4$\frac{1}{2}$			6			5$\frac{5}{8}$			3$\frac{1}{2}$
$\frac{1}{2}$ of a Pistareen		0·10				7$\frac{1}{5}$			9$\frac{3}{5}$			9			5$\frac{3}{5}$
				Vir.		8									
$\frac{1}{9}$ of a Dollar		0·11	$\frac{1}{9}$			8			10$\frac{2}{3}$			10			6$\frac{2}{9}$
$\frac{1}{8}$ of ditto		0·12	$\frac{1}{2}$			9		1	0			11$\frac{1}{4}$			7
A Pistareen		0·20			1	2$\frac{2}{5}$		1	6		1	6			11$\frac{1}{5}$
An Eng. Shilling		0·22	$\frac{2}{9}$		1	4		1	7$\frac{3}{5}$		1	8		1	0$\frac{4}{9}$
$\frac{1}{4}$ of a Dollar		0·25			1	6		2	0		1	10$\frac{1}{2}$		1	2
Half ditto		0·50			3	0		4	0		3	9		2	4
A Dollar		1·00			6	0		8	0		7	6		4	8
French Crown		1·10			6	7		8	9$\frac{1}{2}$		8	3		5	1$\frac{3}{5}$
pwt.	gr.														
Fr. Guinea 5	5	4·54	$\frac{1}{3}$	1	7	3$\frac{1}{4}$	1	16	4$\frac{1}{2}$	1	14	0	1	1	2$\frac{1}{2}$
En. Guinea 5	6	4·66	$\frac{2}{3}$	1	8	0	1	17	4	1	15	0	1	1	3$\frac{1}{3}$
$\frac{1}{2}$ Johann. 9	0	8·00		2	8	0	3	4	0	3	0	0	1	17	4
Pistole 4	5	3·53	$\frac{1}{2}$	1	1	2$\frac{1}{4}$	1	8	3	1	6	6		16	5
Moidore 6	18	6·00		1	16	0	2	8	0	2	5	0	1	8	0
Doubloon 17	0	14·66	$\frac{2}{3}$	4	8	0	5	16	0	5	12	0	3	10	0

TABLE OF REFINER'S WEIGHTS.

Blanks
24 = 1 Perrot.
480 = 20 = 1 Mite.
9600 = 400 = 20 = 1 Grain.

Note. What they denominate a carat is the $\frac{1}{24}$ of a ℔ an oz. or any other weight.

DUTCH WEIGHTS FOR GOLD AND SILVER.

Note, 32 aces=1 engel, 20 engels=1 ounce, 8 ounces=1 mark, for gross gold. Also, 24 parts=1 grain, 12 grains=1 carat, 24 carats=1 mark, for fine gold.

The mark weights are 1 per cent. lighter than our Troy weight.

A TABLE OF COMMISSION OR BROKERAGE.

oods or ck sold.	at ½ per cent.			at 1 per cent.			at 1½ per cent.			at 2 per cent.			at 2½ per cent.			at 3 per cent.		
hill. 1	£	s.	d.	0	0	0	0	0	0	0	0	0	0	0	0¼	0	0	0¼
2	0	0	0¼	0	0	0¼	0	0	0¼	0	0	0¼	0	0	0½	0	0	0½
3	0	0	0¼	0	0	0¼	0	0	0½	0	0	0½	0	0	0¾	0	0	1
4	0	0	0¼	0	0	0½	0	0	0½	0	0	0¾	0	0	1	0	0	1¼
5	0	0	0¼	0	0	0½	0	0	0¾	0	0	1	0	0	1½	0	0	1¾
6	0	0	0¼	0	0	0¾	0	0	1	0	0	1¼	0	0	1¾	0	0	2
7	0	0	0½	0	0	0¾	0	0	1¼	0	0	1½	0	0	2	0	0	2¼
8	0	0	0½	0	0	1	0	0	1¼	0	0	1¾	0	0	2¼	0	0	2¾
9	0	0	0½	0	0	1	0	0	1½	0	0	2	0	0	2½	0	0	3
10	0	0	0½	0	0	1¼	0	0	1¾	0	0	2¼	0	0	3	0	0	3¼
11	0	0	0½	0	0	1½	0	0	1¾	0	0	2½	0	0	3¼	0	0	3¾
12	0	0	0½	0	0	1½	0	0	2	0	0	2¾	0	0	3½	0	0	4¼
13	0	0	0¾	0	0	1½	0	0	2¼	0	0	3	0	0	3¾	0	0	4½
14	0	0	0¾	0	0	1¾	0	0	2½	0	0	3¼	0	0	4	0	0	5
15	0	0	0¾	0	0	1¾	0	0	2½	0	0	3½	0	0	4½	0	0	5¼
16	0	0	0¾	0	0	2	0	0	2¾	0	0	3¾	0	0	4¾	0	0	5¾
17	0	0	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	6
18	0	0	1	0	0	2¼	0	0	3	0	0	4¼	0	0	5¼	0	0	6¼
19	0	0	1	0	0	2¼	0	0	3¼	0	0	4½	0	0	5½	0	0	6¾
unds 1	0	0	1¼	0	0	2½	0	0	3½	0	0	4¾	0	0	6	0	0	7
2	0	0	2½	0	0	5	0	0	7½	0	0	9½	0	1	0	0	1	2½
3	0	0	3¾	0	0	7	0	0	10¾	0	1	2¼	0	1	6	0	1	9½
4	0	0	5	0	0	9½	0	1	2¼	0	1	7	0	2	0	0	2	4¾
5	0	0	6	0	1	0	0	1	6	0	2	0	0	2	6	0	3	0
6	0	0	7¼	0	1	2¼	0	1	9½	0	2	4¾	0	3	0	0	3	7
7	0	0	8½	0	1	4¾	0	2	1	0	2	9½	0	3	6	0	4	2¼
8	0	0	9¾	0	1	7	0	2	4¾	0	3	2¼	0	4	0	0	4	9½
9	0	0	10¾	0	1	9½	0	2	8¼	0	3	7	0	4	6	0	5	4¼
10	0	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	6	0
20	0	2	0	0	4	0	0	6	0	0	8	0	0	10	0	0	12	0
30	0	3	0	0	6	0	0	9	0	0	12	0	0	15	0	0	18	0
40	0	4	0	0	8	0	0	12	0	0	16	0	1	0	0	1	4	0
50	0	5	0	0	10	0	0	15	0	1	0	0	1	5	0	1	10	0
60	0	6	0	0	12	0	0	18	0	1	4	0	1	10	0	1	16	0
70	0	7	0	0	14	0	1	1	0	1	8	0	1	15	0	2	2	0
80	0	8	0	0	16	0	1	4	0	1	12	0	2	0	0	2	8	0
90	0	9	0	0	18	0	1	7	0	1	16	0	2	5	0	2	14	0
100	0	10	0	1	0	0	1	10	0	2	0	0	2	10	0	3	0	0
200	1	0	0	2	0	0	3	0	0	4	0	0	5	0	0	6	0	0
300	1	10	0	3	0	0	4	10	0	6	0	0	7	10	0	9	0	0
400	2	0	0	4	0	0	6	0	0	8	0	0	10	0	0	12	0	0
500	2	10	0	5	0	0	7	10	0	10	0	0	12	10	0	15	0	0
600	3	0	0	6	0	0	9	0	0	12	0	0	15	0	0	18	0	0
700	3	10	0	7	0	0	10	10	0	14	0	0	17	10	0	21	0	0
800	4	0	0	8	0	0	12	0	0	16	0	0	20	0	0	24	0	0
900	4	10	0	9	0	0	13	10	0	18	0	0	22	10	0	27	0	0
1000	5	0	0	10	0	0	15	0	0	20	0	0	25	0	0	30	0	0

A TABLE

OF THE RETURNS OF THE NEAT PROCEEDS OF AN ACCOUNT OF SALES FROM A FACTOR TO HIS EMPLOYER, RESERVING HIS COMMISSIONS FOR REMITTANCE.

Neat Proceeds.			Sum to be remitted, reserving 2½ per ct. commism.			Sum to be remitted, reserving 5 per cent commism.			Neat proceeds.			Sum to be remitted, reserving 2½ per ct. commission.			Sum to be remitted, reserving 5 per ct. commission.		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
		3			3			2¾	6	0	0	5	17	0¾	5	14	5½
		4			4			3¾	7	0	0	6	16	7	6	13	4
		5			5			4¾	8	0	0	7	16	1¼	7	12	4½
		6			5¾			5¾	9	0	0	8	15	7¼	8	11	5¼
		7			6¾			6¾	10	0	0	9	15	1½	9	10	5¾
		8			7¾			7½	20	0	0	19	10	3	19	0	11½
		9			8¾			8½	30	0	0	29	5	4¼	28	11	5¼
		10			9¾			9½	40	0	0	39	0	5¾	38	1	10¾
		11			10¾			10½	50	0	0	48	15	7¼	47	12	4½
	1	0			11¾			11¼	60	0	0	58	10	8¾	57	2	10¼
	2	0		1	11½		1	10¾	70	0	0	68	5	10	66	13	4
	3	0		2	11¼		2	10¼	80	0	0	78	0	11½	76	3	9¼
	4	0		3	10¾		3	9¾	90	0	0	87	16	1	85	14	3½
	5	0		4	10½		4	9¼	100	0	0	97	11	2¾	95	4	9
	6	0		5	10¼		5	8½	200	0	0	195	2	5¼	190	9	6¼
	7	0		6	10		6	8	300	0	0	292	13	8	285	14	3¼
	8	0		7	9¾		7	7½	400	0	0	390	4	10½	380	19	0½
	9	0		8	9¼		8	6½	500	0	0	487	16	1¼	476	3	9½
	10	0		9	9		9	6¼	600	0	0	585	7	3¾	571	8	6¾
1	0	0		19	6¼		19	0½	700	0	0	682	18	4¾	666	13	4
2	0	0	1	19	0¾	1	18	1¼	800	0	0	780	9	9	761	18	1
3	0	0	2	18	6½	2	17	1¾	900	0	0	878	0	11¼	857	2	10
4	0	0	3	18	0½	3	16	2¼	1000	0	0	975	12	2¼	952	7	7¼
5	0	0	4	17	6¼	4	15	2¾									

Suppose I have the neat proceeds, or balance of an account of sales 325l. 17s. 9d. in my hands and would make remittance to my employer, reserving my commission at 2½ per cent. What sum must be remitted, so that my employer's account may be closed?

	£	s.	d.		£	s.	d.
Against	300	0	0	stands.	292	13	8
	20	0	0		19	10	3
	5	0	0		4	17	6¾
		10	0			9	9
		7	0			6	10
			9				8¾

To be remitted £317 18 9½ Answer.

A TABLE,

SHEWING THE NUMBER OF DAYS FROM ANY DAY IN ANY MONTH TO THE SAME DAY IN ANY OTHER MONTH THROUGH THE YEAR.

From	Jan	Feb	Mar	Ap	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
To Jan.	365	334	306	275	246	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
Apr.	90	59	31	365	335	304	274	243	212	182	151	121
May.	120	89	61	30	365	334	304	273	242	212	181	151
June.	151	120	92	61	31	365	335	304	273	243	212	182
July.	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

The Use of the preceding Table of number of days, will easily appear from the following examples.

Suppose the number of days between the first, or 10th, or 30th, &c. of January, and the 1st, or 10th or 30th, &c. of October, were required: Look in the column under January for October, and against that month you will find 273, which is the number of days between the said times; and so for the days between any other two months.

If the *given days* be *different*, it is only adding or subtracting their inequality to or from the *tabular number*.

How many days from the 6th of April to the 12th of January? From the 6th of April to the 6th of January is 275, and adding the 6 overplus days, it makes 281 days. And from the 5th of June to the 1st of February is 240 days.

Note. After February 31 (in leap years) increase each number with an unit or 1.

A TABLE *of the Measure of Length of the principal places in Europe, compared with the American yard.*

100	Aunes or Ells of England,	= 125
100	—— of Holland or Amsterdam, Hærlem, Leyden the Hague, Rotterdam, Nuremberg, and other cities of Holland,	= 75
100	—— of Barbant, or Antwerp,	= 76
100	—— of France and Oznaburg,	= 128½
100	—— of Hamburg, Frankfort, Leipsick, Bern, & Basil,	= 62½
100	—— of Breslau,	= 60
100	—— of Dantzick,	= 66¾
100	—— of Bergen and Drontheim,	= 68¼
100	—— of Sweden and Stockholm,	= 65¾
100	—— of St. Gall, for linens,	= 87½
100	—— of ditto for cloths,	= 67
100	—— of Geneva,	= 124¾
100	Canes of Marseilles and Montpelier,	= 214½
100	—— of Thoulouse and High Languedoc,	= 200
100	—— of Genoa, of 9 palms,	= 245¼
100	—— of Rome,	= 227¼
100	Varas of Spain,	= 93¾
100	—— of Portugal,	= 123
100	Cavidos of Portugal,	= 75
100	Brasses of Venice,	= 73½
100	—— of Bergamo,	= 71¼
100	—— of Florence and Leghorn,	= 64
100	—— of Milan,	= 58½

The use of the following Table, directing how to buy and sell by the hundred.

If you buy or sell any thing by the great hundred (112℔) and desire to know, by the ℔, what the hundred is valued at, observe the following examples.

1. If you buy sugar at 6¾d. per ℔, look for 6¾d. in the left hand column of the Table, against it in the second column, you will find £3 3s. which is the value of 1cwt. at that rate.

2. If 1cwt. (112℔) cost £9 4s. 4d. to know how much it is per ℔, look £9 4s. 4d. in the fourth column, and against it in the next left hand column, you will find 1s. 7¾d. which is the price per ℔.

Again, If you buy one hundred weight of goods for 9l. 4s. 4d. and retail it at 1s. 9¾d. per ℔, it comes at that rate to 10l. 3s.; then take 9l. 4s. 4d. from 10l. 3s. and, by the remainder, you will find that you have gained 18s. 8d.

And in this manner you may, with ease, calculate any quantity by the following Table.

A TABLE DIRECTING HOW TO BUY AND SELL BY THE HUNDRED.

d.	£	s.	d.	s.	d.	£	s.	d.	s.	d.	£	s.	d.
¼	0	2	4	1	0¼	5	14	4	2	0¼	11	6	4
½	0	4	8	1	0½	5	16	8	2	0½	11	8	8
¾	0	7	0	1	0¾	5	19	0	2	0¾	11	11	0
1	0	9	4	1	1	6	1	4	2	1	11	13	4
1¼	0	11	8	1	1¼	6	3	8	2	1¼	11	15	8
1½	0	14	0	1	1½	6	6	0	2	1½	11	18	0
1¾	0	16	4	1	1¾	6	8	4	2	1¾	12	0	4
2	0	18	8	1	2	6	10	8	2	2	12	2	8
2¼	1	1	0	1	2¼	6	13	0	2	2¼	12	5	0
2½	1	3	4	1	2½	6	15	4	2	2½	12	7	4
2¾	1	5	8	1	2¾	6	17	8	2	2¾	12	9	8
3	1	8	0	1	3	7	0	0	2	3	12	12	0
3¼	1	10	4	1	3¼	7	2	4	2	3¼	12	14	4
3½	1	12	8	1	3½	7	4	8	2	3½	12	16	8
3¾	1	15	0	1	3¾	7	7	0	2	3¾	12	19	0
4	1	17	4	1	4	7	9	4	2	4	13	1	4
4¼	1	19	8	1	4¼	7	11	8	2	4¼	13	3	8
4½	2	2	0	1	4½	7	14	0	2	4½	13	6	0
4¾	2	4	4	1	4¾	7	16	4	2	4¾	13	8	4
5	2	6	8	1	5	7	18	8	2	5	13	10	8
5¼	2	9	0	1	5¼	8	1	0	2	5¼	13	13	0
5½	2	11	4	1	5½	8	3	4	2	5½	13	15	4
5¾	2	13	8	1	5¾	8	5	8	2	5¾	13	17	8
6	2	16	0	1	6	8	8	0	2	6	14	0	0
6¼	2	18	4	1	6¼	8	10	4	2	6¼	14	2	4
6½	3	0	8	1	6½	8	12	8	2	6½	14	4	8
6¾	3	3	0	1	6¾	8	15	0	2	6¾	14	7	0
7	3	5	4	1	7	8	17	4	2	7	14	9	4
7¼	3	7	8	1	7¼	8	19	8	2	7¼	14	11	8
7½	3	10	0	1	7½	9	2	0	2	7½	14	14	0
7¾	3	12	4	1	7¾	9	4	4	2	7¾	14	16	4
8	3	14	8	1	8	9	6	8	2	8	14	18	8
8¼	3	17	0	1	8¼	9	9	0	2	8¼	15	1	0
8½	3	19	4	1	8½	9	11	4	2	8½	15	3	4
8¾	4	1	8	1	8¾	9	13	8	2	8¾	15	5	8
9	4	4	0	1	9	9	16	0	2	9	15	8	0
9¼	4	6	4	1	9¼	9	18	4	2	9¼	15	10	4
9½	4	8	8	1	9½	10	0	8	2	9½	15	12	8
9¾	4	11	0	1	9¾	10	3	0	2	9¾	15	15	0
10	4	13	4	1	10	10	5	4	2	10	15	17	4
10¼	4	15	8	1	10¼	10	7	8	2	10¼	15	19	8
10½	4	18	0	1	10½	10	10	0	2	10½	16	2	0
10¾	5	0	4	1	10¾	10	12	4	2	10¾	16	4	4
11	5	2	8	1	11	10	14	8	2	11	16	6	8
11¼	5	5	0	1	11¼	10	17	0	2	11¼	16	9	0
11½	5	7	4	1	11½	10	19	4	2	11½	16	11	4
11¾	5	9	8	1	11¾	11	1	8	2	11¾	16	13	8
12	5	12	0	2	0	11	4	0	3	0	16	16	0

A Comparison of the American Foot with the Feet of other Countries.

The American Foot being divided into 1000 parts, or into 12 inches, the feet of several other countries will be as follow.

	Parts.	Inch. Dec.
America,	1000	12·
London,	1000	12·
Antwerp,	946	11·352
Bologna,	1204	14·448
Bremen,	964	11·568
Cologne,	954	11·448
Copenhagen,	965	11·580
Amsterdam,	942	11·304
Dantzick,	944	11·328
Dort,	1184	14·208
Frankfort on the Main,	948	11·376
The Greek,	1007	12·084
Lorrain,	958	11·496
Mantua,	1569	18·828
Mecklin,	919	11·028
Middleburg,	991	11·892
France,	1066	12·792
Prague,	1026	12·312
Rhyneland or Leyden,	1033	12·396
Riga,	1831	21·972
Roman,	967	11·604
Old Roman,	970	11·640
Scotch,	1005	12·060
Strasburgh,	920	11·040
Toledo,	899	10·788
Turin,	1062	12·744
Venice,	1162	13·944

A Table *representing the Conformity of the weights of the principal trading Cities of Europe with those of America.*

lb.		of America.	
100	of England, Scotland and Ireland,	= 100lb.	Ooz.
100	of Amsterdam, Paris, Bordeaux, &c.	= 109	8
100	of Antwerp, or Brabant,	= 104	2⅗
100	of Rouen, the Viscounty,	= 113	14
100	of Lyons, the city,	= 94	3
100	of Rochelle,	= 110	9
100	of Thoulouse, and Upper Languedoc,	= 92	6
100	of Marseilles and Provence,	= 88	11
100	of Geneva,	= 123	
100	of Hamburg,	= 107	4
100	of Frankfort,	= 111	11
100	of Leipsick,	= 104	5
100	of Bremen,	= 110	0
100	of Russia,	= 88	4
100	of Vienna and Trieste,	= 123	0

A TABLE *representing the Conformity of the Weights of the principal trading Cities of Europe with those of America.*

lb.		of America.	
100 of Genoa,	=	73	
100 of Leghorn,	=	77	4
100 of Milan,	=	65	3
100 of Venice,	=	65	11
100 of Naples,	=	64	10
100 of Seville, Cadiz, &c.	=	103	2
100 of Portugal,	=	77	4
100 of Liege,	=	104	
100 of Spain,	=	97	
Note, The Spanish Arrobe is 25 Spanish pounds,	=	24	4

A TABLE to cast up wages, or expenses, for a year, at so much per day, week, or month.

Day.		*by week.*			*by month*			*by year.*		
s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
0	1	0	0	7	0	2	4	1	10	5
0	2	0	1	2	0	4	8	3	0	10
0	3	0	1	9	0	7	0	4	11	3
0	4	0	2	4	0	9	4	6	1	8
0	5	0	2	11	0	11	8	7	12	1
0	6	0	3	6	0	14	0	9	2	6
0	7	0	4	1	0	16	4	10	12	11
0	8	0	4	8	0	18	8	12	3	4
0	9	0	5	3	1	1	0	13	13	9
0	10	0	5	10	1	3	4	15	4	2
0	11	0	6	5	1	5	8	16	14	7
1	0	0	7	0	1	8	0	18	5	0
2	0	0	14	0	2	16	0	36	10	0
3	0	1	1	0	4	4	0	54	15	0
4	0	1	8	0	5	12	0	73	0	0
5	0	1	15	0	7	0	0	91	5	0
6	0	2	2	0	8	8	0	109	10	0
7	0	2	9	0	9	16	0	127	15	0
8	0	2	16	0	11	4	0	146	0	0
9	0	3	3	0	12	12	0	164	5	0
10	0	3	10	0	14	0	0	182	10	0
11	0	3	17	0	15	8	0	200	15	0
12	0	4	4	0	16	16	0	219	0	0
13	0	4	11	0	18	4	0	237	5	0
14	0	4	18	0	19	12	0	255	10	0
15	0	5	5	0	21	0	0	273	15	0
16	0	5	12	0	22	8	0	292	0	0
17	0	5	19	0	23	16	0	310	5	0
18	0	6	6	0	25	4	0	328	10	0
19	0	6	13	0	26	12	0	346	15	0
20	0	7	0	0	28	0	0	365	0	0

Note. In these two Tables the month is only 28 days

A TABLE to find wages or expenses for a month, week or day, at so much by the year.

by yr	*by month.*			*by week.*			*by day.*		
£.	£.	s.	d.	£.	s.	d.	£.	s.	d.
1	0	1	6½	0	0	4½	0	0	0¾
2	0	3	0¾	0	0	9¼	0	0	1¼
3	0	4	7¼	0	1	1¾	0	0	2
4	0	6	1¼	0	1	6½	0	0	2¾
5	0	7	8	0	1	11	0	0	3¼
6	0	9	2½	0	2	3½	0	0	4
7	0	10	9	0	2	8¼	0	0	4½
8	0	12	3¼	0	3	0¾	0	0	5¼
9	0	13	9¾	0	3	5½	0	0	6
10	0	15	4	0	3	10	0	0	6½
11	0	16	10½	0	4	2¾	0	0	7¼
12	0	18	5	0	4	7¼	0	0	8
13	0	19	11¼	0	4	11¾	0	0	8½
14	1	1	5¾	0	5	4½	0	0	9¼
15	1	3	0¼	0	5	9	0	0	9¾
16	1	4	6½	0	6	1¼	0	0	10½
17	1	6	1	0	6	6¼	0	0	11¼
18	1	7	7½	0	6	10¾	0	0	11¾
19	1	9	1¾	0	7	3½	0	1	0½
20	1	10	8¼	0	7	8	0	1	1¼
30	2	6	0¼	0	11	6	0	1	7¾
40	3	1	4½	0	15	4	0	2	2¼
50	3	16	8½	0	19	2¼	0	2	9
60	4	12	0¾	1	3	0¼	0	3	3½
70	5	7	4¾	1	6	10¼	0	3	10
80	6	2	9	1	10	8¼	0	4	4½
90	6	18	1	1	14	9¼	0	4	11¼
100	7	13	5	1	18	4¼	0	5	5¼
200	15	6	10¼	3	16	3½	0	10	11½
300	23	0	3¼	5	15	0¾	0	16	5¼
400	30	13	8½	7	13	5	1	1	11
500	38	7	1½	9	11	9¼	1	7	4¾
1000	76	14	3	19	3	6¾	2	14	9½

1787	Thursday
1788	*Saturday*†
1789	Sunday
1790	Monday
1791	Tuesday
1792	*Thursday*†
1793	Friday
1794	Saturday
1795	Sunday
1796	*Tuesday*†
1797	Wednesday
1798	Thursday
1799	Friday
1800	Saturday
1801	Sunday
1802	Monday
1803	Tuesday
1804	*Thursday*†
1805	Friday
1806	Saturday
1807	Sunday
1808	*Tuesday*†
1809	Wednesday
1810	Thursday
1811	Friday
1812	*Sunday*†
1813	Monday
1814	Tuesday
1815	Wednesday
1816	*Friday*†
1817	Saturday
1818	Sunday
1819	Monday

PERPETUAL ALMANACK:

February March November	*February* August.*	*May.*	*January October.*	*January* April July.*	*September December.*	*June.*
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

To find on what day of the week any given day in any month will fall, and the contrary.

EXAMPLE.

On what day of the week will the 31st day of January, 1810, fall?

Observe the day of the week, annexed to the year, in the outer column; then, in the Table, under the given month, in the upper row of figures, you will find the day of the month on which that day falls. According to this direction, I find that, in January, 1810, Thursday is the 4th, 11th, 18th, and 25th, then reckoning on, Friday 26, Saturday 27th, &c. I find the 31st day falls on Wednesday; or, that the last Wednesday in January is the 31st day.

Note. In leap years, January and February must be taken in the columns marked thus, *. Leap years are marked, in the outer columns, thus, †.

The years 1800, 1900, and all other 100th years, not to be leap years, except the years 2000, 2400, 2800, and every 400th year following, which must be leap years.

1820	*Wednesday*†
1821	Thursday
1822	Friday
1823	Saturday
1824	*Monday*†
1825	Tuesday
1826	Wednesday
1827	Thursday
1828	*Saturday*†
1829	Sunday
1830	Monday
1831	Tuesday
1832	*Thursday*†
1833	Friday
1834	Saturday
1835	Sunday
1836	*Tuesday*†
1837	Wednesday
1838	Thursday
1839	Friday
1840	*Sunday*†
1841	Monday
1842	Tuesday
1843	Wednesday
1844	*Friday*†
1845	Saturday
1846	Sunday
1847	Monday
1848	*Wednesday*†
1849	Thursday
1850	Friday
1851	Saturday
1852	*Monday*†
1853	Tuesday

A TABLE for reducing Troy wt. to Avoirdupois.

Tr.	Av.	Troy	Avoirdupois.		
gr.	drm.	oz.	lb.	oz.	dr.
1	·04	1		1	1·55
2	·07	2		2	3·11
3	·11	3		3	4·66
4	·15	4		4	6·22
5	·18	5		5	7·77
6	·22	6		6	9·32
7	·26	7		7	10·89
8	·29	8		8	12·44
9	·33	9		9	14
10	·36	10		10	15·56
11	·40	11		22	1·09
12	·44	lb.			
13	·47	1	0	13	2·65
14	·51	2	1	10	5·3
15	·55	3	2	7	8
16	·58	4	3	4	10·6
17	·62	5	4	1	13·25
18	·66	6	4	14	15·9
19	·69	7	5	12	2·56
20	·73	8	6	9	5·21
21	·77	9	7	6	7·86
22	·80	10	8	3	10·52
23	·84	20	16	7	5·03
pw.		30	24	10	15·54
1	0·88	40	32	14	10·05
2	1·75	50	41	2	4·57
3	2.63	60	49	5	15·08
4	3·51	70	57	9	9·6
5	4·39	80	65	13	4·11
6	5·27	90	74	0	13·62
7	6·14	100	82	4	9·15
8	7·02	200	164	9	2·28
9	7·9	300	246	13	11·42
10	8·78	400	329	2	4·57
11	9·65	500	411	6	13·71
12	10·53	600	493	11	6·85
13	11·41	700	576	0	0
14	12·29	800	658	4	9·14
15	13·16	900	740	9	2·28
16	14·04	1000	822	13	11·42
17	14·92	2000	1645	11	6·84
18	15·79	3000	2528	9	2·26
19	16·67	4000	3291	6	13·68

A TABLE for reducing Avoirdudois wt. info Troy.

Av.	Troy,				Avoir.	Troy.			
dr.	lb.	oz.	pw.	gr.	lb.	lb.	oz.	pw.	gr.
½				13·67	1	1	2	11	16
¾			0	20·51	2	2	5	3	8
1			1	3·34	3	3	7	15	0
2			2	6 68	4	4	10	6	16
3			3	10 02	5	6	0	18	8
4			5	13·36	6	7	3	10	0
5			6	16·7	7	8	6	1	16
6			7	20 04	8	9	8	13	8
7			8	23·38	9	10	11	5	0
8			9	2·72	10	12	1	16	16
9			10	6·06	20	24	3	13	8
10			11	9·4	30	36	5	10	0
11			12	12·74	40	48	7	6	16
12			13	15·08	50	60	9	3	8
13			14	19·42	60	72	11	0	0
14			15	22·76	70	85	0	16	16
15			17	2·1	80	97	2	13	8
oz					90	109	4	10	0
1		0	18	5·5	100	121	6	6	16
2		1	16	11	200	243	0	13	8
3		2	14	16·5	300	364	7	0	0
4		3	12	22	400	486	1	6	16
5		4	11	3·5	500	607	7	13	8
6		5	9	9	600	729	2	0	0
7		6	7	14·5	700	850	8	6	16
8		7	5	20	800	972	2	13	8
9		8	4	1·5	900	1093	9	0	0
10		9	2	7	1000	1215	3	6	16
11		10	0	12·5	2000	2430	6	13	8
12		10	18	18	3000	3645	10	0	0
13		11	16	23·5	4000	4861	1	6	16
14	1	0	15	5	5000	6076	4	13	8
15	1	1	13	10·5	6000	7291	8	0	0

I. TABLE

OF THE MONEY OF COMMERCIAL COUNTRIES, WITH THEIR VALUE IN STERLING AND FEDERAL MONEY.

MONEY is of two kinds, *Real*, and *Imaginary*, or *coin* and *money of account*.

Real Money is the coin of a country, whose value is established at a certain amount.

Imaginary money is merely a denomination to express a certain sum, which is not represented by any coin, as a *pound*, a *livre*; a *mill*. The denominations of imaginary money are employed with or without those of real money in keeping accounts.

UNITED STATES.

	Sterling. £.	s.	d.	Dolls.	Cts.
Eagle, gold coin,	2	5	0*	10	00
Half Eagle, do.	1	2	6	5	00
Quarter do. do.	0	11	3	2	50
Dollar, silver,	0	4	6	1	00
Half dollar, do.	0	2	3	0	50
Quarter doll. do.	0	1	1½	0	25
Dime, do.	0	0	5⅖	0	10
Half dime, do.	0	0	2 7/10	0	05
Cent, copper,	0	0	0 27/50	0	01
Half cent, do.	0	0	0 27/100	0	00½

ENGLAND AND SCOTLAND.

	£.	s.	d.	$	Cts. Deci
Pound†=20 shillings,	1	0	0	4	44·44 4/9
Shilling=12 pence,	0	1	0	0	22·22 2/9
Penny=4 farthings,	0	1	1	0	01·85 5/27
9 pence,	0	0	9	0	16·66⅔
6 pence,	0	0	6	0	11,11 1/9
4 pence or 1 groat,	0	0	4	0	07·40 20/27
3 pence,	0	0	3	0	05·55 5/9
2 pence,	0	0	2	0	03 70 5/9
6¼ pence,	0	0	6¼	0	12·50
3⅛ pence,	0	0	3⅛	0	06·25
Crown, silver,	0	5	0	1	11.11 1/9
Guinea, gold,	1	1	0	4	66·66⅔

The English crown passes in the U. S. at, $1 10.

* Estimating the dollar at four shillings and six pence sterling, this is the sterling value of the Eagle; but the value of the Eagle in English gold is only £2 3s. 9¾d. or 525¾d. sterling.

† The pound was anciently a pound Troy of silver; it now contains only one-third as much. The pound sterling in the time of William the Conqueror or 1066, A. D. was to the present pound sterling as 31 to 10. Articles were then about ten times cheaper than at present. So that his revenue of £400,000 sterling, was equal to about £12,400,000 sterling at present.

IRELAND.

Denominations are the same as in England, but sterling is to Irish money as 13 to 12, or £12 sterling= £13 Irish.

	£.	s.	d.	D.	c.	dec.
Pound=20 shillings,	0	18	$5\frac{7}{13}$	4	10·25$\frac{23}{117}$	
Shilling=12 pence,	0	0	$11\frac{1}{13}$	0	20·51$\frac{1}{4}$—	
13 pence,	0	1	0	0	22·22$\frac{2}{9}$	
22s. 9 pence=Eng. guinea,	1	1	0	4	66·66$\frac{2}{3}$	
4s. 10$\frac{1}{2}$d.	0	4	6	1	00	

FRANCE.

	£.	s.	d.	D.	c.	dec.
Livre=20 sols,	0	0	10	0	18·51$\frac{23}{27}$	
Sol=12 deniers,	0	0	$0\frac{1}{2}$	0	00·92$\frac{16}{27}$	
Denier,	0	0	$0\frac{1}{24}$	0	00 07$\frac{4}{10}$+	
Crown or Ecu=6 livres,	0	5	0	1	11·11$\frac{1}{9}$	
Pistole=10 livres,	0	8	4	1	85·18$\frac{14}{27}$	
Louis d'or=24 do.	1	0	0	4	44·44$\frac{4}{9}$	
Ecu of exchange=60 sols,	0	2	6	0	55.55	
The livre or livre tournois is estimated in the United States, at				0	18$\frac{1}{2}$	

NEW COINS.

	£.	s.	d.	D.	c.	dec.
Franc=$\frac{81}{80}$ livre tournois,	0	0	$10\frac{1}{8}$	0	18·75—	
Decim=$\frac{1}{10}$ franc,	0	0	$1\frac{1}{80}$	0	01·87$\frac{1}{2}$—	
Centim=$\frac{1}{100}$ franc,	0	0	$0\frac{81}{800}$	0	00 18$\frac{3}{4}$—	
Five franc piece, silver,	0	4	$2\frac{5}{8}$	0	93·75—	
2 francs,	0	1	$8\frac{1}{4}$	0	37·50—	
Crown of exchang=3 livres,	0	2	6	0	55 55$\frac{5}{9}$	
In the United States the five franc pieces are estimated at				0	93·30	
And the franc at				0	18·73$\frac{1}{8}$	

There are silver coins of the value of 1 franc, and of $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ franc. The franc is to weigh $18\frac{841}{1000}$ grs. composed of $\frac{9}{10}$ of pure silver, and $\frac{1}{10}$ alloy. The gold coins are also to contain $\frac{1}{10}$ of alloy.

	£.	s.	d.	D.	c.
20 francs, gold,	0	16	$10\frac{1}{2}$	3	75
40 do. do.	1	13	9	7	50

SPAIN.

Accounts are kept in *money of vellon* or *current* dollars, and money of *plate*, or hard or plate dollars.

	£.	s.	d.	D.	c.
Dollar, plate=10 rials plate,	0	4	6	1	00
Doll. current or piastre=8 do.	0	3	$7\frac{1}{5}$	0	80
Pistarine=2 rials plate,	0	0	$10\frac{4}{5}$	0	20
Rial, plate=34 maravedies,	0	0	$0\frac{2}{5}$	0	10
Quarto=4 do.	0	0	$5\frac{54}{85}$	0	01·17$\frac{11}{17}$

	Sterling. £	s.	d.	$	c.
Maravedi,	0	0	0 27/170	0	00·29 7/17
100 rials plate=188 4/17 rials vel.	2	5	0	10	00·00
100 rials vellon=53 1/8 plate,	1	3	10 7/8	5	31·25
Rial vellon,	0	0	2 139/160	0	05·31 1/4
32 rials vellon=17 rials plate.					
Ducat of Exchange=375 mar.	0	4	11 19/34	1	10·29 7/17
Pistole=36 rials plate,	0	16	2 2/5	3	60·

In the United States, the rial vellon is estimated at 5 cents, or half a rial plate, and equal to 20 to a Spanish Dollar.

The lower denominations are different in some parts of Spain. Thus, accounts are sometimes kept in the following denominations at

CADIZ.

	£	s.	d.	$	c.
Plate dollar=20 rials vellon,	0	4	6	1	00
Rial vellon=8 1/2 quartos,	0	0	2 7/10	0	05
Quarto,	0	0	0 27/85	0	00·58 14/17

BARCELONA, VALENCIA, SARAGOSSA.

	£	s.	d.	$	c.
Plate dollar=37 1/2 sols,	0	4	6	1	00
Livre=20 sols,	0	2	4 4/5	0	53 1/3
Sol=12 deniers,	0	0	1 11/25	0	02 2/3
Denier,	0	0	0 3/25	0	00 2/9

BILBOA.

	£	s.	d.	$	c.
Plate dollar=20 rials vellon,	0	4	6	1	00·00
Rial vellon=34 maravedies,	0	0	2 7/10	0	05·00
Maravedie,	0	0	0 27/340	0	00·14 12/17

ST. LUCAR.

	£	s.	d.	$	c.
Plate dollar=10 rials plate,	0	4	6	1	00·00
Rial plate=16 quartos,	0	0	5 4/10	0	10·00
Quarto,	0	0	0 27/80	0	00·62 1/2

PORTUGAL.

	£	s.	d.	$	c.
Millrea=1000 reas=10 testoons,	0	5	7 1/2	1	25·00
Rea,	0	0	0 27/400	0	00·12 1/2
Vintin=20 reas,	0	0	1 7/20	0	02·50
Testoon=5 vintins=100 reas,	0	0	6 3/4	0	12·50
Crusade of exchange=4 testoons,	0	2	3	0	50·00
New crusade=4 4/5 do.	0	2	8 2/5	0	60·00
Moidore=48 testoons,	1	7	0	6	00·00
Joanese or 1/2 Johan.=64 testoons,	1	16	0	8	00·00

The millrea is 125 cents in the United States.

HOLLAND.

	Sterling.				
	£	s.	d.	$	c.
Guilder or Florin=20 stivers,	0	1	$9\frac{6}{10}$	0	40·00
Stiver=2 grotes,	0	0	$1\frac{8}{100}$	0	02·00
Grote=8 pennings,	0	0	$0\frac{54}{100}$	0	01·00
Penning,	0	0	$0\frac{27}{400}$	0	00·12½
Shilling=6 stivers=12 grotes,	0	0	$6\frac{48}{100}$	0	12·00
Pound Flemish=20s.=6 guilders,	0	10	$9\frac{6}{10}$	2	40·00
Rix dollar=2½ guilders,	0	4	6	1	00·00
Ducat,	1	16	0	8	00·00
Sovereign,	1	7	0	6	00·00

The florin is 40 cents in the United States.

HAMBURGH.

	£	s.	d.	$	c.
Pound=20 shillings Flemish,	0	11	3	2	50
Fl. shil.=12 grotes or pence Fl.	0	0	6¾	0	12·500
Grote or penny=6 deniers,	0	0	$0\frac{9}{16}$	0	$01\frac{1}{24}$
Mark banco=32 grotes or 2⅔ shil.	0	1	6	0	33⅓
Rix dollar=3 marks,	0	4	6	1	00
Stiver or shilling lubs,	0	0	1⅛	0	$02\frac{1}{12}$
Ducat=6½ marks,	0	9	9	2	16⅔

The Bank money of Hamburgh is superior to the currency; the *agio*, or rate, varies, and is sometimes 20 per cent. or more in favor of the Bank money.

BREMEN.

	£	s.	d.	$	c.
Rix dollar=72 grotes=2¼ marks,	0	3	4½	0	75
Grote,	0	0	$0\frac{9}{16}$	0	$01\frac{1}{24}$

ANTWERP.

	£	s.	d.	$	c.
Guilder=3⅓ shillings=40 grotes,	0	1	9⅗	0	40
Shilling=12 grotes,	0	0	$6\frac{12}{25}$	0	12
Grote,	0	0	$0\frac{27}{50}$	0	01
Stiver=2 grotes,	0	0	$1\frac{4}{50}$	0	02

VIENNA AND TRIESTE.

	£	s.	d.	$	c.
Florin=60 cruitzers,	0	2	4	0	$51·85\frac{5}{27}$
Cruitzer=4 fenings,	0	0	$0\frac{7}{15}$	0	00·866—
Batzen=4 cruitzers,	0	0	$1\frac{13}{15}$	0	03·46
Rix dollar=1½ florins,	0	3	6	0	77$\frac{7}{9}$
Livre=20 soldi,	0	0	5⅓	0	09·87
Specie dollar=30 batzen,	0	4	8	1	03·7
Ducat=60 batzen,	0	9	4	2	07·4

The value of these denominations is the same generally through Austria, Swabia, Franconia, Bohemia, Silesia, and Hungary.

HANOVER AND SAXONY.

	Sterling. £	s.	d.	$	c.
Rix dollar=24 groshen,	0	3	6	0	77$\frac{7}{9}$
Grosh=12 fening,	0	0	1$\frac{3}{4}$	0	03$\frac{13}{54}$
Gould or guilder=16 groshen,	0	2	4	0	51·85$\frac{5}{27}$
Ducat=4 guilders or goulds,	0	9	4	2	07·40
Specie dollar or hard dollar,	0	4	8	1	03·7

POLAND AND PRUSSIA.

	£	s.	d.	$	c.
Rix dollar=90 groshen,	0	3	6	0	77$\frac{7}{9}$
Florin =30 do.	0	1	2	0	25$\frac{25}{27}$
Grosh=3 shelons,	0	0	0$\frac{7}{15}$	0	00·866—
Ducat=8 florins,	0	9	4	2	07·4
Frederic d'or=5 rix dollars,	0	17	6	3	88$\frac{8}{9}$

LIVONIA.

	£	s.	d.	$	c.
Rix dollar=90 groshen,	0	3	6	0	77$\frac{7}{9}$
Florin =30 do.	0	1	2	0	25$\frac{25}{27}$
Marc = 6 do.	0	0	2$\frac{1}{3}$	0	05$\frac{5}{27}$
Grosh=6 blackens,	0	0	0$\frac{7}{15}$	0	00·866—

RUSSIA.

	£	s.	d.	$	c.
Ruble=100 copecs,	0	4	6	1	00
Poltin= 50 do.	0	2	3	0	50
Copec=4 poluscas,	0	0	0$\frac{27}{50}$	0	01
Zervenitz=2 rubles,	0	9	0	2	00

DENMARK AND NORWAY.

	£	s.	d.	$	c.
Rix dollar=6 marks,	0	4	6	1	00
Crown =4 do.	0	3	0	0	66$\frac{2}{3}$
Marc=16 skillings,	0	0	9	0	16$\frac{2}{3}$
Skilling,	0	0	0$\frac{9}{16}$	0	01$\frac{1}{27}$
Ducat=11 marcs,	0	8	3	1	83$\frac{1}{5}$

SWEDEN AND LAPLAND.

	£	s.	d.	$	c.
Rix dollar=3 silver dollars,	0	4	8	1	03$\frac{19}{27}$
Silver dollar=3 copper dollars,	0	1	6$\frac{2}{3}$	0	34$\frac{46}{81}$
Copper dollar=4 copper marcs,	0	0	6$\frac{2}{9}$	0	11$\frac{127}{243}$
Copper marc=8 runstics,	0	0	1$\frac{5}{9}$	0	02$\frac{214}{243}$
Ducat=2 rix dollars,	0	9	4	2	07$\frac{11}{27}$

SWITZERLAND.

	£	s.	d.	$	c.
Rix dollar=108 cruitzers,	0	4	6	1	00
Cruitzer=4 fenings,	0	0	0$\frac{1}{3}$	0	00·92$\frac{1}{27}$

	Sterling. £	s.	d.	$	c.
Fening=3 rap,	0	0	$0\frac{1}{8}$	0	$00·23\frac{4}{27}$
Sol=12 fenings,	0	0	$1\frac{1}{2}$	0	$02·77\frac{7}{9}$
Livre or gulden=20 sols,	0	2	6	0	$55\frac{5}{9}$

GENOA, CORSICA, &c.

	£	s.	d.	$	c.
Pezzo of exchange=115 soldi,	0	4	2	0	$92·59\frac{7}{27}$
Soldi=12 denari,	0	0	$0\frac{10}{23}$	0	00·30
Lire=20 soldi,	0	0	$8\frac{16}{23}$	0	16·10
Testoon=30 soldi,	0	1	$0\frac{44}{46}$	0	24·15
Pistole=20 lires,	0	14	$5\frac{1}{23}$	3	22·22

SARDINIA, TURIN, AND SAVOY.

	£	s.	d.	$	c.
Scudi=6 florins,	0	4	6	1	00
Florin=12 soldi,	0	0	9	0	$16\frac{2}{3}$
Soldi=12 denari,	0	0	$0\frac{3}{4}$	0	$01\frac{7}{18}$
Lire=20 soldi,	0	1	3	0	$27\frac{7}{9}$
Ducatoon=7 florins,	0	5	3	1	$16\frac{2}{3}$
Pistole=13 lires,	0	16	3	3	$61\frac{1}{9}$
Louis d'or=16 lires,	1	00	0	4	$44\frac{4}{9}$

ROME AND CIVITA VECCHIA.

	£	s.	d.	$	c.
Current crown=10 julios	0	5	0	1	$11\frac{1}{9}$
Julio=8 bayocs	0	0	6	0	$11\frac{1}{9}$
Bayoc=5 quatrini	0	0	$0\frac{3}{4}$	0	$01\frac{7}{18}$
Stamped Julio=10 bayocs	0	0	$7\frac{1}{2}$	0	$13\frac{8}{9}$
Stamped crown=12 julios	0	6	0	1	$33\frac{1}{3}$
Pistole=31 julios	0	15	6	3	$34\frac{1}{9}$

FLORENCE AND LEGHORN.

	£	s.	d.	$	c.
Piastre=6 lires	0	4	0	0	$88\frac{8}{9}$
Lire=20 soldi	0	0	8	0	$14·81\frac{1}{3}$
Soldi=12 denari	0	0	$0\frac{2}{3}$	0	01·23
Ducat=$7\frac{1}{2}$ lires	0	5	0	1	$11\frac{1}{9}$
Pistole=22 lires	0	14	8	3	25·93

NAPLES.

	£	s.	d.	$	c.
Ducat=100 grains	0	3	4	0	$74·07\frac{11}{27}$
Grain=3 quatrini	0	0	$0\frac{4}{10}$	0	00·74
Carlin=10 grains,	0	0	4	0	07·40
Ounce=3 ducats	0	10	0	2	$22·22\frac{6}{27}$
Pistole=46 carlins	0	15	4	3	$33·33\frac{1}{3}$

VENICE.

	£	s.	d.	$	c.
Ducat=24 gros,	0	4	4	0	$96\frac{8}{27}$
Gros or grosi=$5\frac{1}{6}$ soldi,	0	0	$2\frac{1}{6}$	0	$04\frac{1}{81}$
Soldi=12 denari,	0	0	$0\frac{13}{31}$	0	$00·77\frac{2}{3}$
Lire=20 soldi,	0	0	$8\frac{17}{31}$	0	15·53

In Leghorn, a piastre = 20 soldi of Genoa.
Naples, a ducat = 86 " "
Milan, a crown = 80 " "
Sicily, a crown = 127 $\frac{3}{5}$ " "

PALERMO IN SICILY.

	Sterling. £	s.	d.	$	c.
Once or Onge=30 tarins,	0	10	9 $\frac{3}{5}$	2	40
Tarin=20 grains,	0	0	4 $\frac{8}{25}$	0	08
Grain,	0	0	0 $\frac{27}{125}$	0	00·4
Dollar of Sicily=240grs.=12 tarin,	0	4	3 $\frac{21}{25}$	0	96
Spanish dollar=252 grains,	0	4	6	1	00

TURKEY.

	£	s.	d.	$	c.
Piastre=80 aspers,	0	4	0	0	88 $\frac{8}{9}$
Asper=4 mangars,	0	0	0 $\frac{3}{5}$	0	01·11 $\frac{1}{9}$
Parac=3 aspers,	0	0	1 $\frac{4}{5}$	0	03·33 $\frac{1}{3}$
Ostic=10 do.= $\frac{1}{2}$ solota,	0	0	6	0	11·11 $\frac{1}{9}$
Caragrouch=100 aspers,	0	5	0	1	11·11 $\frac{1}{9}$
Xeriff=10 solotas=200 aspers,	0	10	0	2	22·22 $\frac{2}{9}$

SMYRNA.

	£	s.	d.	$	c.
Piastre=100 aspers or 40 paras,	0	1	3 $\frac{15}{17}$	0	29·41 $\frac{3}{17}$
Asper,	0	0	0 $\frac{27}{170}$	0	00·29
136 Paras=Spanish dollar,	0	4	6	1	00 00
Para,	0	0	0 $\frac{27}{68}$	0	00·73 $\frac{9}{17}$
10 Spanish dollars=34 piastres,	2	5	0	10	00

ARABIA.

	£	s.	d.	$	c.
Piastre=60 comashees=80 caveers,	0	4	6	1	00
Comashee=7 carrets,	0	0	0 $\frac{9}{10}$	0	01·66 $\frac{2}{3}$
Larin=80 do.	0	0	10 $\frac{2}{7}$	0	19 05
Sequin=100 comashees,	0	7	6	1	66 66 $\frac{2}{3}$
Tomond=80 Larins,	3	7	6	15	00

EGYPT.

	£	s.	d.	$	c.
Piastre=80 aspers,	0	4	0	0	88 $\frac{8}{9}$
Asper,	0	0	0 $\frac{6}{10}$	0	01 11 $\frac{1}{10}$
Medin=3 aspers,	0	0	1 $\frac{4}{5}$	0	03·33 $\frac{1}{10}$
Dollar=30 medins,	0	4	6	1	00
Ecu or crown=96 aspers,	0	5	0	1	11·11 $\frac{1}{9}$
Sultanin=2 ecu,	0	10	0	2	22·22 $\frac{2}{9}$
Pargo dollar=70 medini,	0	10	6	2	33·33 $\frac{1}{3}$

ALGIERS, TUNIS AND TRIPOLI.

	Sterling. £	s.	d.	$	c.
Dollar=4 doubles,	0	4	6	1	00
Double=2 rials,	0	1	$1\frac{1}{2}$	0	25
Rial=10 aspers,	0	0	$6\frac{3}{4}$	0	$12\frac{1}{2}$
Pistole=15 doubles,	0	16	$10\frac{1}{2}$	2	75
Zequin=180 aspers,	0	10	$1\frac{1}{2}$	2	25
Pataca Chica,	0	0	$11\frac{1}{2}$	0	$21\frac{8}{27}$
Sultanin=$8\frac{1}{2}$ patacas chicas,	0	8	$1\frac{3}{4}$	1	81
Piastre=3 " "	0	2	$10\frac{1}{2}$	0	64

MOROCCO, FEZ, TANGIERS, AND SALLEE.

	£	s.	d.	$	c.
Dollar=28 blanquils,	0	4	6	1	00
Blanquil=24 fluces,	0	0	$1\frac{13}{14}$	0	$03·57\frac{1}{7}$
Ounce=4 blanquils,	0	0	$7\frac{5}{7}$	0	$14·28\frac{4}{7}$
Quarto=14 "	0	2	3	0	50
Zequin=56 "	0	9	0	2	00
Pistole=100 "	0	16	$0\frac{6}{7}$	3	$57\frac{1}{7}$

PERSIA.

	£	s.	d.	$	c.
Bovello=12 abashees,	0	16	0	3	$55·55\frac{5}{9}$
Abashee=4 shahees,	0	1	4	0	29·63—
Shahee=10 coz,	0	0	4	0	07·41—
Larin =25 do.	0	0	10	0	18·51
Or=5 abashees,	0	6	8	1	48·14

BOMBAY.

	£	s.	d.	$	c.
Rupee=4 quarters,	0	2	3	0	50
Quarter=20 pices=100 reas,	0	0	$6\frac{3}{4}$	0	$12\frac{1}{2}$
Rea,	0	0	$0\frac{27}{400}$	0	00·125
Pagoda=14 quarters,	0	7	$10\frac{1}{2}$	1	75
Rupee, gold=60 do.	1	13	9	7	50
Current mohuss=15 current rupees,	1	1	$1\frac{1}{8}$	4	$68\frac{3}{4}$
Current rupee=50 pice,	0	1	$4\frac{7}{8}$	0	$31\frac{1}{4}$

MADRAS AND PONDICHERRY.

	£	s.	d.	$	c.
Pagoda=36 fanams,	0	8	$3\frac{9}{25}$	1	84
Fanam=8pice=30 cash,	0	0	$2\frac{3}{4}$	0	$05\frac{1}{9}$
Pice=2 viz=10 cash,	0	0	$0\frac{11}{32}$	0	00·64—
Rupee=10 fanams,	0	2	$3\frac{3}{5}$	0	$51\frac{1}{9}$
Crown=2 rupees,	0	4	$7\frac{1}{5}$	1	$02\frac{2}{9}$
Rupee,* gold=4 pagodas,	1	13	$1\frac{11}{25}$	7	36
Bengal, or new Sicca rupee,	0	2	3	0	50

* *Lack of rupees* is 100000, and 340 Sicca rupees pass currently for 100 Star pagodas.

Pagoda is 184 cents in the United States.

Sea shells, called *cowries* are used for change; their value varies with their quality.

CALCUTTA AND CALLICUT.

	Sterling. £	s.	d.	$	c.
Rupee=16 annas,	0	2	3	0	50
Anna=12 pices,	0	0	$1\frac{11}{16}$	0	$03\frac{1}{8}$
Fanam=4 do.	0	0	$6\frac{3}{4}$	0	$12\frac{1}{2}$
Crown or Ecu=2 rupees,	0	4	6	1	00
Pagoda=56 annas,	0	7	$10\frac{1}{2}$	1	75

100 Sicca rupees=116 current rupees.

PEGU, JAVA, SUMATRA, &c.

	£	s.	d.	$	c.
Dollar=900 fettees,	0	4	6	1	00
Fettee=10 cori,	0	0	$0\frac{3}{50}$	0	$00\frac{1}{9}$
Tical=500 fettees,	0	2	6	0	$55\frac{5}{9}$
Rial, crown or ecu=2 ticals,	0	5	0	1	$11\frac{1}{9}$

CHINA.

	£	s.	d.	$	c.
Tale=10 maces,	0	6	8	1	$48\frac{4}{27}$
Mace=10 candereens,	0	0	8	0	14·81
Candereen=10 cash,	0	0	$0\frac{4}{5}$	0	01·48
The Spanish dollar passes at 72 candereens, which makes the candereen worth,				0	01·38$\frac{8}{9}$
But the tale is reckoned in the United States,				1	48·00

JAPAN.

	£	s.	d.	$	c.
Tale=10 mace=rix dollar,	0	3	$4\frac{1}{2}$	0	75
Mace=10 candereens,	0	0	$4\frac{1}{20}$	0	$07\frac{1}{2}$
Candereen=2 pitis,	0	0	$2\frac{1}{40}$	0	03·75

MANILLA.

	£	s.	d.	$	c.
Dollar=8 reals,	0	4	6	1	00
Real=12 quartos,	0	0	$6\frac{3}{4}$	0	$12\frac{1}{2}$
Quarto,	0	0	$0\frac{9}{16}$	0	$01\frac{1}{24}$

BATAVIA.

	£	s.	d.	$	c.
Spanish dollar=64 stivers,	0	4	6	1	00
Stiver,	0	0	$0\frac{27}{32}$	0	$01\frac{9}{16}$
Rix dollar=48 stivers	0	3	$4\frac{1}{2}$	0	75
Ducatoon=80 do.	0	5	$7\frac{1}{2}$	1	25

COLOMBO IN CEYLON.

	£	s.	d.	$	c.
Spanish dollar=64½ stivers,	0	4	6	1	00
Stiver,	0	0	$0\frac{36}{43}$	0	01·55
Rupee=30 stivers,	0	2	$1\frac{5}{43}$	0	46·53½
Rix dollar=48 do.=8 shillings,	0	3	$4\frac{8}{43}$	0	74·45

ENGLISH WEST INDIES.

The principal difference is in the number of shillings in the Spanish dollar, while a pound is 20 shillings, and a shilling is 12 pence, at Jamaica and Bermudas, the Spanish dollar is 6 shillings and 8 pence. Hence the

	£	s.	d.	$	c.
Pound=20 shillings=3 dollars,	0	13	6	3	00
Shilling=12 pence,	0	0	$8\frac{1}{10}$	0	15
Penny,	0	0	$0\frac{27}{40}$	0	01·25

At Barbadoes, the Spanish dollar is 6 shillings and 3 pence. Hence a

	£	s.	d.	$	c.
Pound=20 shillings	0	14	$4\frac{4}{5}$	3	20
Shilling=12 pence	0	0	$8\frac{16}{25}$	0	16
Penny	0	0	$0\frac{36}{50}$	0	01⅓

FRENCH WEST INDIES.

In some of the Islands the Spanish dollar passes for 8 livres and 5 sols, and in others for 9 livres. In the former, the

	£.	s.	d.	D.	c.
Livre=20 sols	0	0	$6\frac{6}{11}$	0	$12\frac{4}{33}$
Sol=12 deniers	0	0	$0\frac{18}{55}$	0	$00·60\frac{20}{33}$
8 livres 5 sols	0	4	6	1	00
In the latter, the					
Livre=20 sols	0	0	6	0	$11\frac{1}{9}$
Sol=12 deniers	0	0	$0\frac{3}{10}$	0	$00·55\frac{5}{9}$
9 livres	0	4	6	1	00

IN MARTINICO, TOBAGO AND ST. CHRISTOPHERS,

The English inhabitants keep their accounts in the denominations of English money, and the French, in those of France. But the *round* dollar passes for 9 shillings, and the *current* dollar at 8 shillings and three pence, or, the *round* is to the *current* dollar as 12 to 11. So that 99 livres=11 *round* dollars or =12 current dollars.

SPANISH WEST INDIES.

	£.	s.	d.	$	c.
Dollar=8 reals	0	4	6	1	00
Real	0	0	6¾	0	12½

TABLE II.

OF THE MONEY OF THE JEWS, GREEKS, AND ROMANS, WITH THE VALUE IN STERLING AND FEDERAL MONEY.*

OF THE JEWS.

	£.	s.	d.	$	c.
Talent=50 manch	342	3	9	1520	$83\frac{1}{3}$
Manch, or Hebrew mina } =60 shekels	6	16	$10\frac{1}{2}$	30	$41\frac{2}{3}$
Shekel=2 becah	0	2	$3\frac{3}{8}$	0	$50\frac{25}{36}$
Becah=10 gerah	0	1	$1\frac{11}{16}$	0	$25\frac{25}{72}$
Gerah	0	0	$1\frac{59}{160}$	0	$1\frac{77}{144}$
Sextula	0	12	$0\frac{1}{2}$	2	$67\frac{7}{36}$
Siclus aureus	1	16	6	8	$11\frac{1}{9}$
Talent of Gold	5475	0	0	25533	$33\frac{1}{3}$

In this estimate the value of gold is to that of silver nearly as 16 to 1.

OF THE GREEKS.

	£	s.	d.	$	c.
Drachma=$1\frac{1}{2}$ tetrobolum	0	0	$7\frac{3}{4}$	0	$14\frac{19}{54}$
Tetrobolum=2 diobolum	0	0	$5\frac{1}{6}$	0	$09\frac{46}{81}$
Diobolum=2 obolus	0	0	$2\frac{7}{12}$	0	$04\frac{127}{162}$
Obolus=2 hemiobolum	0	0	$1\frac{7}{24}$	0	$02\frac{127}{324}$
Hemiobolum=4 chalcus	0	0	$0\frac{31}{48}$	0	$01\frac{127}{648}$
Chalcus=7 septon	0	0	$0\frac{31}{192}$	0	$00\frac{1}{3}$ nearly.
Didrachmon=2 drachma	0	1	$3\frac{1}{2}$	0	$28\frac{19}{27}$
Tetrard statu=2 didrachma	0	2	7	0	$57\frac{11}{27}$
Mina=100 drachmæ	3	4	7	14	$35\frac{5}{27}$
Talent=60 minæ	193	15	0	861	$11\frac{1}{9}$
100 Talents	19375	0	0	86111	$11\frac{1}{9}$
Statu aureus=25 drachma	0	16	$1\frac{3}{4}$	3	$58\frac{43}{54}$
Statu daricus=50 do. according to Josephus,	1	12	$3\frac{1}{2}$	7	$17\frac{16}{27}$

OF THE ROMANS.

	£	s.	d.	$	c.
Denarius=2 quinarii	0	0	$7\frac{3}{4}$	0	$14\frac{19}{54}$
Quinarius=2 sestertii	0	0	$3\frac{7}{8}$	0	$07\frac{19}{108}$
Sestertius=$2\frac{1}{2}$ asor libella =4 teruncii	0	0	$1\frac{15}{16}$	0	$03\frac{127}{216}$
Teruncius	0	0	$0\frac{31}{64}$	0	$00\frac{9}{10}$ nearly.
Sestertium=1000 sestertii	8	1	$5\frac{1}{2}$	35	87

* Authors differ respecting the precise value of ancient money. The common estimate is here given, which is, at least sufficiently near the truth.

	£	s.	d.	D.	c.
Decius sestertium=1000 sestertia,	8072	18	4	35879	63
Centies sestertium, or centies HS. was 10000 sestertia, or	80729	3	4		
Millies HS. was 100000 sestertia, or	807291	13	4		
And the millies centies HS. was the sum of the last two, or	888020	16	8		
Aureus=25 denarii	0	16	1¾	3	58¼¼

This ratio of the aureus to the denarius is that mentioned by Tacitus.

TABLE III.

OF MEASURES OF LENGTH AND CAPACITY AND WEIGHTS OF VARIOUS COUNTRIES.

THE table of English and American measures has been given under compound addition.

Compared with the French measure, the English inch is 0·253994053958532382123 5+ of a French metro.

	French metres.
A Foot English	0·30479286+
Yard=3 feet	0·91437859+
Rod or pole=5½ yards	5·02908227+
Mile=320 rods	1609·30632588+
League=3 miles	4827·91997764+
Ell English=5 quarters of a yard	1·142973
Ell Flemish=3 do.	0·685784
Ell French=6 do.	1·371568
	French litres.
Wine gallon=231 cubic inches English	3·3735
Ale do.=282 do.	4·6208
Gallon, dry measure=268·8 do.	4.4043
Bushel=32 quarts=2150·42 do.	35·2343
Wine quart=57·75 do.	0·9463
Ale do.=70·5 do.	1·1552
Dry do.=67·2 do.	1·1011
Wine Hogshead=63 gallons	238·4509

SCOTLAND.

	English.
3 Feet make 1 ell,	37·2 inches.
1 Mile=5760 feet,	5952 feet.
30 Scotch ells are equal to	31 yards.
30 Scotch miles,	31 miles.

1 Fall=6 ells,	223½ inches.
48 Scotch acres are very nearly	61 acres.

For the measure of Wheat, Peas, Beans, Rye, and White Salt,
100 Bolls equal 409 bushels, Winchester measure.

For Barley, Oats, and Malt.
100 Bolls equal 596 bushels, Winchester measure.

Note. The Boll varies in different parts of Scotland.

IRELAND.

The Irish and English foot and yard are equal.

The Irish mile,	=2240 yards.
11 miles Irish,	14 miles.
121 acres "	196 acres.
The Irish bushel contains	1740·8 cubic inches.

FRANCE.*

	French feet.	English feet.
Metre,	3·078444	3·2809167
Deca-metre=10 metres,	30·78444	32·809167
Hecto-metre=100 "	307·8444	328·09167
Kilo-metre=1000 "	3078·444	3280·9167
Myria-metre=10000 "	30784·44	32809·167
	French inches.	**English inches.**
Deci-metre=$\frac{1}{10}$ metre,	3·6941328	3·93710004
Centi-metre=$\frac{1}{100}$ "	0·36941328	0·393710004
Milli-metre=$\frac{1}{1000}$ "	0·036941328	0·0393710004
	French lines.	**English lines.**
Metre,	443·295936	472·4520048

Quadrant of the Meridian, 100 French degrees, 90 English degrees.

* France is the only nation, which has established an invariable standard of measure. The linear unit of the French measure is the *metre*. By accurate observations and calculations the length of the meridian from the Equator to the pole, which passes through the city of Paris, was ascertained to be 5130740 toises of six feet each of the ancient French measure. This number of toises is equal to 30784440 French feet, or 32809167 English feet. The *metre* is one *ten millionth part* of this arc of the meridian, or 3·078444 feet, which is 3 feet 11 lines, and $\frac{295936}{1000000}$ of a line of the former French measure. All other measures are derived from the metre.

In England, it has been proposed to make the length of the pendulum to vibrate seconds at London, the standard of measure. At the level of the sea, and when the temperature is 62d. Far. and in lat. 51d. 31′ 8·4″ N. the length of the pendulum to vibrate in a second is 39·1386 inches, English, as very accurately determined by Capt. Kater. According to Capt. Kater's measure, the French metre is 39·37076 inches English, at the same temperature, and may be taken with sufficient accuracy to be 39·371 inches. These measures will vary a little according to the scale on which they are estimated. If Troughton's scale of 36 inches be taken as the standard, General Roy's Scale is 36·00036 inches, and Bird's Parliamentary standard of 1758, is 36·00023 inches. And if the scale of 1758 be the standard, Troughton's scale is 35·99977 inches. According to Mr. Hassler, the French metre is 3·28168733 feet on Troughton's Scale.

	French feet.	English feet.
Degree=54 min. Eng. 100 min. Fr.	307844·4	328091·67
Minute=32·4 sec. Eng. 100 sec. Fr.	3078·444	3280·9167

The *are* is the square of the *deca-metre*, and is the unit for square measure.

	French square feet.	English square feet.
Are,	947·681746113	1076·44143923
Square metre or Centiare,	9·47681746113	10·7644143923

The *litre* is the cube of the *decimetre*, and is the unit for dry and liquid measure.

	French cubic inches.	Eng. cubic inches.
Litre,	50·412416	61·028
Hectolitre=13 veltes $3\frac{1}{3}$ pints,		6102·8

Paris pint is $\frac{9313177}{10000000}$ litre or 46·95 cubic inches.

The *stere* is a cubic metre, and is the unit for Cubic or Solid Measure.

	French cubic feet.	English cubic feet.
Stere,	20·17385	35·317

OF THE OLD FRENCH MEASURES.

	French feet.	English feet.
The Toise,	6	6·3946266+
Aune or Ell,	$3·63\frac{8}{9}$	3·8782+
Foot=12 inches,	1·	1·0657711+*
Inch=12 lines,	$0\frac{1}{12}$	0·0888142+

A French League is nearly $2\frac{3}{4}$ English miles, or about $\frac{11}{12}$ of an English League.

By a decree of 1812, the Toise, Aune, Foot, &c. are allowed to be the denominations of measure for the common people of France, in the following ratios to the *metre*.

Toise=2 metres,	6 56 English feet nearly.
Foot=$\frac{1}{6}$ metre,	0·5468
Inch=$\frac{1}{12}$ metre,	0·2734
Aune or Ell=$1\frac{1}{5}$ metre,	3·9371
Bushel=$\frac{1}{8}$ Hectolitre.	762·854 cubic inches.

The old litron=$40·393455\frac{17}{36}$ French cubic inches, by statute, but the common litron is 48·8224 English cubic inches.

Weight.	English.
Paris pound or 16oz.=2 marcs,	7560grs.
Ounce,	472·5

16 pounds are	21lbs. Troy.
63oz.	64oz. Troy.
100 pounds	108lbs. Avoirdupois,
$92\frac{16}{27}$ do.	100 do.
25 do.	27 do.
1 Kilogramme	45·35 do.
1 Hectogramme	453.50 do.

* The ratio of the French to the English foot here assigned is very little different from 1 to $1\frac{66}{1000}$, which was formerly considered as the true ratio.

HAMBURGH.

	English.
100 Ells,	$62\frac{1}{2}$ yards.
16 do.	10 do.
1 German mile,	4 miles.
100 lbs.	$107\frac{1}{4}$ lbs.
$93\frac{103}{429}$,	100 do.
1 Shippound,	280 do.

SWEDEN.

1026·275 feet,	1000 feet.
1000 do.	974·397 do.
1 Can or Kann,	159·864 cub. inch.
877·7125 Victualievigt,	1000lbs. Troy.
1333·4935 Skulpounds,	1000 do. Av.

HOLLAND.

100 Ells,	75yds.
$133\frac{1}{3}$ do.	100 do.
1 Dutch mile.	$3\frac{1}{4}$ miles.
1 ℔=2marcs=16oz.	5775grs.
100 lbs.	$109\frac{1}{8}$lbs.
$91\frac{71}{219}$ do.	100 do.
219 do.	200 do.

ANTWERP.

100 Brabant Ells,	74yds.
$135\frac{5}{27}$ do.	100 do.
100 lbs.	$104\frac{1}{8}$lbs.
96 do.	100 do.
Quintal=10 myriogrammes,	$204\frac{1}{4}$ do.
100 pots of Brabant,	$36\frac{1}{2}$ gallons.

BREMEN.

100 ℔,	110℔.
$90\frac{10}{11}$ do.	100 do.
1 Last,	80 bushels.

DENMARK.

96 lbs.	100lbs.
100 lbs.	$104\frac{1}{6}$ do.
1 Shippound=320lbs. or 20 lispounds,	$333\frac{1}{3}$ do.
1000 Feet,	1049

RUSSIA.

1 Arsheen,	28 inches.
9 do.	7 yards.

		English.
100 lbs.		88¼lbs.
1 Pood=40lbs.		33 3/16 do.
1 Borquit=10 poods,		333 do.
Russian Verst,		¾ mile.

SPAIN.

Vara or Barra=17/24 of French ell,		2·7471 feet.
Vara, of Catalonia=1½ do.		5·8173 do.
100℔,		97℔.
Arobe=25lbs.		24¼ do.
Spanish league,		3⅔ miles.

BILBOA.

100 Varas,		108yds.
100 ℔,		106¼lbs.
100 ℔ of iron,		100 ℔.
32 velts,		66 gallons.
100 fanagues,		152 bushels.

PORTUGAL.

Cavedo=26⅔ Eng. in. or accurately,		26·5933 inches.
Vara 43½ do. do.		44·66
32℔=1 arobe,	nearly	33℔.
Quintal=4 arobes or 128℔		132℔.
60 Alquiers or 1 Moy,		24 bushels.
Fanga=4 alquiers,		1⅔ do.
Canado=4 quarteels,		3 pints.
Almude=12 canados,		4½ wine gallons.

NAPLES.

Cane,	=7 feet, or	84 inches.
Pound of silk,		12oz.
100lbs.		64⅘lbs.
Cantar=100 rotolos,		196lbs.

LEGHORN.

145 lbs.		112℔.
100 brasses,		64yds.
Cane=4 brasses,		2 14/25yds.
116 sacks,	100 quarters,	800 bushels.

TRIESTE.

Brace,	27 inches, or	¾ yard.
100 ℔ of Vienna,		123 ℔ Av.
3⅓ Staros,	1 quarter,	8 bushels.
Staro,		2⅖ do.
Barrel of wine,		18 gallons.

PALERMO IN SICILY.

	English.
Cantar,	176lbs. Av.
Rottoli=$\frac{1}{100}$ cantar,	1¾lb nearly.
Pound weight,	¾lb.
Salm,	485 lb.
Caffis,	3¼ gallons.
Cantar of oil,	25 do.
Barrel,	9 do.

SMYRNA.

Pike,		¾ yd. nearly.
45 Okes,	nearly	123¾ lb Av.
40¾ do.		112 do.

BOMBAY.

Maud,	¼ Cwt.
Surat maud,	37⅓lbs.
Surat Candy=21 S. mauds,	784 do.
Bombay Candy=21 B. mauds,	588 do.
The lb is the English lb.	

MADRAS.

Picul=100 cattas,	133⅓lbs.
Catta,	1⅓lb.
Maud,	25lb Troy.
Candy=20 mauds,	500lbs. do.

CALCUTTA.

Cavid,	½yd.
Bazar maud,	¾Cwt.
Factory do.	75 lb.
1 Maud=40 Seers, and	
1 Seer=16 chittacks.	

CHINA.

Covid,	14$\frac{9}{10}$ inches.
Picul=100 cattas,	133⅓ lbs.
Catta=16 tales,	1⅓ lb.

BATAVIA.

Picul = 100 cattas = 125lb Dutch = 133⅓lb.

MANILLA.

100lb,	104lb.
Arobe or 25lb,	26 do.
Picul=5½ arobes,	143 do.

JAPAN.

	English.
Hichey or Ichan,	$3\frac{1}{2}$ feet.
Catta=16 mace,	$13\frac{1}{3}$lb.
Mace=10 tales,	$\frac{5}{6}$ do.
Balec,	$16\frac{1}{4}$ gallons.

FRENCH WEST INDIES.

104lb,	112 lbs.
100lb,	$107\frac{9}{13}$lbs.

AN ACCOUNT OF THE GREGORIAN OR NEW STYLE, TOGETHER WITH SOME CHRONOLOGICAL PROBLEMS, FOR FINDING THE EPACT, GOLDEN NUMBER, MOON'S AGE, &c.

POPE GREGORY the XIIIth made a reformation of the calendar. The Julian calendar, or old style, had, before that time, been in general use all over Europe. The year, according to the Julian calendar, consists of three hundred and sixty five days and six hours; which six hours being one fourth part of a day, the common years consisted of three hundred and sixty five days, and every fourth year, one day was added to the month of February, which made each of those years three hundred and sixty six days, which are usually called leap years.

This computation, though near the truth, is more than the solar year by eleven minutes, which, in one hundred and thirty one years, amounts to a whole day. By which the Vernal Æquinox was anticipated ten days, from the time of the general council of Nice, held in the year 325 of the Christian Æra, to the time of Pope Gregory; who therefore caused ten days to be taken out of the month of October in 1582, to make the Æquinox fall on the 21st of March, as it did at the time of that council. And, to prevent the like variation for the future, he ordered that three days should be abated in every four hundred years, by reducing the leap year at the close of each century, for three successive centuries, to common years, and retaining the leap year at the close of each fourth century only.

This was at that time esteemed as exactly conformable to the true solar year; but Dr. Halley makes the solar year to be three hundred and sixty five days, five hours, forty eight minutes, fifty four seconds, forty one thirds, twenty seven fourths, and thirty one fifths: According to which, in four hundred years, the Julian year of three hundred and sixty five days and six hours will exceed the solar by three days, one hour and fifty five minutes, which is near two hours, so that in fifty centuries it will amount to a day.

Though the Gregorian calendar, or new style, had long been used throughout the greatest part of Europe, it did not take place in Great Britain and America till the first of January, 1752; and in September following, the eleven days were adjusted by calling the third day of that month the fourteenth, and continuing the rest in their order.

CHRONOLOGICAL PROBLEMS.

Problem I.

As there are three leap years to be abated in every four centuries: to shew how to find in which century the last year is to be a leap year, and in which it is not.

Rule.

Cut off two cyphers, and divide the remaining figures by 4; if nothing remain, the year is a leap year.

Examp. 1. The year 18|00.

```
4)18(4
  16
  --
   2
```

Examp. 2. The year 19|00.

```
4)19(4
  16
  --
   3
```

Examp. 3. The year 20|00.

```
4)20(5
  20
  --
   0
```

Examp. 4. The year 40|00.

```
4)40(10
  40
  -
   0
```

The first and second examples, having remainders, shew the years to be common years of three hundred and sixty five days; but the third and fourth, having no remainders, are leap years of three hundred and sixty six days.

Problem II.

To find, with regard to any other years, whether any given year be leap year, and the contrary.

Rule.

Divide the proposed year by 4, and if there be no remainder, after the division, it is leap year; but if 1, 2 or 3 remain, it is the first, second or third after leap year.

Examp. 1. For the year 1784.

```
4)1784(446
  16
  --
   18
   16
   --
    24
    24
    --
     0
```

Examp. 2. For the year 1786.

```
4)1786(446
  16
  --
   18
   16
   --
    26
    24
    --
     2 } second after leap year.
```

Problem III.

To find the Dominical Letter for any year, according to the Julian method of calculation.

Rule.

Add to the year its fourth part and 4, and divide that sum by 7: if nothing remain, the Dominical Letter is G; but if there be any

remainder, it shews the letter in a retrograde order from G, beginning the reckoning with F; or, if it be subtracted from 7, you will have the index of the letter from A, accounting as follows:

A	B	C	D	E	F	G
1	2	3	4	5	6	7

Examp. For the year 1786.

Add { Given year=1786
Its fourth = 446
And 4

```
7)2236(319
  21
  --
   13
    7
   --
   66
   63
   --
```

And 7—3=4=D, reckoning from A.

Problem IV.

To find the Dominical Letter for any year according to the Gregorian computation.

Rule.

Divide the year and its fourth part, less 1 (for the present century) by 7; subtract the remainder after the division, from 7, and this remainder will be the index of the Dominical Letter, as before: if nothing remain it is G.

Examp. 1. For the year 1810.

Add { Given year=1810
Its fourth = 452

```
         2262
Subtract    1
         ----
    7)2261(323
      21
      --
       16
       14
       --
        21
        21
        --
```

And 7—0=7=G.

Examp. 2. For the year 1812.*

```
    1812
     453
    ----
    2265
       1
    ----
 7)2264(323
   21
   --
    16
    14
    --
     24
     21
     --
```

And 7—3=4=D.

* Here it is to be observed, that every leap year has two Dominical Letters; that, found by this rule, is the Dominical Letter from the twenty fifth day of

PROBLEM V.

To find the Prime, or Golden Number.

RULE.

Add 1 to the given year; divide the sum by 19, and the remainder, after the division, will be the Prime; if nothing remain, then 19 will be the Golden Number.

EXAMP. For the year 1786.

To the given year 1786
Add 1

19)1787(94
171

77
76

1 Golden Number.

The Golden Number, or Lunar Cycle, is a period of 19 years, invented by *Meton*, an *Athenian*, and from him called the *Metonick* Cycle. The use of this cycle is to find the change of the moon; because after 19 years, the changes of the moon fall on the same days of the month as in the former 19 years; though not at the same time of the day, there being an anticipation of one hour, twenty seven minutes, forty one seconds, and thirty two thirds; which, in 312 years, amount to a whole day. Hence, the Golden Number will not show the true change of the moon for more than three hundred and twelve years, without being varied. But the golden number is not so well adapted to the *Gregorian*, as the *Julian* calendar: The epact being more certain in the new style, to find which, the golden number is of use.

PROBLEM VI.

To find the Julian Epact.

RULE.

First find the Golden Number, which multiply by 11, and the product, if less than 30, will be the number required; if the product exceed 30, then divide it by 30, and the remainder is the epact.

EXAMP. 1. For the year 1786.

February to the end of the year; and the next in the order of the alphabet serves from the first of January to the twenty fourth of February.

In the 2d Example, D is the Dominical Letter for the year; but E, the next in the order of the alphabet, is the Dominical Letter for January and February. From this interruption of the Dominical Letter every fourth year, it is twenty eight years before the Dominical Letter returns to the same order, which, were it not for the leap years, would return to the same every seven years.

This Cycle of twenty eight years is called the Cycle of the Sun.

To the given year 1786
Add 1

19)1787(94
171

77
76

Golden Number=1 and 1×11=11 the Julian Epact.

Examp. 2. For the year 1791.

1791
1

19)1792(94
171

82
76

6=Golden Number, and 6×11=66, therefore 30)66(2
60

Epact 6

Problem VII.

To find the Gregorian Epact.

Rule. Subtract 11 from the Julian Epact: If the subtraction cannot be made, add 30 to the Julian Epact; then subtract, and the remainder will be the Gregorian Epact: if nothing remain, the Epact is 29.

Or, take 1 from the Golden Number, and divide the remainder by 3; if 1 remain, add 10 to the dividend, which sum will be the Epact; if 2 remain, add 20 to the dividend; but if nothing remain, the dividend is the Epact.

Examp. 1. For the year 1786.
The Julian Epact being 11
Subtract 11

0

Because nothing remains, the Epact is 29.

Or,

Examp. 2. For the year 1786.
The Golden number being 1
Take from it 1

Divide by 3)0(0

There being no remainder, the Epact is 29, as before.

Examp. 3. For the year 1791.
The Julian Epact being but 6
Add to it 30

36
Subtract 11

Gregorian Epact=25

Or,

Examp. 4. For the year 1791.
The Golden number being 6
Take from it 1

3)5(1
3

2

Therefore, as 2 remains, add 20 to the dividend, and it gives the Epact 25, as before.

A general Rule for finding the Gregorian Epact forever.

Divide the *centuries* of any year of the Christian Era by 4, (rejecting the subsequent numbers;) multiply the remainder by 17, and to this product add the quotient multiplied by 43; divide this sum plus 86 by 25, multiplying the Golden Number by 11, from which subtract the last quotient, and rejecting the *thirties*, the remainder will be the Epact.

Examp. For the year 1786.

Rejecting the subsequent numbers 86, it will be 17.

```
            4)17(4
              16
              —
               1
Multiply by   17
              —
              17
Add 4×43=    172
             ———
             189
      Add     86
             ———
         25)275(11
             25
             ——
              25
              25
```

```
    Golden Number=  1
        Multiply by 11
                    —
                    11
Subtract the last quotient=11
                    —
                    00
```

Therefore, as nothing remains, the Epact is 29, as before.

A TABLE OF THE NINETEEN EPACTS FOR THE JULIAN AND GREGORIAN ACCOUNTS, BY THE GOLDEN NUMBER.

G. N.	Julian Epact.	Greg. Epact.	G. N.	Julian Epact.	Greg. Epact.	G. N.	Julian Epact.	Greg. Epact.
1	11	29	7	17	6	13	23	12
2	22	11	8	28	17	14	4	23
3	3	22	9	9	28	15	15	4
4	14	3	10	20	9	16	26	15
5	25	14	11	1	20	17	7	26
6	6	25	12	12	1	18	18	7
						19	29	18

Problem VIII.

To calculate the Moon's age on any given day.

Rule. To the given day of the month, add the Epact and number of the month: If the sum be less than 30, it is the Moon's age, but if it exceed 30, then take 30 from it, and the remainder will be the Moon's age.

Note. The numbers to be added to the following months, are as follow:

To	January	0	July	5
	February	2	August	6
	March	1	September	8
	April	2	October	8
	May	3	November	10
	June	4	December	10

EXAMPLE. For January 25th, 1786.

Add	Given day	=25
	Epact	=29
	No. of the month	=00
		54
	Subtract	30
		24=Moon's age.

PROBLEM IX.

To find the times of the New and Full Moon, and the first and last Quarters.

RULE. Find the Moon's age on the given day, then, if it be 15, the Moon will be full on that day, and by counting $7\frac{1}{2}$ days backward and forward you will have the first and last quarters, and by counting backward and forward 15 days, you will have the times of the last and next change; but if the age of the Moon be greater than 15, take 15 from it, and the remainder will shew how many days have passed since the last full moon, and counting these backward, you will have the day the last full moon happened on, and by knowing that, we can find the change, or either of the quarters, as before.

Again, if the age of the moon, on the assumed day, be less than 15, then take that from 15, and the remainder will shew how many days are to run till the next full moon, which you will have by adding the remainder to the assumed day; and proceeding as before, you will have the days of the change, and either quarter as above.

EXAMP. For January 25th, 1786.

Add	Assumed day	=25
	Epact	=29
	Number of the month	=00
		54
	Subtract	30
	Moon's age	=24
	Subtract	15
	Take the days since the last full moon	= 9
	From the assumed day	=25
	To the day of the full moon	=16th
	Add	15

New Mooon	31st
From the full Moon	16
Take	$7\frac{1}{2}$
First quarter	9th
To the full Moon=	16
Add	$7\frac{1}{2}$
Last quarter=	23

Problem X.

The time of the Moon's coming to the South, after the Sun, being given, to find the age of the Moon.

Rule. As 24 hours, the whole difference of time, are to 30, the number of days from change to change, so is the difference of time, to the Moon's age.

Example. I observed the Moon to be on the meridian, or due south, at 6 o'clock in the afternoon: What is the Moon's age?

24 ; 30 :: 6 : $7\frac{1}{2}$ days, Ans.

Problem XI.

To find the time of the Moon's southing.

Rule. Multiply the Moon's age, on the given day, by 48 minutes, and divide the product by 60, the minutes in an hour, (or multiply by 4 and divide by 5) and the quotient will show how many hours and minutes the moon is later in coming on the meridian, than the sun, and counting so many hours and minutes forward from 12 o'clock, we have the time of the Moon's southing: if the hours and minutes, found as above, be less than 12, then, that will be the time of the Moon's southing after noon; but, if greater than 12, then, take 12 from them, and the remainder will be the time of the Moon's southing in the morning.

Examp. 1. Required the time of the Moon's southing on the 25th day of January 1786?

		Or,
	Moon's age=24	24
h. m.	48	4
From 19 12	——	—
Take 12 00	192	5)96
——	96	— h. m.
7 12	—— h. m.	$19\frac{1}{5}$=19 12 as before.
Hence the Moon souths at 12 minutes past 7 in the morning.	60)1152(19 12	
	60	
	——	
	552	
	540	
	——	
	12	

EXAMP. 2. For the 9th of February 1786?

```
Moon's age=10
          48
          ---- h. m.
       60)480(8 0 afternoon, is the time of the Moon's
          48                                [southing.
```

Note. From the change to the full, the Moon comes to the south afternoon; but from the full to the change, before noon.

PROBLEM XII.

To find on what day of the week, any given day in any month will fall.

As one of the first seven letters of the alphabet is prefixed to every day in the year beginning with A, which is always prefixed to the first day of January: And as, in common years, the letter, annexed to the first Sunday in January, shews the Dominical Letter for that year; but every leap year having two Dominical Letters, the first of which serving to the twenty fourth of February, and the other for the rest of the year, consequently, in any common year, the Dominical Letter being known, the first of January may be easily found, reckoning from A according to the natural order of the letters: and in any leap year, the first of its two Dominical Letters will shew as above, counting from A 1, B 2, C 3, &c. and by counting backward, you may have the day of the week, on which the first of January will happen.

RULE. Find the day of the week answering to the first of January that year, then add together the days contained in each month from the beginning of the year to the proposed day of the month inclusively; divide this sum by 7, and if any thing remain, after the division, then count so many forward, beginning with that day on which the first of January falls, and you will have the day of the week, on which the proposed day will fall: but if nothing remain, then the day of the week, preceding that day on which the first of January falls, answers to the proposed day.

EXAMPLE.

On what day of the week will the 5th day of May 1786 fall?

By the preceding observations, and by Prob. 4th, the first of January is found to fall on Sunday.

Jan.	31
Feb.	28
March	31
April	30
May	5th

Now, counting forward six days from Sunday, the first of January (inclusively) the 5th of May falls on Friday.

```
7)125(17
  7
  --
  55
  49
  --
   6 from Jan 1.
```

Problem XIII.

To find the Cycle of the Sun.

Rule. Add 25* to the given year; divide the sum by 28, and the remainder, after division, is the Cycle required; but if nothing remain, the Cycle is 28.

Example.

For the year 1807?

To 1807
Add 25

28)1832(65
168
152
140
12=Cycle required.

The use of this Cycle is to find the Dominical Letter by the following Table.

A TABLE OF THE DOMINICAL LETTERS FOR THE NEW STYLE, ACCORDING TO THE CYCLE OF THE SUN.

Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.	Cycle.	Letter.
1	D C	8	B	15	G	22	E
2	B	9	A G	16	F	23	D
3	A	10	F	17	E D	24	C
4	G	11	E	18	C	25	B A
5	F E	12	D	19	B	26	G
6	D	13	C B	20	A	27	F
7	C	14	A	21	G F	28	E

Problem XIV.

To find the year of the Dionysian Period.

Rule. Add to the given year 457; divide the sum by 532, and the remainder will be the number required.

Example.

Required the year of the Dionysian Period for the year 1786?

To 1786
Add 457

532)2243(4
2128
115=Dionysian Period.

* From the commencement of this century, 9+16=25 must be added to the given year. The leap year having been omitted in the year 1800, makes it necessary to add 25 to the date of the year, and then dividing by 28, it will give the Cycle right during the present century. And this is a general rule to be observed, that when a leap year has been abated, add 16 to the number which was

PROBLEM XV.

To find the year of Indiction.

RULE. Add 3 to the given year; divide the sum by 15, and the remainder, after division, will be the Indiction; if nothing remain, it will be 15.

EXAMPLE.

Required the year of Indiction for 1786?

```
To 1786
Add    3
   ————
15)1789(119
   15
   ——
    28
    15
    ———
    139
    135
    ———
      4=Indiction.
```

PROBLEM XVI.

To find the Julian Period.

RULE. Add 4713 to the given year, and the sum will be the Julian Period.

EXAMPLE.

What year of the Julian Period will answer to the year 1786?

```
To  1786
Add 4713
    ————
    6499 Ans.
```

PROBLEM XVII.

To find the Cycle of the Sun, Golden Number, and Indiction, for any current year.

RULE. To the current year add 4729;* divide the sum by 28, 19 and 15, respectively, and the several remainders will be the numbers required; when nothing remains, the divisor is the number required.

EXAMPLE.

What are the Cycle of the Sun, Golden Number, and Indiction, for the year 1807?

before added to the year, rejecting 28 when it exceeds it, and this number being added to the year, and the sum divided by 28, the remainder after division, will be the Cycle for finding the Dominical Letter. Thus in the nineteenth century, it will be $9+16=25$, and in the twentieth century $\overline{25+16}-28=13$, which number will serve two centuries, for the year 2000 is a leap year.

* For any year in the nineteenth century add $4713+16=4729$.

1807	19)6536(344	15)6536(435
4729	57	60
28)6536(232	83	53
56	76	45
93	76	86
84	76	75
96	0	Indiction=11
84	Golden Number=19	
12 Cycle of the Sun.		

PROBLEM XVIII.

To find the time of High Water.

RULE. Find the Moon's southing, to which add the point of the compass making full sea, on the full and change days, for the place proposed, and the sum will be the time required.

EXAMPLE.

I demand the time of high water at Boston, January 25th, 1786, admitting the tide to flow and ebb N. W. and S. E. on the days of change and full?

We have before found the Moon's southing to be 7h. 12m. in the morning.

	h.	m.	
Therefore to	7	12	
Add	4	0	=the point of the compass, and it
Gives	11	12	in the morning, for the time of high water.

PROBLEM XIX.

To find on what day Easter will happen.

It was ordered by the Nicene Council, that Easter Sunday should be kept on the first *Sunday* after the first full moon, which happened upon or after the twenty first day of March, the day on which they thought the Vernal Equinox happened. Though this was a mistake, for the Vernal Equinox, that year, fell on the twentieth of March. But yet, the full moon, which fell on, or next after the twenty first of March, they called the Paschal full moon. And by the introduction of the Gregorian, or New Style, the Equinox will now always happen on the twentieth or twenty first of March. And the feast of Easter is now to be kept on the next Sunday after the Paschal full moon, or the full moon which happens after the twenty first of March; but, if the full moon happens on a Sunday, Easter day is to be the next Sunday after.

RULE. Find the age of the moon on the 21st of March, in the given year, and if it be 14, then find the day of the week answering to it, and the Sunday following is Easter Sunday; but if the

moon's age on the 21st day of March be not 14, then reckon forward to the day on which the moon's age is 14, and find the day of the week answering to that day; the Sunday following will be the day required.

N. B. On leap year take the 20th of March.

Examp. When does Easter happen in the year 1786?

	21	of March	Jan. 31
	29	Epact.	Feb. 28
	1	No. of the month.	March 31
	—		April 13th
	51		—
Subt.	30		7)103(14
	—		7
	21	Moon's age.	—
Add	23	No. of days to the Moon's being 14 days old.	33
			28
	—		—
	44		5
Take	31	=days in March.	
	—		

13th of April, the day of the full moon, or Easter limit.

There-fore the first of January being Sunday, reckon forward 5 days, including Sunday, and you will find the 13th of April falls on Thursday, consequently the next Sunday is the 16th, which is Easter Sunday.

Easter may be found, for any future time, by the following Table which is calculated from 1753, the time of the commencement of the New Style in America, and which shews, by the Golden Number, the days of the Paschal full moons; by which, and the Dominical Letter, the day on which Easter will fall, may be found.

The Use of the Table.

First, find the Golden Number as before taught, which seek in the column of Golden Numbers under the time in which the given year is included; right against the Golden Number of the year, in the last column but one, you have the day of the month on which the Paschal full moon happens, which is the limit of Easter; from thence run your eye down among the Dominical Letters, till you come to the Letter of the given year, and against it you have the day of the month, on which Easter falls that year.

Example. To know when Easter falls in 1786.

The Golden Number for the year being one, and the Dominical Letter A; therefore seek in the first column (the given year being included between the years 1753 and 1899) for the Golden Number: then cast your eye along to the last column but one, under the title Paschal full ●, and you will find the thirteenth of April to be the day of the full moon; against which, in the last column, stands E, which shews it to be Thursday, therefore the next Sunday following is Easter Sunday, which, by going down the column of Letters to the next A, you will find to be the sixteenth of April.

GOLDEN NUMBERS FROM 1753 TO 1899, AND SO ON TO 4199, INCLUSIVELY.

1753 to 1899.	1900 to 2199.	2200 to 2299.	2300 to 2399.	2400 to 2499.	2500 to 2599.	2600 to 2899.	2900 to 3099.	3100 to 3399.	3400 to 3499.	3500 to 3599.	3600 to 3699.	3700 to 3799.	3800 to 4099.	4100 to 4199.	Paschal Full ● Days of the Moon.	Dom. Letter.
14	—	6	17	6	17	—	9	—	1	17	1	12	—	4	March 21	C
3	14	—	6	—	6	17	—	9	—	9	—	1	12	—	22	D
—	3	14	—	14	—	6	17	—	9	—	9	—	1	12	23	E
11	—	3	14	3	14	—	6	17	—	9	—	9	—	1	24	F
—	11	—	3	—	3	14	—	6	17	—	17	—	9	—	25	G
19	—	11	—	11	—	13	14	—	6	17	6	17	—	9	26	A
8	19	—	11	—	11	—	3	14	—	6	—	6	17	—	27	B
—	8	19	—	19	—	11	—	3	14	—	14	—	6	17	28	C
16	—	8	19	8	19	—	11	—	3	14	3	14	—	6	29	D
5	16	—	8	—	8	19	—	11	—	3	—	3	14	—	30	E
—	5	16	—	16	—	8	19	—	11	—	11	—	3	14	31	F
13	—	5	16	5	16	—	8	19	—	11	—	11	—	3	April 1	G
2	18	—	5	—	5	16	—	8	19	—	19	—	11	—	2	A
—	2	13	—	13	—	5	16	—	8	19	8	19	—	11	3	B
10	—	2	13	2	13	—	5	16	—	8	—	8	19	—	4	C
—	10	—	2	—	2	13	—	5	16	—	16	—	8	19	5	D
18	—	10	—	10	—	2	13	—	5	10	5	16	—	8	6	E
7	18	—	10	—	10	—	2	13	—	5	—	5	16	—	7	F
—	7	18	—	18	—	10	—	2	13	—	13	—	5	16	8	G
15	—	7	18	7	18	—	10	—	2	13	2	13	—	5	9	A
4	15	—	7	—	7	18	—	10	—	2	—	2	13	—	10	B
—	4	15	—	15	—	7	18	—	10	—	10	—	2	13	11	C
12	—	4	15	4	15	—	7	18	—	10	—	10	—	2	12	D
1	12	—	4	—	4	15	—	7	18	—	18	—	10	—	13	E
—	1	12	—	12	—	4	15	—	7	18	7	18	—	10	14	F
9	—	1	12	1	12	—	4	15	—	7	—	7	18	—	15	G
—	9	—	1	—	1	12	—	4	15	—	15	—	7	18	16	A
17	17	9	—	9	—	1	12	12	4	15	4	15	15	7	17	B
6	6	17	9	17	9	9	1	1	12	4	12	4	4	15	18	C
															19	D
															20	E
															21	F
															22	G
															23	A
															24	B
															25	C

PLANE GEOMETRY.

DEFINITIONS.

1. A *POINT* in the Mathematicks is considered only as a mark, without any regard to dimensions.

2. A *Line* is considered as length, without regard to breadth or thickness.

3. A *Plane* or *Surface* has two dimensions, length, and breadth, but is not considered as having thickness.

4. A *Solid* has three dimensions, length, breadth and thickness, and is usually called a *Body*.

5. A line is either *straight*, which is the nearest distance between two Points; or *crooked*, called a Curve Line, whose ends may be drawn further asunder.

6. If two Lines are at equal distance from one another in every part, they are called *parallel* Lines, which, if continued infinitely, will never meet.

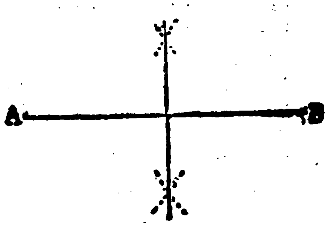
7. If two lines incline one towards another, they will, if continued, meet in a point: by which meeting is formed an *Angle*.

8. If one Line fall directly upon another, so that the Angles on both sides are equal, the Line, so falling, is called a *perpendicular*, and the Angles so made, are called *right* Angles, and are equal to 90 degrees, each.

9. All Angles, except right Angles, are called oblique Angles, whether they are *acute*, that is, less than a right Angle; or *obtuse*, that is, greater than a right Angle.

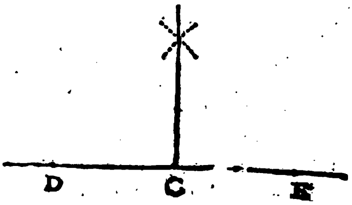
GEOMETRICAL PROBLEMS.

PROBLEM I. *To divide a Line* AB *into two equal parts.*



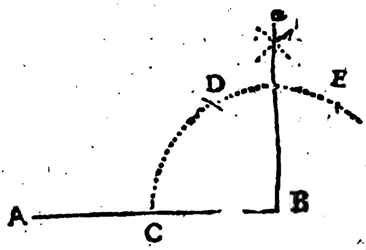
Set one foot of the compasses in the point A, and, opening them beyond the middle of the line, describe arches above and below the line; with the same extent of the compasses, set one foot in the point B, and describe two arches crossing the former: draw a line from the intersection of the arches above the line, to the intersection below the line, and it will divide the line AB into two equal parts.

PROBLEM II. *To erect a perpendicular on the point* C *in a given line.*



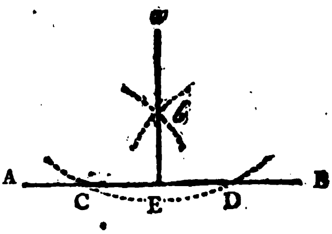
Set one foot of the compasses in the given point C, extend the other foot to any distance at pleasure, as to D, and with that extent make the marks D, and E. With the compasses, one foot In D, at any extent above half the distance of D and E, describe an arch above the line, and with the same extent, and one foot in E, describe an arch crossing the former; draw a line from the intersection of the arches to the given point C, which will be perpendicular to the given line in the point C.

PROBLEM III. *To erect a perpendicular upon the end of a line.*



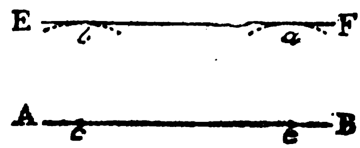
Set one foot of the compasses in the given point B, open them to any convenient distance, and describe the arch CDE; set one foot in C, and with the same extent, cross the arch at D: with the same extent cross the arch again from D to E; then with one foot of the compasses in D, and with any extent above the half of ED, describe an arch *a*; take the compasses from D, and, keeping them at the same extent with one foot in E, intersect the former arch *a* in *a*; from thence draw a line to the point B, which will be a perpendicular to AB.

PROBLEM IV. *From a given point, a, to let fall a perpendicular to a given line* AB.



Set one foot of the compasses in the point *a*, extend the other so as to reach beyond the line AB, and describe an arch to cut the line AB in C and D; put one foot of the compasses in C, and, with any extent above half CD, describe an arch *b*; keeping the compasses at the same extent, put one foot in D, and intersect the arch *b* in *b*; through which intersection, and the point *a*, draw *a* E, the perpendicular required.

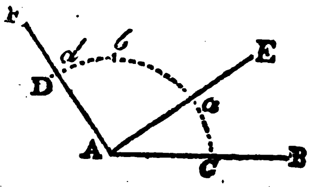
Problem V. *To draw a Line parallel to a given Line* AB.



Set one foot of the compasses in any part of the line, as at *c*; extend the compasses at pleasure, unless a distance be assigned, and describe an arch *b*; with the same extent in some other part of the line AB, as at *e*, describe the arch *a*; lay a ruler to the extremities of the arches, and draw the line E F, which will be parallel to the line A B.

Problem VI. *To make an Angle equal to any number of degrees.*

It is required to lay off an acute Angle of 35° on a given line AB.

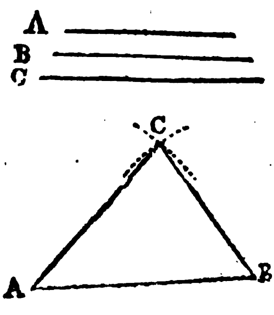


Take 60 degrees from the line of chords in the compasses, set one foot of the compasses in the point A, describe an arch CD, at pleasure; then set one foot of the compasses in the brass centre, in the beginning of the line of chords, and bring the other to 35 on the line; with this extent set one foot in C, with the other intersect the arch CD, in *a*, and through *a* draw the line AE, so will EAB be an angle of 35 degrees.

If the angle had been obtuse, suppose 125°, then take 90° from the line of chords; set one foot in C, and intersect the arch in *b*; then take 35° from the same line of chords, and set them from *b* to *d*: a line drawn from A through *d* to F will make an angle, FAB, of 125°.

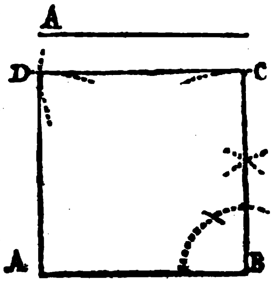
To measure an angle by the line of chords, is only to take the distance on the arch between the lines AB and AE, or AB and AF, and lay it on the line of chords.

Problem VII. *To make a Triangle, whose sides shall be equal to three given lines, provided any two of them be longer than the third.*



Let A,B,C, be the three given lines; draw a line AB, at pleasure; take the line C in the compasses, set one foot in A, and with the other make a mark at B; then take the given line B in the compasses, and setting one foot in A, draw the arch C; then take the line A in the compasses, and intersect the arch C in C; lastly, draw the lines AC and BC, and the triangle will be completed.

PROBLEM VIII. *To make a Square, having equal sides, equal to any given line.*

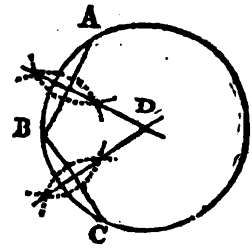


Let A be the given line; draw a line AB equal to the given line; from B raise a perpendicular to C equal to AB, with the same extent, set one foot in C and describe the arch D; also with the same extent, set one foot in A and intersect the arch D; lastly, draw the lines AD and CD, and the square will be completed.

In like manner may a Parallelogram be constructed, only attending to the difference between the length and breadth.

PROBLEM IX. *To describe a Circle, which shall pass through any three given Points, which are not in a straight line.*

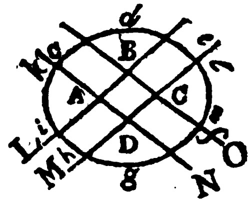
Let the three given points be A,B,C, through which the circle is to pass Join the points AB and BC with right lines, and bisect these lines; the point D, where the bisecting lines cross each other, will be the centre of the circle required. Therefore, place one point of the compasses in D, extending the other to either of the given points, and the circle, described by that radius, will pass through all the points.



Hence, it will be easy to find the centre of any given circle; for, if any three points are taken in the circumference of the given circle, the centre will be readily found as above. The same may also be observed, when only a part of the circumference is given.

PROBLEM X. *To describe an Ellipsis or Oval mechanically.*

Draw two parallel lines, as L and M, at a moderate distance, by Prob. 5; then draw two others at the same distance, across the former, as N and O; by the crossing of these lines will be made a figure ABCD, of four sides; extend the compasses at pleasure, and setting one foot in D, describe the arch *cde*; with the same extent, set one foot in B, and describe the arch *fgh*; then set one foot in C, and contract them so as to reach the point *e*, and describe the arch *lm*; with the same extent, and one foot in A, describe the arch *ik*, and the oval will be completed. In the same manner, with a greater or less extent of the compasses, may a greater or less oval be made by the same four sided figure ABCD.



MENSURATION

OF SUPERFICIES AND SOLIDS.

Section I. Of Superficies.

SUPERFICIES, or surfaces, are measured by the superficial inch, foot, yard, &c. according to the measures peculiar to different artists.

The superficial inch, foot, &c. is one inch, foot, &c. in length and breadth; and, because 12 inches make one foot of Long Measure, therefore $12\times12=144$ inches make 1 superficial foot, $3\times3=9$ feet, a yard, &c.

The superficial content of every surface is found by the proper rule of its figure, whether square, triangle, polygon, or circle.

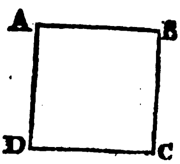
Article 1. *To measure a Square, having equal sides.*

Rule.

Multiply the side of the square into itself, and the product will be the area or superficial content, of the same name with the denomination taken, either in inches, feet, or yards, respectively.

Let ABCD represent a square, whose side is 12 inches or 12 feet. Multiply the side 12 by itself, thus,

	12 inches.	12 feet.
	12 inches.	12 feet.
Area=	144 inches.	144 feet.



By the Sliding Rule.

Set 1 to the length on B, then, find the breadth on A, and opposite to this on B, you will have the content.

By Gunter's Scale.

Extend the dividers from 1, on the line of numbers, to the length; that distance, laid the same way from the breadth, will point out the answer.

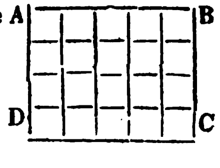
Art. 2. *To measure a Parallelogram or long Square.*

Rule.

Multiply the length by the breadth, and the product will be the area, or superficial content.*

Let ABCD represent a parallelogram, whose length is 5 feet, and breadth, 4 feet. Multiply 5 by 4.

Length	5
Breadth	4
Area	20 Ans.



* If the parallelogram be divided into squares by drawing lines as in the figure, it is obvious on inspection, that the number of squares must always be equal to the product of the length and breadth. The same may be shown on the square also. The area of a Rhombus or Rhomboides is equal to that of a parallelogram of the same base and altitude.

The content of this figure is found on the sliding rule and scale, as the former.

ART. 3. *When the breadth of a Superficies is given, to find how much in length will make a square foot, yard, &c.*

RULE.

As the breadth is to a foot, yard, &c. so is a foot, yard, &c. to the length required to make a foot, yard, &c. Or divide 144 by the breadth, and the quotient will be the length required.

How much, in length, of a board 2½ feet wide, will make a square foot?

In. br. In. leng. In. br. In. leng.
As 30 : 12 :: 12 : 4·8
12

30)144(4·8 inches, length required.
120

240
240 In.
Breadth=30)144(4·8 inches, Ans.

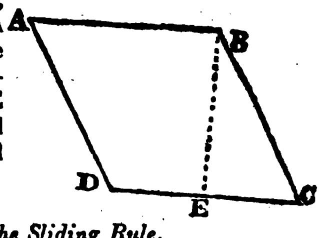
ART. 4. *To measure a Rhombus.*

Definition. A rhombus is a figure with four equal sides, in the form of a diamond on cards, having two angles greater and two less than the angles of a square: the former are called *obtuse* angles, and the latter, *acute*, or sharp, angles.

RULE.

Multiply the side by the length of a perpendicular, let fall from one of the obtuse angles to the side opposite such angle.

Let ABCD represent a rhombus, each of whose sides is 16 feet: A perpendicular let fall from the obtuse angle, at B, on the side DC, will intersect it in the point E, so will BE be 12 feet; and this being multiplied into the given side, the product will be the area of the rhombus.



Side=16
Per.=12

192 area.

By the Sliding Rule.

Set 1 on A to the length on B; find the perpendicular height on A, against which on B is the content.

By Gunter.

The extent from 1 to the perpendicular height will reach from the length to the content.

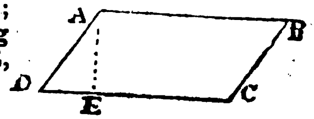
Art. 5. *To find the Area of a Rhomboides.*

Definition. A rhomboides is a figure, whose opposite sides and opposite angles are equal.

Rule.

Multiply one of the longest sides by the perpendicular let fall from one of the obtuse angles on one of the longest sides.

Let ABCD represent a rhomboides; the longest sides AB and CD being 16·5 feet, and the perpendicular AE, 9·7 feet.



Side=16·5
Perp. 9·7

1155
1485

Ans. 160·05 feet.

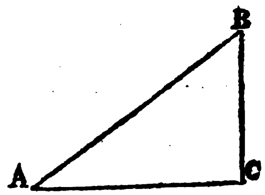
The content is found on the sliding rule, and scale, as in the last figure.

Art. 6. *To measure a Triangle.**

Rule.

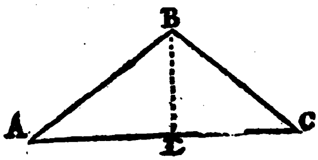
If it be a right angled triangle, multiply the base by half the perpendicular, or half the base by the perpendicular, and the product will be the area: but if it be an oblique angled triangle, (whether obtuse, or acute,) multiply half the base by the length of the perpendicular let fall on the base from the angle opposite to it, and the product will be the area. The longest side of a triangle is usually called the base, except in a right angled triangle, where the longest of the two legs, which include the right angle, is called the base.

In the right angled triangle ABC right angled at C; the base AC is 18·8 feet; and the perpendicular BC=12·6.



Base =18·8	Or, Perp.=12·6
½ Perp.= 6·3	½ Base = 9·4
564	504
1128	1134
118·44 area.	118·44 area.

The oblique angled triangle ABC being given, let fall a perpendicular from the angle at B on the base AC, and that perpendicular is the height of the triangle. The base AC being 15·6, and the perpendicular BD=9, to find the area.



* A triangle is half a parallelogram of the same base and altitude; hence the rule. In a right angled triangle, the longest side is called the hypotenuse; the next longest, the base; and the shortest side, the perpendicular.

7·8=half the base.
9=height of the angle.

70·2=area.

By the Sliding Rule.

Set 1 on A to the length of the base on B, and opposite to half the length of the perpendicular, on A, you will have the content on B.

By Gunter.

The extent from 1 to half the length of the perpendicular will reach from the length of the base to the content.

In this place it may be proper to instruct the learner in one of the properties of a right angled triangle: viz. That the square of the longest side of a right angled triangle, usually called the hypotenuse, is equal to the sum of the squares of the two other sides, usually called the legs; which is of great use, for by this mean, any two sides of a right angled triangle being given, the other may be found by common Arithmetick. Thus, in the right angled triangle ABC, the base AC and perpendicular BC being given, the hypotenuse AB may be found by extracting the square root of the sum of the squares of the base and perpendicular.

Base 18·8	Perp. 12·6	353·44=square of the base.
18·8	12·6	158·76=square of the perp.
1504	756	· · ·
1504	252	512·20(22·63 hypotenuse.
188	126	4
353·44	158·76	42)112
		84
		446)2820
		2676
		4523)14400
		13569
		831

And, if the hypotenuse and one of the legs be given, the other may be found by subtracting the square of the given leg from the square of the hypotenuse.

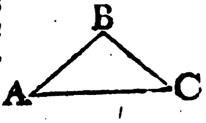
There are some numbers, the sum of whose squares make a perfect square, of which sort are 3 and 4, whose squares, being added together, make 25, which is the square of 5; therefore, if the base of a triangle be 4, and the perpendicular 3, the hypotenuse will be 5; and if any of these numbers be multiplied by any other number, those products will be the sides of right angled triangles, as 6, 8, 10, and 15, 20, 25, &c. Thus artificers, when they set off the corner of a building, usually measure 6 feet on one side, and 8 feet on the other, then laying a 10 feet pole across, it makes the corner a true right angle.

Art. 7 *There is another method of finding the area of triangles, the three sides being given.*

Rule. Add the three sides together, then take the half of that sum, and out of it subtract each side severally; and multiply the half of the sum and these remainders continually, and the square root of this product will be the area of the triangle.

In the oblique triangle ABC, the base AC is given 15·6, the side AB is 10·4, and the side BC is 9·2, to find the area.

15·6	17·6	17·6	17·6
10·4	—15·6	—10·4	— 9·2
9·2	2·	7·2	8·4
35·2 sum			
17·6=half the sum.			



17·6
2
35·2
7·2
704
2464
253·44
8·4
101376
202752
2128·896

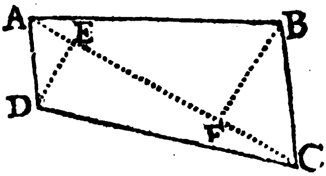
2128·8960(46·139=area.
16
86)528
516
921)1289
921
9223)36860
27669
92269)919100
830421
88679

Art. 8. *To measure a Trapezium.*

Definition. A trapezium is an irregular figure of four unequal sides, and unequal angles.

Rule. Draw a diagonal line from one of the angles to the opposite angle, as AC, and then will the trapezium be divided into two triangles, of which the diagonal is the common base: then, letting fall perpendiculars from the other opposite angles on the diagonal, add those perpendiculars together, and multiply half that sum into the diagonal, or half of the diagonal into the sum of the perpendiculars, and that product will be the area of the trapezium.

In the trapezium ABCD, the diagonal AC is 24, the perpendicular DE 6, and the perpendicular BF 10. The sum of the perpendiculars is 16, whose half is 8, which being multiplied into 24, will give the area.



24
8
—
192=area.

By the sliding Rule.

Set 1 on A to ½ the sum of the perpendiculars on B, and opposite the length of the diagonal on A, you will have the area on B.

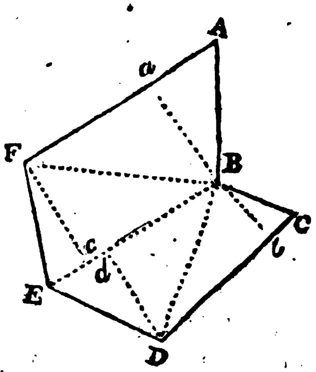
By Gunter.

The extent from 1 to ½ the sum of the perpendiculars will reach from the length of the diagonal to the area.

ART. 9. *To measure any irregular figure.*

RULE. Divide the figure into triangles, by drawing diagonals from one angle to another; then measure all the triangles by either of the rules, already taught, at Article 6 or 7, and the sum of the several areas of all the triangles will be the area of the given figure.

The irregular figure ABCDEF being given, divide it into triangles by the diagonals FB, EB, and DB: then may the triangles be measured by letting fall perpendiculars on their respective bases, as B*a*, B*b*, D*c*, F*d*, and multiplying those perpendiculars by half their respective bases.

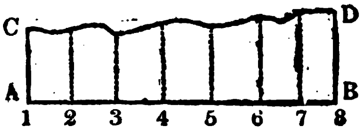


In the triangle AFB the base FA is 100, and the perpendicular B*a* 49; in the triangle FBE the base BE is 92, and the perpendicular F*d* 52; in the triangle EBD, the base BE is the same as before, and the perpendicular D*c* 44; and in the triangle DCB, the base DC is 80, and the perpendicular B*b* 38; by which the area of each may be found by Art. 6. as follows.

50=half AF.	46=half BE.	2450
49=perp. *a*B.	52=perp. F*d*.	2024
——	——	2392
2450=area of AFB.	92	1520
	230	——
46=half BE.	——	8386=area of the
44=perp. D*c*.	2392=area of FBE.	figure ABCDEF.
——		
184	38=perp. B*b*.	
184	40=half DC.	
——	——	
2024=area of EBD.	1520=area of DCB.	

In dividing any irregular figure into triangles, the triangles will be less, by two, and the diagonals less, by three, than the number of the sides of the figure.

If there be a long, irregular figure like the following, the mean breadth may be found very nearly, by measuring the breadth at certain equal distances along AB, and dividing the sum of the breadths by their number.



Let the length, AB, be 16 rods, the 1st breadth AC 3·9 rods, the 2d 4 rods, the 3d 3·95 rods, the 4th 4·3 rods, the 5th 4·25 rods, the 6th 4·5 rods, the 7th 4·8 rods, and the 8th 4·9 rods; what is the area?

$$\frac{3{\cdot}9+4{\cdot}+3{\cdot}95+4{\cdot}3+4{\cdot}25+4{\cdot}5+4{\cdot}8+4{\cdot}9}{8}=\frac{34{\cdot}6}{8}$$ the mean breadth. Then $\frac{34{\cdot}6}{8}\times16=69{\cdot}2$ rods, *Ans.*

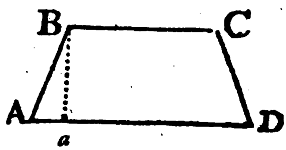
Art. 10. *To measure a Trapezoid.*

Definition. A trapezoid is the segment of a triangle, cut by a line parallel to the base.

Rule. Add the parallel sides together, and multiply half that sum by the perpendicular breadth.

In the trapezoid	24=AD
ABCD, the side AD	16=BC
is 24, the side BC is	—
16, and the perpen-	40=sum.
dicular breadth B*a*	—
is 10, to find the	20=½ sum.
area by adding the	10=B*a*.
sides BC and AD	—
and multiplying half	200=area.

their sum by the perpendicular breadth B*a*.



By the Sliding Rule.

Set 1 on A to the equated length on B, and against the breadth on A you will have the area on B.

By Gunter.

The extent from 1 to the breadth will reach from the equated length to the area.

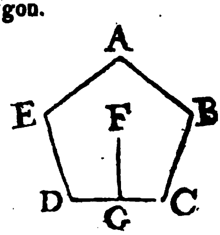
Art. 11. *To measure any regular Polygon.*

Definition. A regular polygon is a figure whose sides and angles are all equal; they are usually denominated from the number of their sides.

Thus, A figure having	3	equal sides and angles is a	Trigon.
	4		Tetragon.
	5		Pentagon.
	6		Hexagon.
	7		Heptagon.
	8		Octagon.
	9		Enneagon.
	10		Decagon.
	11		Endecagon.
	12		Dodecagon.

RULE. Multiply the length of one of the sides by the number of sides; then, this product by the half of a perpendicular let fall from the centre of the figure to the middle of one of the sides, and the product will be the area of the polygon.

In the pentagon ABCDE, each side is 95, and the perpendicular FG 65·36, to find the area.



95=length of a side.
5=number of sides.

475=sum of the sides.
32·68=½ of the perpendicular.

3800
2850
950
1425

15523·00=area of the pentagon.

By the Sliding Rule.

Set 1 on A to ½ the perpendicular on B, and against the sum of the sides on A you will have the area on B.

By Gunter.

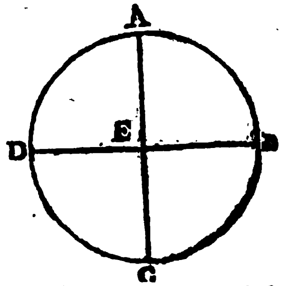
The extent from 1 to half the length of the perpendicular, will reach from the sum of the sides to the content.

But for the more ready measuring regular polygons, the following Table, containing multipliers for all regular figures from the triangle to the dodecagon, will be of use to the learner.

Number of sides.	Names.	Multipliers.	Number of sides.	Names.	Multipliers.
3	Trigon.	·433013	8	Octagon.	4·828427
4	Tetragon.	1·	9	Enneagon.	6·181827
5	Pentagon.	1·720477	10	Decagon.	7·694209
6	Hexagon.	2·589076	11	Endecagon.	9·361
7	Heptagon.	3·633959	12	Dodecagon.	11·196

If the square of the side of a polygon be multiplied by the multiplier of the like figure, the product will be the area of the figure sought.

To measure a Circle and its Parts.



In the annexed circle ABCD, the arch line ABCD is called the *periphery*, the length of which is called the *circumference:* Any line, as DB or AC, passing through the centre E, cuts the circle into two equal parts, called *semicircles*, or half circles; and such lines are called diameters of the circle: If two diameters be drawn through a circle, at right angles to each other, then, the four equal divisions of the circle are called *quadrants:* half the diameter as EB, is called the *radius*, or semidiameter.

Art. 12. *The Diameter of a Circle being given, to find the Circumference.**

Rule. This may be done by either of the following proportions in whole numbers, as 7 is to 22, or more exactly, as 113 is to 355; or in decimals, as 1 is to 3·14159; so is the diameter of a circle to the circumference.

* *Note.* 1. If the diameter of any circle

be	multiplied	by	3·14159, the product	is the circumference.
	divided		·31831, the quotient	
			2. If the diameter of any circle	
be	multiplied	by	·886227, the product	is the side of an equal square.
	divided		1·128379, the quotient	
			3. If the diameter of any circle	
be	multiplied	by	·866024, the product	is the side of the equilateral
	divided		·1547, the quotient	triangle inscribed.
			4. If the diameter of any circle	
be	multiplied	by	·707016, the product	is the side of the square
	divided		1·414213, the quotient	inscribed.
			5. If the square of the diameter of any circle	
be	multiplied	by	·785398, the product	is the area.
	divided		1·273241, the quotient	
			6. If the circumference of any circle	
be	multiplied	by	·31831, the product	is the diameter.
	divided		3·14159, the quotient	
			7. If the circumference of any circle	
be	multiplied	by	·282094, the product	is the side of the
	divided		3·544907, the quotient	square equal.
			8. If the circumference of any circle	
be	multiplied	by	·2756646, the product	is the side of the equilateral
	divided		3·6275939, the quotient	triangle inscribed.
			9. If the circumference of any circle	
be	multiplied	by	·225079, the product	is the side of the
	divided		4·442877, the quotient	square inscribed.
			10. If the square of the circumference of any circle	
be	multiplied	by	·079577525, the product	is the area.
	divided		12·56636217, the quotient	
			11. If the area of any circle	
be	multiplied	by	1·273241, the product	is the square of
	divided		·785398, the quotient	the diameter.

Examp. A circle whose diameter is 12, to find the circumference.

As 7 : 22 :: 12	As 113 : 355 :: 12	As 1 : 3·14159 :: 12
12	12	12
7)264(37·71 = circumference.	113)4260(37·699 cir.	37·69908 cir.
21	339	
54	870	
49	791	
56	790	
49	678	
10	1120	
7	1017	
3	103	

Note. 3·14159 may be contracted to 3·1416 without any sensible difference.

Art. 13. *The Circumference of a Circle being given, to find the Diameter.*

Rule. As 22 is to 7; or 355 to 113; or as 1 to ·31831, so is the circumference of a circle to the diameter.

Examp. The circumference of a circle being 326, to find the diameter.

12. If the area of any circle be { multiplied / divided } by { 12·56636217, the product / ·0795775 25, the quotient } is the square of the circumference.

13. When the diameter of 1 circle is 1, and the diameter of another is 2, the circumference of the first is equal to the area of the second, =3·141592.

14. If the circumference be 4, the diameter and area are equal, =1·273241.

15. If the diameter be 4, the circumference and area are equal, =12·566368.

Hence, because circles are the most capacious of all figures, if the *fourth* part of a circle be *squared*, it will not be equal to the *area* of that circle (as is commonly supposed) although the *four* sides added together are equal to the *circumference* of that circle.

In a circle whose diameter is 24, circumference 75·4, and area 452·4, the *fourth* part of the circumference is 18·85, the *square* of which is only 355·3225, that is, 97·0775 less than the truth: and the larger the circle is, the greater will the errour be.

For further proof of this matter: If a cylindrical pint, beer measure, whose content is 35·25 cubick inches, be beaten into a perfectly *square form*, it will contain only 28·902 cubick inches, which is less than the truth by 6·3484+; the area of the circle is 8·7615859288, and the area of the square only 6·8813320653076624.

Hence appears the reason, why taking the *fourth part* of the girth in measuring a cylinder (or a round stick of timber) is false.

16. If the diameter of one circle be double to *that* of another, the *area* of the first circle will be *four times* the area of the second, because the areas of circles are as the squares of their diameters; see Art. 15.

As 22 : 7 :: 326
7
———
22)2282(103·72 diam.
22
——
82
66
——
160
154
——
60
44
—
16

355 : 113 :: 326
326
——
678
226
339
———
355)36838(103·76 diam.
355
———
1338
1065
———
2730
2485
———
245

1 : ·31831 :: 326
326
———
190986
63662
95493
———
103·76906 = diameter. This proportion is the most accurate.

Art. 14. *To find the Area of a Circle.*

Rule. Multiply half the diameter by half the circumference and the product is the area.

If the diameter be given, find the circumference by Art. 12.

If the circumference be given, find the diameter by Art. 13.

Examp. A circle whose diameter is 12, and circumference is 37·7, given, to find the area?

18·85=half the circumference.
6=half the diameter.
———
113·10=area of the given circle.

Note. A *circular ring* is the figure contained between the *peripheries* of two concentric circles. Hence, the area of a circular ring must be the difference of the areas of the two circles.

Art. 15. *The Diameter being given to find the Area of a Circle without finding the Circumference.*

Rule. Multiply the square of the diameter by ·7854,* and the product will be the area of the circle, whose diameter was given.

Examp. The diameter of a circle being 12, to find the area?

·7854
12×12= 144
———
31416
31416
7854
———
113·0976=area.

* When the diameter is 1, the area is found to be ·7854, and as the areas of circles are as the squares of their diameters, the rule is evident.

By the Sliding Rule.

Set 1 on A to the diameter on B, then find ·7854 (which expresses the area of a circle whose diameter is 1) on A, against which on B is a 4th number, then find this 4th number on A, against which on B is the area.

By Gunter.

The extent from 1 to the length of the diameter reaches from ·7854 to a 4th number, and from that 4th number to the area.

Art. 16. *The Circumference of a Circle being given, to find the Area without finding the Diameter.*

Rule. Multiply the square of the circumference by ·07958, and the product will be the area of the circle.

Examp. The circumference of a circle being 37·7, to find the area.

```
   37·7            1421 29
   37·7              ·07958
  ------           ---------
   2639            1137032
  2639             710645
 1131             1279161
                  994903
 --------        ----------
 1421·29=square.  113·1062582=area of the circle.
```

Art. 17. *The Dimensions of any of the parts of a Circle being given, to find the side of a Square equal to the Circle.*

Rule. If the area of the circle be given, extract the square root of the area, which will be the side of a square equal to the circle: If the diameter or circumference be given, find the area by Art. 15 or 16, and then extract the square root, as before. And this is a *general rule* to find the side of a square equal to any superficial figure, regular or irregular: for the square root of the area of any figure whatever, is the side of a square equal to the given figure. But with regard to circles, if the diameter be given; multiply it by ·886, and the product will be the side of an equal square: or, as 13·545 is to 12, or 1354 to 1200: so is the diameter of a circle to the side of a square equal to the given circle. And, if the circumference be given, multiply it by ·282 for the side of an equal square. Or, divide it by 3·545, and the quotient will be the side of an equal square.

Examp. 1.

Let the diameter of a circle be 12, to find the side of a square equal to the circle?

·886×12=10·632=side of the spuare.

Or, as 13·545 : 12 :: 12 : 10 631 =the side.

Examp. 2.

The circumference being 37·7 to find the side of an equal square?

37·7×·282=10·631=side of the square.

Or, 37·7÷3·545=10·634.

Art. 18. *The Area of a Circle being given, to find the Diameter.*

Rule. Multiply the given area by 1·2732, and the product will be the square of the diameter; then, extracting the square root of the product, you will have the diameter.*

Examp. The area of a circle being 113·09, to find the diameter.

```
   1·2732          143·986188(11·999=12=diameter.
   113·09          1
  --------        ---
   114588        21)43
  381960            21
 12732            ----
12732            229)2298
---------            2061
143·986188          -----
                 2389)23761
                      21501
                     ------
                23989)226088
                      215901
                      ------
                       10187 remainder.
```

Art. 19. *The Area of a Circle being given, to find the circumference.*

Rule. Multiply the given area by 12·566, and extract the square root of the product, which root will be the circumference required.

Examp. The area of a circle being 113·03 to find the circumference.

```
   12·566          1420·3349(37·68=circumference.
   113·03          9
  -------         ---
   37698         67)520
  376980            469
 12566             ----
12566            746)5133
---------             4476
1420·33498           -----
                 7528)65749
                      60224
                      -----
                       5525 remainder.
```

Art. 20. *The Side of a Square being given, to find the Diameter of a circle equal to the Square, whose Side is given.*

Rule. Multiply the given side by 1·128, and the product will be the diameter of a circle, whose area is equal to the area of the

* As the area of a circle, whose diameter is 1, is ·7854, the area divided by ·7854 must give the square of the diameter; but as 1·2732 is the reciprocal of ·7854, the rule is evident.

given square. Or, if the side of the square be divided by ·886, the quotient will be the diameter. Or, as 12 to 13·54, so is the side of any square to the diameter of an equal circle.

EXAMP. The side of a square being 10·635, to find the diameter of a circle equal to that square?

10·635×1·128=12 nearly. Or, 10·635÷·886=12=diameter.

Or, as 12 : 13·54 :: 10·635 : 12 nearly.

ART. 21. *The Side of a Square being given, to find the circumference of a Circle equal to the given Square.*

RULE. Multiply the given side by 3·545 and the product will be the circumference required. Or, divide it by 282, and the quotient will be the circumference.

EXAMP. The side of a square being 10·631, to find the circumference of a circle equal to that square.

10·631×3 545=37·686=circum. Or, ·282)10·631(37·698 circum.

ART. 22. *To find the Area of a Semicircle, the Diameter being given.*

RULE. Find the area of the circle by Art. 15, and take the half of it.

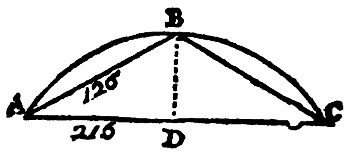
In the same manner may the area of a quadrant, or a quarter of a circle, be found, by taking a fourth part of the area of the whole circle.

But with regard to measuring a sector, or a segment of a circle, it will be necessary first to show how to find the length of the arch line of a sector, and the diameter of the circle to a given segment.

ART. 23. *A Segment of a Circle being given, to find the length of the Arch Line.*

RULE. Divide the segment into two equal parts; then measure the chord of the half arch, from the double of which subtract the chord of the whole segment; and one third of that difference, being added to the double of the chord of the half arch, will give the length of the arch line.

EXAMP. In the segment ABCD, the whole chord ADC is 216, and the chord AB or BC 126, to find the arch line ABC.



126=chord AB or BC.
2

252=double.
216=ADC, to be subtracted.

3)36=difference.

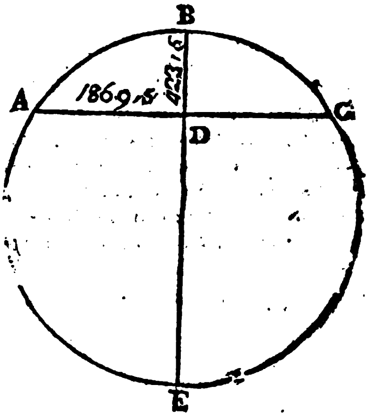
12=⅓ difference.

252=double of AB.
12=⅓ difference added.

264=length of the arch ABC.

Art. 24. *The Chord and versed Sine of a Segment being given, to find the Diameter of a Circle.*

Rule. Multiply half the chord by itself, and divide the product by the versed sine; then add the quotient to the versed sine, and the sum will be the diameter of the circle.



Example. In the segment ABCD, the chord AC is 1869·5, and the versed sine BD 423·5, to find the diameter.

934·75 } half the chord AC
934·75

467375
654325
373900
280425
841275

423·5)873757·5625(2063·1=DE.
8470   423·5=BD, add.

26757   2486·6=diameter BDE.
25410

13475
12705

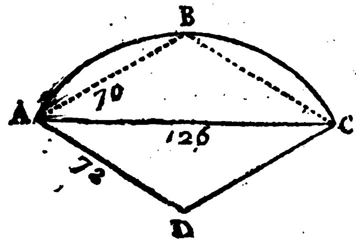
7706
4235

3471

Art. 25. *To measure a Sector.*

Definition. A sector is a part of a circle, contained between an arch line, and two radii or semidiameters of the circle.

Rule. Find the length of half the arch by Art. 23: Then multiply this by the radius or semidiameter, and the product will be the area.



Examp. 1. In the sector ABCD, given the radius AD or DC 72 feet, the chord AC=126 feet, and the chord AB or BC=70, to find the area of the sector.

First.

70=chord AB or BC.
2

Carried over.

140 Brought over.
126=AC, subtract.

3)14

4·66
140

144·16=length of the arch [ABC, by Art. 23.

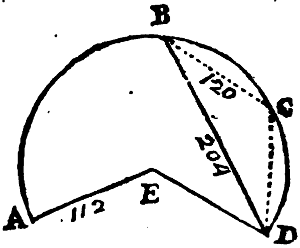
72·33

Secondly.
72·33=half the arch.
72=radius.

14466
50631

5207·76=area.

EXAMP. 2. In the sector ABCD, greater than a semicircle, given the radius AE or ED=112, the chord BD (of half the arch ABD)=204, and the chord BC (of half the arch BCD)=120, to find the area of the sector.



120=BC.
2

240
204 subtract.

3)36

12
240 Add.

252= { Length of the arch BCD, by Art. 23.

252=half the arch ABD.
112=radius.

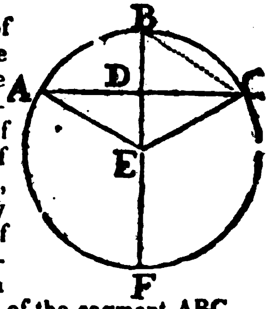
504
252
252

28224=area of the sector.

ART. 26. *To find the Area of a Segment of a Circle.*

Definition. A segment of a circle is any part of a circle cut off by a right line drawn across the circle, which does not pass through the centre, and is always greater or less than a semicircle.

EXAMP. 1. To find the area of the segment ABC, whose chord AC is 172, the chord of half the arch ABC, viz. BC=104, and the versed sine BD=58·48.



RULE. By Art. 23, find the length of the arch line ABC, and by Art. 24, the diameter FB; then multiply half the chord of the arch ABC by half the diameter, and the product will be the area of the sector ABCE: then find the area of the triangle AEC, whose base AC is 172, and perpendicular height 34, found by subtracting the versed sine BD from half the diameter; and the area of the triangle AEC, being subtracted from the area of the sector ABCE, will leave the area of the segment ABC.

104=BC.
2
—
208
172=AC, subtract.
—
3)36
—
12
208 add.
—
220=arch line ABC.
—
110=half arch.

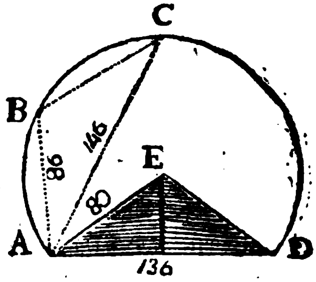
92·475=radius.
110
—
924750
92475
—
10172·25=area of the sector.
86=half the base=AD.
34=perpendicular DE.
—
344
258
—
2924=area of the triangle.

86=half ADC.
86
—
516
688
—
58·48)7396·00(126·47=DEF.
5848
—
15480
11696
—
37840
35088
—
27520
23392
—
41280
40936
—
344

58·48=BD, add.
—
184·96=diameter BF.
—
92·475= { radius or semi-diameter.

10172·25=area of the sector.
2924 =area of the triangle.
—
7248·25=area of the segment.

EXAMP. 2. In the segment ABCD greater than a semicircle, given the chord of the whole segment AD=136, the chord AC of half the arch ACD=146, the chord AB or BC one fourth of the arch ACD =86, and the radius AE or ED= 80, to find the area of the segment ABCD.



First find the area of the sector ABCDE, by Art. 25, at the second Example; then find the area of the triangle AED, by Art. 6, and, adding the area of the triangle to the area of the sector, you will have the area of the segment.

86=chord AB.
2
—
172
146=chord AC, subtract.
—
3)26

8·666
172 =double of AB, add.
—
180·666=arch line ABC.
80=radius.
—
14453·280=area of the sector.

Carried over.

Brought over.

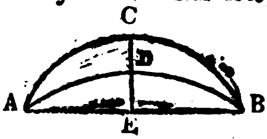
68=half the base AD.
42=perpendicular E 136.

136
272

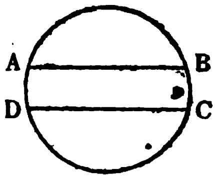
2856=area of the triangle AED.
14453·28=area of the sector, [add.

17309·28=area of the segment.

Note 1. The area of a *Lune* or *Crescent*, is calculated by the preceding rule. A Lune is a figure made by two circular arcs, which intersect each other, as ACBD. The area of the Lune is the difference of the two segments, which are contained by the arcs and the chord. Thus the difference of the segments ACBE and ADBE is the area of the crescent ACBD.



Note 2. A *Circular Zone* is a figure contained between two parallel chords. If the chords be equal, it is called a *middle* zone, as ABCD. The area of a zone is evidently the difference between the area of the circle and the areas of the two segments.

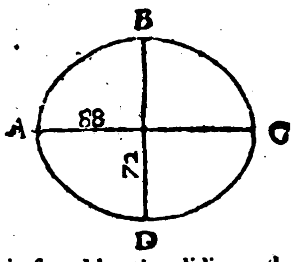


Art. 27. *To find the Area of an Ellipsis.*

Definition. An ellipsis, or oval, is a curve which returns into itself like a circle, but has two diameters, one longer than the other, the longest of which is called the transverse, and the shortest the conjugate diameter.

Rule. Multiply the two diameters of the ellipsis together; then multiplying the product by ·7854, this last product will be the area of the ellipsis.

Examp. In the ellipsis ABCD, the transverse diameter AC is 88, and the conjugate diameter BD is 72, to find the area.



88
72

176
616

6336
·7854

25344
31680
50688
44352

4976·2944=area.

The content is found by the sliding rule and Gunter, in the same way as the circle, only using the product of the two diameters as the square of the diameter of a circle.

Mensuration of Superficies is easily applied to *Surveying:* thus, take the angles of the plot with a good compass, then measure the

sides with Gunter's chain, which note down in links (or chains and links, which is done by separating the two right hand figures of your links by a comma, your chain being 100 links) then cast up the contents, according to the rule of the figure, cutting off the five right hand figures of the product, and those at the left hand, if any, are acres; then multiply the five figures cut off, by 4, by 40, and by 272¼, cutting off as before, and those at the left hand, will be roods, poles, and feet, respectively.

Section II. Of Solids.

Solids are measured by the solid inch, foot, or yard, &c. 1728 of these inches, that is 12×12×12, make one cubick or solid foot.

The solid content of every body is found by rules adapted to their particular figures.

Art. 28. *To measure a Cube.**

Definition. A cube is a solid of six equal sides, each of which is an exact square.

* Here follows a Table of the Proportions, which the following *Solids* have to the *Cube* and *Cylinder*, having the same *Base* and *Altitude*.

	Solid Inches.
1. A *Cube* whose side is 12 inches, contains	1728
2. A *Prism*, having an equilateral triangle, whose side is 12 inches from its *Base*, and its *Altitude* 12 inches, contains	784·24
3. A *Square Pyramid*, whose height and the side of its base, are each 12 inches, is ⅓ of the above cube, and therefore contains	576
4. A *Triangular Pyramid*, whose height and side of its triangular base are each 12 inches, is near $\frac{1}{7}$ of the cube, and contains	249·413
5. A *Cylinder*, whose diameter and height are each 12 inches, is $\frac{11}{14}$ of the above cube, and contains	1357·17
6. A *Sphere* or *Globe*, whose axis or diameter is 12 inches, equal to the side of the cube, is $\frac{11}{21}$ of it, and contains	904·78
7. A *Cone*, whose base and altitude are each 12 inches, equal to the side of the cube, is $\frac{5}{19}$ of it, and contains	452·38829
8. A *Parabolick Conoid*, whose diameter at the base and height, are each 12 inches, being ½ its circumscribing cylinder, contains	678·583
9. A *Hyperbolick Conoid*, whose height, and diameter at the base, are each 12 inches, is $\frac{5}{12}$ of its circumscribing cylinder, and contains	565·49
10. A *Parabolick Spindle*, whose height and middle diameter are each 12 inches, is $\frac{8}{15}$ of its circumscribing cylinder, and contains	723·824

Hence arises a different method of finding their contents.

General Rule. If the base of the solid, whose contents you would find, be rectilinear, consider it as *Parallelopipedon*; if curved, as a *Cylinder*, and find the content accordingly: then take such a part of the content, thus found, as is specified in the preceding Table, which if the parts be taken in inches, will be the solid content of the given figure, in inches, which, divided by 1728, will give the cubick feet.

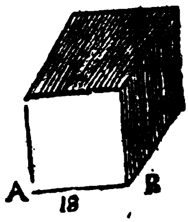
Examp. 1. There is a triangular prism, the side of whose base is 48 inches, and whose perpendicular height is 108 inches: what is its solid content?

The base being right lined, I consider it as a *parallelopipedon*, the side of whose base is 48 inches, and whose length is 108 inches, and as 784·24 is con-

The solid foot is composed of 1728 inches; for a solid, that is 1 foot, or 12 inches every way, that is 12×12×12, contains 1728 inches.

Rule. Multiply the side by itself and that product by the same side, and this last product will be the solid content of the cube.†

Examp. The side of a cube AB, being 18 inches, or 1 foot and 6 inches, to find the content?



1 foot 6 inches=1·5 foot.		18 inches.
	1·5	18
	75	144
	15	18
Carried over.	2·25	324

tained 2·20340712 times in a cubick foot; 2·20340712 is a divisor, to divide the content of the parallelopipedon by; therefore 48×48×108÷2·20340712= 112930·56 solid inches=65·353 solid feet.

Had the dimensions been given in feet, it would have been 4×4×9÷2·20340712=65·353 feet.

Examp. 2. There is a square pyramid, whose height is 12 feet, and the side of whose base is 3·5 feet; what is its content?

3·5×3·5×12÷3=49 feet, Ans.

Examp. 3. There is a triangular pyramid, whose height is 15 feet, and the side of whose base is 5 feet: what is its content?

5×5×15÷7=53·57 feet, Ans.

Examp. 4. There is a cylinder whose diameter is 2·5 feet, and whose length is 24 feet; what is its content?

Here, the diameter is to be considered as the side of the base of a parallelopipedon. Therefore, 2·5×2·5×24×11÷14=117·857 feet, Ans.

Examp. 5. There is a spherical balloon, whose diameter is 50 feet; how many cubick feet of air does it contain?

Here, the diameter is to be considered as the side of a cube. Therefore,

50×50×50×11÷21=65476·19 feet, Ans.

Examp. 6. There is a cone, whose height is 15 feet, and the diameter of whose base is 5 feet; what is its content?

Here, the diameter of the base is to be considered as the side of the base of a parallelopipedon, and its height, as the length. Therefore,

5×5×15×5÷19=98·684 feet, Ans.

Examp. 7. There is a parabolick conoid, whose diameter at the base is 2·9 feet, and whose height is 6 feet; what is the content?

This solid being ½ of a cylinder; we must first find the content as of that of a cylinder, and then halve it. Therefore,

2·9×2·9×6×11÷14=39·647, and 39·647÷2=19·823, Ans.

Examp. 8. There is a hyperbolick conoid, whose diameter at the base is 2·9 feet, and whose height is 6 feet; what is the content?

First, find the content of a cylinder.

2·9×2·9×6×11÷14=39·647, and 39·647×$\frac{5}{12}$=16·519 feet, Ans.

Examp. 9. There is a parabolick spindle, whose middle diameter is 2·9 feet, and whose length is 6 feet; required the content?

First, find the content of a cylinder.

2·9×2·9×6×11÷14=39·647, and 39·647×$\frac{8}{15}$=21·145 feet, Ans.

† Multiplying a side by itself, or squaring a side, gives the area of the base, or the number of square inches, feet, &c. in the base; whence one inch, foot, &c. in height would give as many solid inches, feet, &c. as there are squares in the base; two inches, &c. in height, twice as many, and so on, and is the rule, when the sides are equal to each other. In the same way, the rule for the content of the Parallelopipedon is proved.

```
Brought up.  2·25          324
              1·5           18
             ----          ----
             1125          2592
             225           324
             ----          ----
             3·375   1728)5832(3·375
                           5184
                           ----
                           6480
                           5184
                           ----
                          12960
                          12096
                          -----
                           8640
                           8640
```

In this operation, the inches are changed into the decimal parts of a foot.

I have done this two different ways, that the learner may see they come out the same. The content in inches is 5832, which being divided by 1728, the inches in a solid foot, and the division continued by annexing cyphers, it comes out the same as the decimal operation.

Note. The area of the surface, or superficial content of the cube and parallelopipedon is found by adding the areas of the several quadrilateral figures which compose them.

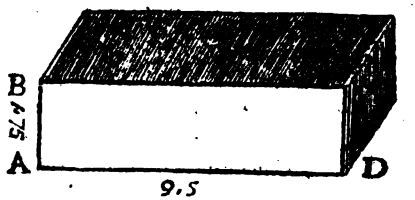
Art. 29. *To measure a Parallelopipedon.*

Definition. A parallelopipedon is a solid of three dimensions, length, breadth and thickness; as a piece of timber exactly squared, whose length is more than the breadth and thickness. The ends are called bases, which are equal.

Rule. Find the area of the base, then multiply that by the length, and it will give the solid content.

Examp. 1. The side AB is 1·75 foot, and the length AD 9·5 feet, to find the solid content?

```
 1·75=1 foot, 9 inches.
 1·75
 ----
  875
 1225
 175
 ------
 3·0625=area of base.
    9·5
 ------
 153125
 275625
 ------
 29·09375=solid content.
```



Examp. 2. A vessel 3·5 feet each side within, and 5 feet deep, to find the content?

```
 3·5
 3·5      12·25
 ---          5
 175      -----
 105      61·25=the content.
```

If a piece of timber, or any other thing, be of an equal bigness through its whole length, though there be a difference between the breadth and thickness, if the breadth and thickness are multiplied together, and that product multiplied by the length, this last product will be the solid content.

Examp. 3. A piece of timber being 1 foot and 6 inches, or 18 inches broad, 9 inches thick, and 9 feet 6 inches, or 114 inches long, to find the content?

```
1 foot 6 inches=1·5 foot
       9 inches=·75 foot.
                ---
                 75
                105
               -----
               1·125
9 feet 6 inches= 9·5
               -----
               5625
              10125
              -------
              10·6875=content.
```

In this operation the inches are changed into the decimal fractions of a foot.

```
Breadth =18 inches.
Depth   = 9 inches.
          ---
          162
Length=114 inches.
          ---
          648
         162
        162
        -----
1728)18468(10·6875=content,
     1728            as before.
     -----
      11880
      10368
      -----
       15120
       13824
       -----
        12960
        12096
        -----
          8640
          8640
```

Note. When the end is given in inches and the length in feet, find the area at the end in inches, multiply that by the length in feet, and divide this product by 144 (the square inches in a foot) and the quotient will be the feet.

Take the last example.

```
  Foot.
  1·5 =18 inches.
  ·75= 9 inches.
       ---
       162 area in inches.
       9·5 feet=length.
       ---
       810
      1458
      ----
144)1539(10·6875=content.
```

By the sliding Rule.

Set 12 inches on the girt line D to the side of the square end on C, then, against the length on D, you will have the answer on C.

By Gunter.

Extend the compasses from 12 inches to the length of the side of the square end; that distance, twice turned over from the length, will reach to the content.

When the side of a square solid is given, in inches, to find how much in length will make a foot solid.

Rule. As the given side is to 12, so is 12 to a fourth number, and so is that fourth number to its required length. Or divide 1728 by the area at the end, and the quotient will be the length making a solid foot.

If the given side is in foot measure, then,

Rule. As the given side is to 1; so is 1 to a fourth number, and so is that fourth number to the required length.

When two sides of an equal square solid (that is, of unequal breadth) are given, to find what length will make any number of solid feet.

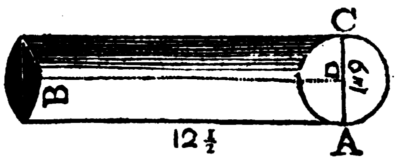
Rule. Multiply the proposed number of feet by 144: divide that product by the product of the breadth and depth, and the quotient will be the length required.

Art. 30. *To measure a Cylinder.*

Definition. A cylinder is a round body, whose bases are circles, like a round column, or a rolling stone of a garden.

Rule. The diameter of the base being given, find the area of the end by Art. 15, then, multiplying the area of the base by the length, that product will be the content of the cylinder.

Examp. The diameter of the base AC being 1 foot and 9 inches, and the length BD 12 feet and 6 inches, to find the content.



1·75=diam. of the base.
1·75

875
1225
175

3·0625
·7854

122500
153125
245000
214375

2·40528750=area of the base.

2·405=area of the base.
12·5=length.

12025
4810
2405

30·0625=content.

If the square of the diameter of a cylinder be multiplied by ·7854, and the solidity divided by that product, the quotient will be the length, and if the content be divided by the length, the quotient will be the area of the end, from which the diameter is found by Art. 13.

The learner may, for his practice, reduce all the dimensions to inches, and find the solid content in inches, which being divided by 1728, the quotient will be the solid content in feet: or, if he finds the area at the end in inches, and multiplies that by the length in feet, and divides by 144; the quotient will be feet.

This is a general rule for finding the content of any straight solid body, of equal bigness from end to end, of whatever form the bases are: for, if the area of the base be multiplied by the length, the product will be the solid content.

By the Sliding Rule.

Set 13·5, the square root of 183·34 (which is a guage point arising from the division of 144 by ·7854) found on D, to the diameter found on C, and opposite to the length, on D, you will find the content on C.

Or, as 42·54 is to the circumference; so is the length in feet to a fourth number, and so is that fourth number to the answer.

Note. The superficial content of a cylinder is found by multiplying the circumference of one of the bases into the length, and to the product adding the areas of the two bases, or ends.

When the diameter is given in inches, to find what length will make a solid foot.

Rule. As the given diameter is to 13·531: so is 12 to a fourth number, and so is that fourth number to the required length. If the diameter be given in foot measure: Rule, as the given diameter is to 1·128: so is 1 to a fourth number, and so is that fourth number to the required length. Or, divide 1728 by the area at the end in inches, and the quotient will be the required length.

To find how much a Cylindrick or round Tree, that is equally thick from end to end, will hew to, when made square.

Rule. Multiply twice the square of its semidiameter by the length, then divide the product by 144, and the quotient will be the answer.

If the diameter of a round stick of timber be 24 inches from end to end, and its length 20 feet: how many solid feet will it contain, when hewn square; and what will be the content of the slabs which reduce it to a square?

$$\frac{12\times12\times2\times20}{144}=40 \text{ feet, the solidity when hewn square.}$$

$$\frac{24\times24\times\cdot7854\times20}{144}=62\cdot8 \text{ feet, or } 2\times2\times\cdot7854\times20=62\cdot8 \text{ the total}$$

solidity, whence 62·8—40=22·8 feet, the solidity of the slabs.

Note. The rule of workmen for measuring round timber is to multiply the square of the quarter girt or one fourth of the circumference, by the length. This rule allows about one fifth, for the bark, waste in hewing, &c. The example above, in which the diameter of the cylinder is 1 foot 9 inches, and the length 12 feet

6 inches, will give the quarter girt 1·3744 feet, and the solid content is $\overline{1\cdot3744}^2\times12\cdot5=23\cdot61$ feet, which is nearly four fifths of 30·6625, the content by the accurate rule.

A rule, nearly correct, is to multiply twice the square of one fifth of the circumference by the length. Thus, in the example, $\frac{1}{5}$ of the circumference is 1·0995, and $2\times\overline{1\cdot0995}^2\times12\cdot5=30\cdot22$ feet.

Art. 31. *To measure a Prism.*

Definition. A prism is a body with two equal or parallel ends, either square, triangular, or polygonal, and three or more sides, which meet in parallel lines, running from the several angles at one end, to those of the other.

Rule. Prisms of all kinds, whether square, triangular or polygonal, are measured by one general rule, viz. Find the superficial content, or area at the base (or end) by the proper rule of Sect. 1. and this multiplied by the length, or height of the prism, will give the solid content.

Examp. The side of a stick of timber, AB, hewn three square, is 10 inches, and the length, AC, is 12 feet, to find the content?

Side= 10 inches.
$\frac{1}{2}$ Perpendicular=4·33 inches.

43·3=area at the end.
12 feet=length.

144)519·6(3·6 feet, content.
432

876
864

12

Note. The superficial content is found by adding the areas of the several quadrilateral and triangular figures which compose it.

Art. 32. *To measure a Pyramid.*

Definition. Solids, which decrease gradually from the base till they come to a point, are generally called pyramids, and are of different kinds, according to the figure of their bases; thus, if it has a square base, it is called a square pyramid: if a triangular base, a triangular pyramid: If the base be a circle, a circular pyramid, or simply a cone. The point, in which the top of a pyramid ends, is called a Vertex, and a line drawn from the vertex, perpendicular to the base, is called the height of the pyramid.

Rule. Find the area of the base, whether triangular, square, polygonal or circular, by the rules in superficial measure: then, multiply this area by one third of the height, and the product will be the solid content of the pyramid.

EXAMP. 1. In a triangular pyramid, the height BE, being 48, and each side of the base 13: the base being a triangle, let the perpendicular height DE be 11; to find the content.

5·5=half ED.
13=base AC.

165
55

71·5=area of the base.
16=⅓ of the height EB.

4290
715

1144·0=content.

EXAMP. 2. In a quadrangular pyramid, the height BE being 48, and each side of the base 13, to find the content.

13
13

39
13

169=area of the base.
16=⅓ of the height EB.

1014
169

2704=content.

EXAMP. 3. *To measure a Cone.*—The diameter AC being 13, and the height BD 48, to find the content.

13
13

39
13

169
·7854

676
845
1352
1183

132·7326=area of the base.

Brought up. 132·7326
16=$\frac{1}{3}$ of the height.

7963956
1327326

2123·7216=content.

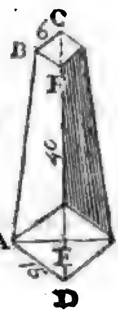
Note. The superficial content of all pyramids is found by taking the sum of the several areas, which compose them. That of a cone, by multiplying the circumference of the base into half the line joining the vertex and any point in that circumference, and adding the area of the base to the product.

Art. 33. *To measure the Frustum of a Pyramid.*

Definition. The frustum of a pyramid is what remains after the top is cut off by a plane parallel to the base, and is in the form of a log greater at one end than the other, whether round, or hewn three or four square, &c.

Rule. If it be the frustum of a square pyramid, multiply the side of the greater base by the side of the less; to this product add one third of the square of the difference of the sides, and the sum will be the mean area between the bases; but if the base be any other regular figure, multiply this sum by the proper multiplier of its figure in the Table, Art. 11. and the product will be the mean area between the bases: lastly, multiply this by the height, and it will give the height of the frustum.

Examp. 1. In the frustum of a square pyramid the side of the greater base AD=15, the side of the less, BC=6, and the height EF=40, to find the content.



15=AD.	15
6=BC.	6
Prod.=90	9=difference.
Add 27	9
117	3)81=square of the difference.
× 40	
4680=content.	27=$\frac{1}{3}$ of the square.

Or, if it be a tapering square stick of timber, take the girth of it in the middle; square $\frac{1}{4}$ of the girth (or multiply it by itself in inches) then say, as 144 (inches) to that product; so is the length, taken in feet, to the content in feet.

Examp. 2. What is the content of a tapering square stick of timber, whose side of the largest end is 12 inches, of the least end, 8, and whose length is thirty feet.

One fourth of the girth in the middle=10, and 10×10=100, the area in the middle; then, as 144 : 100 :: 30 feet : 20·83 feet the content.

By the Sliding Rule.

Set 12 on D to $\frac{1}{4}$ of the circumference on C, and against the length on D is the answer on C.

By Gunter.

The extent from 12 to $\frac{1}{4}$ of the circumference doubled, or twice turned over, will reach from the length to the content.

EXAMP. 3. In the frustum of a triangular pyramid, the side of the greater base AC=15, as before, the side of the less BD=6, and the height EF=40, to find the content.

15=AC.	15
6=BD.	6
—	—
9=difference of the sides.	90
9	Add 27
—	—
3)81=square of the difference.	117
—	·433 multiplier.
27=$\frac{1}{3}$ of the square.	—
	351
	351
	468
	—
	50·661=mean area.
	40=height.
	—
	2026·440=content.

Or, if it be a tapering three square stick of timber, you may find the area midway from end to end, then, as 144 is to that area, so is the length, taken in feet, to the content in feet.

EXAMP. 4. *To measure the Frustum of a Cone.*

RULE. Multiply the diameters of the two bases together, and to the product add one third of the square of the difference of the diameters: then multiplying this sum by ·7854, it will be the mean area between the two bases, which being multiplied by the length of the frustum, will give the solid content.

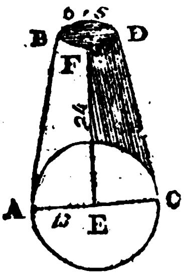
Or, to the areas of the top and bottom add the square root of the product of those areas, and the sum, multiplied by one third of the height of the frustum, will give the solidity.

When figures run uniformly taper; but not to a point (they being considered as portions of the cone or pyramid) we may find the solidity by supplying what is wanting to complete the figure, and then deducting the part cut off.

A general rule for completing every straight sided solid, whose ends are parallel and similar.

As the difference of the top and bottom diameters is to the perpendicular height, (or depth which is the same :) so is the longest diameter to the altitude of the whole cone or pyramid.

EXAMP. 1. The former cone in Art. 32, Examp. 3, being cut off in the middle, the greater diameter AC is 13, the less BD $6\frac{1}{2}$, and height EF 24, to find the content of the frustum.



AC=13 inches.
BD=6·5 inches.

```
   65
  78
 ----
 84·5
Add 14·083
 ------
 98·583
 ·7854
 ------
 394332
 492915
 788664
690081
---------
77·427|0882=mean area.
 24 feet=length.
 ------
309708
154854
 ------
1858·248=content.
```

```
 13
 6·5
 ---
 6·5=difference.
 6·5
 ---
 325
390
 ---
3)42·25= { square of the differ.
 ------
14·083=⅓ of the square.
```

```
144)1858·248(12·9045 feet content.
    144
    ---
    418
    288
    ---
    1302
    1296
    ----
     648
     576
     ---
     720
     720
```

EXAMP. 2. What number of barrels, each 32 gallons of Ale measure, is contained in a cistern whose largest diameter is 6 feet, and smallest diameter 5 feet, and whose depth is 8 feet?

$6 \times 5 = 30$

$\frac{\overline{6-5}|^2}{3} = \frac{1}{3} = 0\frac{1}{3}$

```
  30⅓ mean diameter.
  ·7854
 -------
 23·5620
  ·2618
 -------
 23·3238 mean area.
       8
 -------
190·5904 content in feet.
    1728 inches in a solid foot.
 -------
```

329340·2112 cubic inches, which divided by 9024, the cubic inches in a barrel or 32 gallons, gives 36·5 barrels nearly, Ans.

If the answer had been required in Beer Measure, where the barrel contains 36 gallons, the answer would have been 32·4 barrels.

Note. If when the end diameters of a conical cistern are given, it is required to find the length of the cistern to contain a certain number of barrels; divide the cubic feet contained in the number of barrels by the mean area, and the quotient will be the height.

Let the mean area be as in the last Ex. to find the length of the cistern to contain 50 barrels of 32 gallons of Ale measure.

261·11111 &c.=cubic feet in 50 barrels, which divided by 23·8238, the mean area, gives 10·95 feet, for the length of the cistern, Ans.

To find the diameters of the cistern, when the content, and length, and difference of the diameters, are given, see Art. 53.

Art. 34. *To measure a Sphere or Globe.*

Definition. A sphere or globe is a round solid body, in the middle of which is a point, from which all lines drawn to the surface are equal.

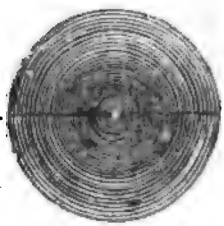
Rule. Multiply the cube of the diameter by ·5236, and the product will be the solid content.

Or, multiply the circumferenee by the diameter, which will give the superficial content; then multiply the surface by one sixth of the diameter, and it will give the solidity.

Or, multiply the cube of the diameter by 11, and the product divided by 21, will give the solidity.

Examp. The diameter, AB, of a globe, is 4·5 feet; to find the solid content.

$$
\begin{array}{r}
4{\cdot}5 \\
4{\cdot}5 \\
\hline
225 \\
180 \\
\hline
20{\cdot}25 \\
4{\cdot}5 \\
\hline
10125 \\
8100 \\
\hline
91{\cdot}125 \\
{\cdot}5236 \\
\hline
546750 \\
273375 \\
182250 \\
455625 \\
\hline
47{\cdot}7130500
\end{array}
$$

A  B

Note. If the circumference, or greatest circle of the sphere, be given, multiply the cube of it by ·016887 for the content.

The surface of the globe may be found by multiplying the square of the diameter by 3·1416; or by multiplying the area of its greatest circle by 4, or the square of the circumference by ·3183.

When the solidity of a globe is given, the diameter may be found by dividing the solidity by ·5236, and extracting the cube root of the quotient.

Or, if the circumference be required, divide the solidity by ·016887, and the cube root of the quotient will give it.

Art. 35. *To measure the Solidity of a Frustum or Segment of a Globe.*

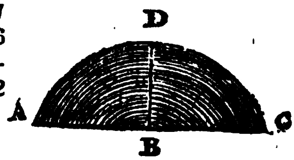
Definition. The frustum of a globe is any part cut off by a plane.

Rule. To three times the square of the semidiameter of the base, add the square of the height; then multiplying that sum by the height, and the product by ·5236, you will have the solid content.

Examp. The height BD being 9 inches, and the diameter of the base AC 24 inches: to find the content.

12=semidiameter.
12
———
144=square.
× 3
———
432
Add 9×9= 81= { square of the height.
———
513
× 9=height.
———
4617

4617
·5236
———
27702
13851
9234
23085
———
2417·4612=solid content.



To measure the Surface of a Frustum or Segment of a Globe.

Rule. Find the diameter of the globe by Art. 24, and the surface of the whole globe, by Art. 34; then, as the diameter of the globe is to the height of the frustum; so is the surface of the globe to the surface of the frustum; then, by Art. 15, find the area of the base; add these two together, and the sum will be the whole surface of the frustum.

Art. 36. *To measure the middle Zone of a Globe.*

Definition. This part of a globe is somewhat like a cask, two equal segments being wanting, one on each side of the axis.

Rule. To twice the square of the middle diameter, add the square of the end diameter; multiply that sum by ·7854, and that product, multiplied by one third of the length, will give the solidity.

Or, to four times the square of the middle diameter add twice the square of the end diameter; that sum multiplied by ·7854, and that product by one sixth of the length, will give the solidity.

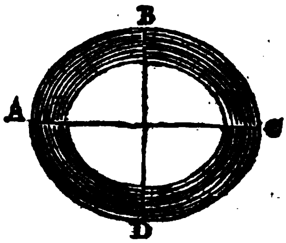
Note. This rule is applicable to the frustum of a cone or pyramid.

If the middle diameter of a zone be 20 inches, the end diameters each 16 inches, and length 12 inches: Required its solidity?

$$\overline{\overline{20\times20\times2}+\overline{16\times16}}\times\cdot7854\times4=3317\cdot5296, \text{ Ans.}$$

ART. 37. *To measure a Spheroid.*

Definition. A spheroid is a solid body like an egg, only both its ends are the same.



RULE. Multiply the square of the diameter of the greatest circle, viz. the diameter of the middle (DB in the figure) by the length AC, and that product by ·5236, and you will have the solidity.

EXAMP. The diameter BD being 20, and the length AC 30, to find the content.

$20\times20\times30\times\cdot5236=6283\cdot2$, Ans.

ART. 38. *To measure the middle Frustum of the Spheroid.*

Definition. This is a cask like solid, wanting two equal segments to complete the spheroid.

RULE. The same as in Article 36.

If the middle and end diameters of the middle frustum of a spheroid be 40 and 30 inches, and its length 50; what is its solidity?

$$50\div3=16\cdot6, \text{then } \overline{\overline{40\times40\times2}+\overline{30\times30}}\times\cdot7854\times16\cdot6=53454\cdot324, \text{Ans.}$$

ART. 39. *To measure a Segment, or Frustum of a Spheroid.*

Definition. This is a part of a spheroid made by a plane, parallel to its greatest circular diameter.

RULE. To four times the square of the middle diameter, add the square of the base diameter, then multiply that sum by ·7354, and the product by one sixth of the altitude, and it will give the solidity.

If the base diameter of the end frustum of a spheroid be 36, diameter at the middle of the height 30, and the height 20 inches; required its solidity?

$$\overline{\overline{30\times30\times4}+\overline{36\times36}}\times\cdot7854\times3\cdot3=12689\cdot55+, \text{ Ans.}$$

ART. 40. *To measure a Parabolick Conoid.*

Definition. This solid may be generated by turning a semiparabola about its abscissa or altitude.

RULE. As a parabolick conoid is half of its circumscribing cylinder, of the same base and altitude; multiply the area of the base by half the height for the solidity.

If the diameter of the base of a parabolick conoid be 40 inches, and its height 42; what is the solidity?

$40\times40\times\cdot7854\times21=26389\cdot44$, Ans.

ART. 41. *To measure the lower Frustum of a Parabolick Conoid.*

Definition. This solid is made by a plane passing through the conoid parallel to its base.

RULE. Multiply the sum of the squares of the diameters of the bases by ·7854, and that product by half the height, for the solidity.

If the diameters of a frustum of a parabolick conoid be 40 and 30 inches, and its height 20 inches; required its solidity.

$\overline{\overline{40\times40}+\overline{30\times30}}\times\cdot7854\times10=19635$, Ans.

ART. 42. *To measure a Parabolick Spindle.*

Definition. This solid is formed by an obtuse parabola, turned about its greatest ordinate.

RULE. This solid being eight fifteenths of its least circumscribing cylinder, multiply the area of its middle or greatest diameter by eight fifteenths of its perpendicular length, and it will give its solidity.

If the diameter at the middle of a parabolick spindle be 20 inches, and its length 60; required its solidity.

$20\times20\times\cdot7854\times32\ (=60\times8\div15)\ =10053\cdot12$, Ans.

ART. 43. *To measure the middle Zone, or middle Frustum, of a Parabolick Spindle.*

Definition. This is a cask like solid, wanting two equal ends of said spindle.

RULE. To the sum and half sum of the squares of the two diameters add three tenths of the difference of their squares, which multiply by a third of the length, and the product will be the solidity.

If the middle and end diameters of the middle frustum of a parabolick spindle be 40 and 30 inches, and its length 60; what is its solidity?

$40\times40=1600$	$1600-900=700$ the difference of the squares.
$30\times30=\ 900$	$700\times\cdot3=210=$three tenths of do. then,
Sum$=2500$	$2500+1250+210\times20\ (=\frac{1}{3}$ of $60)=79200$, Ans.
Half sum$=1250$	

ART. 44. *To measure a Cylindroid, or Prismoid.*

Definition. A cylindroid is a solid somewhat like the frustum of a cone, one base may be an ellipsis, and the other a disproportional ellipsis or circle.

A prismoid is a solid somewhat like the frustum of a pyramid, but its bases are disproportional.

Rule. The same as for the frustum of a cone or pyramid: or, to the areas of both bases, add a mean area, that is, the square root of the product of the two bases, then multiply that sum by a third of the height or length, and it will give the solidity.

If the diameters of the greater base of a cylindroid be 30 and 20 inches, the diameter of the less base 12, and length 60 inches; what is the solidity.

$$\left.\begin{array}{r} 30\times 20=600 \\ 12\times 12=144 \\ \sqrt{144\times 600}=293\cdot 9 \\ \hline 1037\cdot 9 \end{array}\right\} 1037\cdot 9\times\cdot 7854\times 20\ (=60\div 3)= 16303\cdot 33,\ \text{Ans.}$$

If the diameters of the greater base of a prismoid be 30 and 20 inches, the less base 20 by 10 inches, and length 30 inches: What is its solidity?

$$\left.\begin{array}{r} 30\times 20=600 \\ 20\times 10=200 \\ \sqrt{600\times 200}=346\cdot 4 \\ \hline 1146\cdot 4 \end{array}\right\} 1146\cdot 4\times 10\ (=30\div 3)=11464\ \textit{solidity in inches.}$$

Note. To find the solidity of a Wedge, add the length of the edge to twice the length of the base, and multiply the sum by the product of the height of the wedge and the breadth of the base, and one sixth of this product will be the solidity.

Let the base of a wedge be 27 by 8, the edge 36, and the height 42; then $\frac{\overline{2\times 27+36}\times 8\times 42}{6}=5040$, Ans.

Art. 45. *To measure a Solid Ring.*

Rule. Measure the internal diameter of the ring, and its girth, or circumference: then multiply the girth by $\cdot 31831$, and the product will be the diameter of the wire, which add to the internal diameter; multiply this sum by $3\cdot 1416$, and the product will be the length of a cylinder equal to the ring of the same base. Then the area of a section of the ring multiplied by the length of the said cylinder will give the solidity of the ring.

If an iron ring be 12 inches in girth, and its internal diameter be 20 inches; what is its solidity?

$\cdot 31831\times 12=3\cdot 8=$ring's diameter. $\overline{20+3\cdot 8}\times 3\cdot 1416=74\ 77$ the length of a cylinder equal to the ring: And

$3\ 8\times 3\cdot 8\times\cdot 7854\times 74\cdot 77=847\cdot 97=$solidity.

Art. 46. *To measure the Solidity of any irregular Body, whose dimensions cannot be taken.*

Take any regular vessel, either square or round, and put the irregular body into it: pour so much water into the vessel as will exactly cover the body, and measure the dry part from the top of

the vessel to the water, then take out the body, and measure again from the top of the vessel to the water, and subtract the first measure from the second, and the difference is the fall of the water: then, if the vessel be square, multiply the side by itself, and that product by the fall of the water, and you will have the content of the body; but if it be a long square, multiply the length by the breadth, and that product by the fall of the water; or, lastly, if it be a round vessel, multiply the square of the diameter by ·7854, and that product by the fall of the water, and you will have the content.

EXAMP. 1. A body being put into a vessel 18 inches square, on taking out the body, the water sunk 9 inches; required the content of the body?
18 inch.=1·5 foot.
9 inch.=·75 foot.
$1·5 \times 1·5 \times ·75 =$ 1·6875 foot, content.

EXAMP. 2. A body put into a cistern 4 feet by 3, on taking it out, the water fell 6 inches; required the content of the body?
$4 \times 3 \times ·5 = 6$ feet, content.

EXAMP. 3. A body being put into a round tub, whose diameter was 1·5 foot, on taking out the body, the water fell 1·5 foot; what was the content of the body?
$1·5 \times 1·5 \times ·7854 \times 1·5 =$ 2·65 feet, content.

Of the five Regular Bodies.

There are five solids contained under equal regular sides, which by way of distinction, are called *the five regular bodies.*

These are the *Tetraedron*, the *Hexaedron* or *Cube*, the *Octaedron*, the *Dodecaedron*, and the *Eicosiedron*. The measuring of the cube was shewn at Art. 28. I shall now show how to measure the other four by the following Table, which is the shortest method.

A Table of the solid and superficial content of each of the five bodies, the sides being unity, or 1.

Names of the Bodies.	Solidity.	Superficies.
Tetraedron.	0·11785	1·73205
Hexaedron.	1·	6·
Octaedron.	0·4714	3·464
Eicosiedron.	2·181695	8·66025
Dodecaedron.	7·663119	20·6457

All like solid bodies being in proportion to one another as the cubes of their like sides, the solid content of any of these bodies may be found by multiplying the cubes of their sides by the numbers in the second column under Solidity; and their superficies, by multiplying the squares of their sides into the numbers in the third column under Superficies.

OF THE TETRAEDRON.

This solid is contained under four equal and equilateral triangles, that is, it is a triangular pyramid of four equal faces, the side of whose base is equal to the slant height of the pyramid, from the angles to the vertex.

ART. 47. *The side of the Tetraedron being 3, to find the solid and superficial content.*

Cube$=3\times3\times3=27$, and $27\times\cdot11785=3\cdot18195=$solidity.
Square$=3\times3=9$, and $9\times1\cdot73205=15\cdot58845=$superficies.

OF THE OCTAEDRON.

This solid is contained under eight equal and equilateral triangles, which may be conceived to consist of two quadrangular pyramids of equal bases joined together, the sides of whose bases are equal to the given sides of the triangles, under which it is contained.

ART. 48. *The side of an Octaedron being 3, to find the solid and superficial content.*

Cube$=3\times3\times3=27$, and $27\times\cdot4714=12\cdot7278=$solidity.
Square$=3\times3=9$, and $9\times3\cdot464=31\cdot176=$superficies.

OF THE DODECAEDRON.

This solid is contained under 12 equilateral pentagons, and may be conceived to consist of twelve pentagonal pyramids, of equal bases and altitude, whose vertices meet in the centre of the dodecaedron.

ART. 49. *The side of a Dodecaedron being 3, to find the solid and superficial content.*

Cube$=3\times3\times3=27$, and $27\times7\cdot663119=206\cdot904$.
Square$=3\times3=9$, and $9\times20\cdot6457=185\cdot8113$.

OF THE EICOSIEDRON.

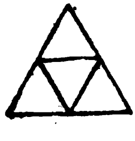
This solid is contained under twenty equal and equilateral triangles, and may be conceived to consist of twenty equal triangular pyramids, whose vertices all meet in the centre.

ART. 50. *The side of an Eicosiedron being 3, to find the solid and superficial content.*

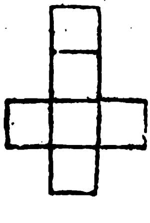
Cube$=3\times3\times3=27$, and $27\times2\cdot18169=58\cdot90563=$solidity.
Square$=3\times3=9$, and $9\times8\cdot66025=77\cdot94225=$superficies.

As the figures of some of these bodies would give but a confused idea of them, I have omitted them; but the following figures, cut out in pasteboard, and the lines cut half through, will fold up into the several bodies.

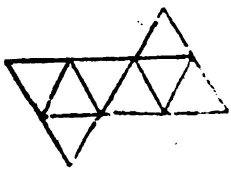
Tetraedron.



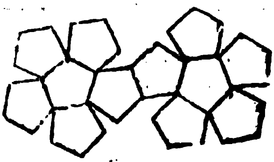
Hexaedron.

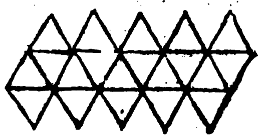


Octaedron.



Dodecaedron. *Eicosiedron.*





OF CASK GAUGING.

Among the many different canons drawn from Stereometry, for Gauging casks, the following is as exact as any.

Take the dimensions of the cask in inches, viz. the diameter at the bung and head, and length of the cask; subtract the head diameter from the bung diameter, and note the difference.

If the staves of the cask be much curved or bulging between the bung and the head, multiply the difference by ·7; if not quite so curve, by ·65; if they bulge yet less, by ·6; and if they are almost or quite straight, by ·55, and add the product to the head diameter; the sum will be a mean diameter, by which the cask is reduced to a cylinder.

Square the mean diameter, thus found, then multiply it by the length; divide the product by 359 for ale or beer gallons, and by 294 for wine gallons.

Note 1. The length is most conveniently taken by callipers, allowing, for the thickness of both heads, 1 inch, 1½ inch, or 2 inches, according to the size of the cask; but if you have no callipers, do thus; measure the length of the stave, then take the depth of the chimes, which with the thickness of the head, being subtracted from the length of the stave, leaves the length within.

Note 2. You must take the head diameter, close to its outside, and, for small casks, add three tenths of an inch: for casks of 30, 40, or 50 gallons, 4 tenths, and for larger casks, 5 or 6 tenths, and the sum will be very nearly the head diameter within. In taking the bung diameter, observe, by moving the rod backward and forward, whether the stave, opposite the bung, be thicker or thinner than the rest, and if it be, make allowance accordingly.

By the Sliding Rule.

On D is 18·94, the gauge point for ale or beer gallons, marked AG, and 17·14, the gauge point for wine gallons, marked WG: set the gauge point to the length of the cask on C, and against the mean diameter, on D, you will have the answer in ale or wine gallons according to which gauge point you make use of.

By the Scale.

Take the extent from the guage point to the mean diameter, set one foot of the dividers in the length, and turning them twice over, they will point out the content.

Art. 51. Required the content in ale and wine gallons, of a cask, whose bung diameter is 35 inches, head diameter 27 inches, and length 45 inches?

Bung diameter=35
Head diameter=27

Difference = 8
·7

5·6
Add the head dia.=27

Mean diameter=32·6
32·6

1956
652
978

Squared 1062·76

Square of the diameter=1062·76
Length= 45

531380
425104

359)47824·20(133·21 [ale gall.
294)47824·2(162·66 wine gall.

Art. 52. A round mash tub is 42 inches diameter at the top, within, and 36 inches at the bottom, and the perpendicular height 48 inches; required the content in beer and wine gallons?

This being the lower frustum of a cone, to the product of the diameters add $\frac{1}{3}$ of the square of their difference; multiply this sum by the length, and it will give the solidity in such parts as the dimensions are taken in. If they be taken in inches, divide by 359 for beer, and 294 for wine gallons.

$$\overline{42\times36+\frac{\overline{42-36}\times\overline{42-36}}{3}\times48}\div\begin{cases}359=203\frac{1}{2}\text{ ale gallons.}\\294=248\frac{3}{4}\text{ wine gallons.}\end{cases}$$

Art. 53. Let the difference of diameters of this tub be 6 inches, the height 48 inches, and the content $203\frac{3}{4}$ gallons, to find the diameters?

Multiply the content, if beer measure, by 359; if wine measure, by 294, and divide the product by the length: from the quotient subtract $\frac{1}{3}$ of the square of the difference of the diameters; to this remainder add the square of $\frac{1}{2}$ the difference of the diameters, and extract the square root of the sum; from the square root subtract $\frac{1}{2}$ the difference of the diameters, and it will give the least diameter to great exactness, to which add the difference of the diameters, and the sum is the greatest diameter.

$$\sqrt{\frac{203\cdot75\times359}{48}-\frac{6\times6}{3}+3\times3}-3=36,\text{ and }36+6=42.$$

The diameters are 36 and 42.

The content of any vessel in gallons, &c. may be thus found: measure the inside of the vessel, according to the rule of the figure, and find the content in cubick inches; then,

Divide by	1728	and the quotient will be the content in	cubick feet.
	282		ale or beer gallons.
	231		wine gallons.
	2150·425		bushels.

ART. 54. *To ullage a Cask, lying on one side, by the Gauging Rod, when the Bung Diameter, and the Content, one, or both are greater or less than the Table on the Rod is made for.*

RULE. As the bung diameter of the cask to be measured, is to the bung diameter that the table is made for; so are the dry inches of the cask, to a fourth number, which find in the table on the rod, and note the number of gallons answering to it. Then as the content of the cask that the table is made for, is to the content of the cask to be measured; so is the number of gallons answering to the aforesaid fourth number, to the number of gallons your cask wants of being full.

ART. 55. *To find a Ship's Burthen, or to Gauge a Ship.*

There is such a diversity in the forms of ships, that no general rule can be applied to answer all varieties; however, the following rules are practised.

RULE 1. Multiply the breadth at the main beam, half the breadth, and length together; divide the product by 94, and the quotient is the tons.

RULE 2. Divide the continued product of the length, breadth, and depth, in feet, by 100, for ships of war, and 95 for merchant ships, in which nothing is allowed for guns, &c. and the quotient is the tons.

RULE 3. Take the length from the stern post to the upper part of the stem; subtract two thirds of her breadth from that length; multiply the remainder by the whole breadth, and that product by half the breadth, in feet, and divide by 100 for war, and 94 for merchant tonnage.

RULE 4. The weight of a ship's burthen is half the weight of water she can hold.

What is the tonnage of a ship, whose length is 97 feet, breadth 31 feet, and depth 15½ feet.

By Rule 1st.		*By Rule 2d.*	
½ breadth	15·5	Length	97
Breadth	31	Breadth	31
	155		97
	465		291
	480·5		3007
Length	97	Depth=	15·5
	33635		15035
	43245		15035
			3007
	94)46608·5(495·83 tons.		95)46608·5(490·61 tons.
	Carried over.		Carried over.

Brought over.

94)46608·5(495·83 tons.
376
———
900
846
———
548
470
———
785
752
———
330
282
———
48

100)46608·5(466 tons.
400
———
660
600
———
608
600
———
85

Brought over.

95)46608·5(490·61 tons.
380
———
860
855
———
585
570
———
150
95
———
55

By Rule 3d.

Length=97
Subtract ⅔ of breadth=20·66
———
76·33
Multiply by the breadth 31
———
7633
22899
———
2366·23
Multiply by ½ breadth 15·5
———
1183115
1183115
236623
———
94)36676·565(390·176 tons.

Allowing the Cubit, as it is found by modern travellers, to be 22 inches, the content of Noah's Ark is as follows, viz.

	Cubits.	
Length of the keel,	300	Its burthen as a man of war 27729 tons.
Breadth by the midship beam,	50	
Depth in the hold,	30	As a merchant ship, 29188·6 ts.

QUESTIONS IN MENSURATION.

1. THE largest of the Egyptian pyramids is square at the base, and measures 693 feet on a side: how much ground does it cover?

$\frac{696 \times 393}{272 \cdot 25}=1764$ poles, and $\frac{1764}{160}=11$ acres and 4 poles, Ans.

2. What difference is there between a floor 20 feet square, and two others, each 10 feet square?

$\overline{20 \times 20}-\overline{\overline{10 \times 10}+\overline{10 \times 10}}=200$ feet, Ans.

3. There is a square of 2500 yards in area: what is each side of the square, and the breadth of a walk along one side and one end, which may take up just one half of the square?

$\sqrt{2500}=50$ yards, each side. $\sqrt{\frac{2500}{2}}=35 \cdot 35$, and $50-35 \cdot 35=14 \cdot 65$ yards, breadth of the walk, Ans.

4. A pine plank is 16 feet and 5 inches long, and I would have just a square yard slit off: at what distance from the edge must the line be drawn?

A square yard=1296 inches, and 16 feet 5 inches=197 inches.

Therefore, $\frac{1296}{197}=6\frac{114}{197}$ inches, Ans.

5. If the area of a triangle be 900 yards, and the perpendicular 40 yards: required the length of the base?

$\frac{900 \times 2}{40}=45$ yards, Ans.

6. If the three sides of a plane triangle be 24, 16, and 12 perches: required its area?

$\frac{24+16+12}{2}=26$; $26-24=2$; $26-16=10$; $26-12=14$, and $\sqrt{26 \times 14 \times 10 \times 2}=85 \cdot 32$ perches, =area. Again, as $24 : 16+12 :: 16-12 : 4 \cdot 6+$, the difference of the segments of the base; then, $12-\frac{4 \cdot 6+}{2}=9 \cdot 6$, and $\sqrt{\overline{12 \times 12}-\overline{9 \cdot 6 \times 9 \cdot 6}}=7 \cdot 11$ the perpendicular on the longest side; whence $24 \div 2 \times 7 \cdot 11=85 \cdot 32$, area as above.

7. Required the area of a circular garden, whose diameter is 12 rods? $12 \times 12 \times \cdot 7854=113 \cdot 0976$ poles, Ans.

8. The wheel of a perambulator turns just once and a half in a rod: what is its diameter?

$16 \cdot 5 \times \frac{2}{3}=11$ circumference, and $11 \times \cdot 31831=3\frac{1}{2}$ feet, Ans.

9. Agreed for a platform to the curb of a round well, at $7\frac{1}{2}$d. per square foot: the inward part, round the mouth of the well, is 36 inches diameter, and the breadth of the platform was to be $15\frac{1}{2}$ inches: what will it come to?

$36+\overline{15\cdot5\times2}=67$ the greatest diam.; $\overline{67\times67\times\cdot7854}-\overline{36\times36\times\cdot7854}$
$=\frac{2507\cdot8722}{144}=17\cdot4157$ square feet, at $7\frac{1}{2}$d. per foot, $=10$s. $10\frac{6}{10}$d.
[Ans.

10. Required the difference between the area of a circle, whose radius (or semidiameter) is 50 yards, and its greatest inscribed square?

$50\times2=100$ the diameter, and $100\times100\times\cdot7854=7854$ the area of the circle; then, $50\times50\times2=5000$ the area of the greatest inscribed square, and $7854-5000=2854$, Ans.

11. There is a section of a tree 25 inches over; I demand the difference of the areas of the inscribed and circumscribed squares, and how far they differ from the area of the section?

$\overline{25\times25}-\overline{12\cdot5\times12\cdot5\times2}=312\cdot5$ the difference of the squares. $\overline{25\times25}-\overline{25\times25\times\cdot7854}=134\cdot125$ the circumscribed square, more than the section, and $\overline{25\times25\times\cdot7854}-\overline{12\cdot5\times12\cdot5\times2}=178\cdot375$ inscribed square, less than the area of the section.

12. Four men bought a grindstone of 60 inches diameter: how much of its diameter must each grind off, to have an equal share of the stone, if one first grind his share, and then another, till the stone is ground away, making no allowance for the eye?

Rule. Divide the square of the diameter by the number of men, subtract the quotient from the square, and extract the square root of the remainder, which is the length of the diameter after the first man has ground his share; this work being repeated by subtracting the same quotient from the remainder, for every man, to the last; extract the square root of the remainders, and subtract those roots from the diameters, one after another; the several remainders will be the answers.

$$\begin{array}{r}60\\60\\\hline 4)3600\\\hline \text{Quot.}=900\end{array}$$

From 3600
Take 900

$\sqrt{2700}=51\cdot9615$, to be taken from 60.
Subt. 900

$\sqrt{1800}=42\cdot4264$, from 51·9615.
Subt. 900

$\sqrt{900}=30$, from 42·4264.

From 60
Take 51·9615
Remains 8·0385 = 1st share.

From 51·9615
Take 42·4264
Rem. 9·5351 = 2d share.

From 42·4264
Take 30·
Rem. 12·4264 = 3d share.
And 30 inches = 4th share.

13. If a cubick foot of iron were hammered, or drawn, into a square bar, an inch about, that is, $\frac{1}{4}$ of an inch square: required its length, supposing there is no waste of metal?

$$\frac{12\times12\times12}{\cdot25\times\cdot25\times4}=6912 \text{ inches},=576 \text{ feet, Ans.}$$

14. Required the axis of a globe, whose solidity may be just equal to the area of its surface?

$$\frac{\cdot7854\times4}{\cdot5236}=6 \text{ inches, Ans.}$$

15. A joist is $7\frac{1}{2}$ inches wide, and $2\frac{1}{4}$ thick; but I want one just twice as large, which shall be $3\frac{3}{4}$ inches thick: what will be the breadth?

$$\frac{7\cdot5\times2\cdot25\times2}{3\cdot75}=9 \text{ inches, Ans.}$$

16. I have a square stick of timber 18 inches by 14; but one of a third part of the timber in it, provided it be 8 inches deep, will serve: how wide will it be?

$$\frac{18\times14}{3}\div8=10\frac{1}{2} \text{ inches, Ans.}$$

17. A had a beam of oak timber, 18 inches square throughout, and 25 feet long, which he bartered with B, for an equilateral triangular beam of the same length, each side 24 inches: required the balance at 1s. 4d. per foot?

$$\frac{18\times18\times25}{144}=56\cdot25, \text{ solidity of the square beam.}$$

The perpendicular let fall on one of the sides of the triangular beam is 20·7846 inches, and the half perp.=10·3923; then $\frac{10\cdot3923\times24}{144}=1\cdot732$ foot, area at the end, and $1\cdot732\times25=43\cdot3$ feet, solidity of the triangular beam; therefore $56\cdot25-43\cdot3=12\cdot95$ feet, at 1s. 4d. per foot=17s. 3·2d. balance due to A, Ans.

18. What is the difference between a solid half foot, and half a foot solid?

$$\frac{12\times12\times6}{6\times6\times6}=4, \text{ therefore, one is but } \tfrac{1}{4} \text{ of the other.}$$

19. A lent B a solid stack of hay, measuring 20 feet every way; sometime afterward, B returned a quantity measuring every way 10 feet: what proportion of the hay remains due?

$$20\times20\times20-10\times10\times10=7000 \text{ feet}=\tfrac{7}{8}, \text{ Ans.}$$

20. A ship's hold is $75\frac{1}{2}$ feet long, $18\frac{1}{2}$ wide, and $7\frac{1}{4}$ deep: how many bales of goods $3\frac{1}{2}$ feet long, $2\frac{1}{4}$ deep, and $2\frac{3}{4}$ wide, may be stowed therein, leaving a gang way the whole length, of $3\frac{1}{4}$ feet wide?

$$\frac{\overline{75\cdot5\times18\cdot5\times7\cdot25}-\overline{75\cdot5\times7\cdot25\times3\cdot25}}{3\cdot5\times2\cdot25\times2\cdot75}=385\cdot44 \text{ bales, Ans.}$$

21. If a stick of timber be $20\frac{1}{2}$ feet long, 16 inches broad, and 8 inches thick, and $3\frac{1}{2}$ solid feet be sawed off one end: how long will the stick then be?

$$20\tfrac{1}{2}-\frac{1728\times3\cdot5}{16\times8}=16 \text{ feet, } 6\tfrac{3}{4} \text{ inches, Ans.}$$

22. The solid content of a square stone is found to be $136\frac{1}{2}$ feet; its length is $9\frac{1}{2}$ feet: what is the area of one end? and if the breadth be 3 feet 11 inches, what is the depth?

$\frac{136\cdot5\times1728}{9\cdot5\times12}$=area 2069·0526 inches, and $\frac{2069\cdot0526}{47}=44\cdot022$ inches, Ans.

23. I would have a cubick box made capable of receiving just 50 bushels, the bushel containing 2150·425 solid inches: what will be the length of the side?

$\sqrt[3]{2150\cdot4\times50}=47\cdot55$ inches.

24. A statute bushel is to be made 8 inches high, and $18\frac{1}{2}$ inches diameter, to contain 2176 cubick inches; (though the content of the dimensions is but 2150·425 inches) I demand what the diameter of the bushel must be, the height being 8 inches; and what the height, the diameter being $18\frac{1}{2}$ inches, to contain 2176 cubick inches?

Solidity.

Height=8)2176 and $\sqrt{272\times1\cdot273}=18\cdot6$ diameter. $18\cdot5\times18\cdot5\times\cdot7854=268\cdot80315$=area, and the solidity

Area= 272 — $2176\div268\cdot8=8\cdot0956$ inches, height.

25. There is a garden rolling stone 66 inches in circumference, and $3\frac{1}{2}$ cubick feet are to be cut off from one end, perpendicular to the axis: where must the section be made?

Area= $\frac{1728\times3\cdot5}{412\cdot5}=14\cdot65$ inches from one end, Ans.

26. I would have a syringe of $1\frac{1}{2}$ inch diameter in the bore, to hold a quart, wine measure: what must be the length of the piston, sufficient to make an injection with?

$1\cdot5\times1\cdot5\times\cdot7854=1\cdot76715$, and $231\div4=57\cdot75$ the cubick inches in a quart, then $\frac{57\cdot75}{1\cdot76715}=32\cdot679$ inches, Ans.

27. If a round pillar, 9 inches diameter, contain 5 feet: of what diameter is that column, of equal length, which measures 10 times as much?

As $5 : 9\times9 :: 5\times10 : 810$, and $\sqrt{810}=28\cdot46$ inches, Ans.

28. There is a square pyramid, each side of whose base is 30 inches, and whose perpendicular height is 120 inches, to be divided by sections parallel to its base into 3 equal parts: required the perpendicular height of each part?

$30\times30\times40=36000$ the solidity in inches, now $\frac{2}{3}$ thereof is 24000, and $\frac{1}{3}$ is 12000. Therefore,

As $36000 : 120\times120\times120 :: \begin{Bmatrix}24000\\12000\end{Bmatrix} : \begin{Bmatrix}1152000\\576000\end{Bmatrix}$ Then,

$\sqrt[3]{1152000}=104\cdot8$ Also, $\sqrt[3]{576000}=83\cdot2$. Then, $120-104\cdot8=15\cdot2$ length of the thickest part, and $104\cdot8-83\cdot2=21\cdot6$ length of the middle part, consequently 83·2 is the length of the top part.

29. Suppose the diameter of the base of a conical ingot of gold to be 3 inches, and its height 9 inches; what length of wire may

be expected from it, without loss of metal, the diameter of the wire being one hundredth part of an inch?

$3\times3\times\cdot7854\times3=21\cdot2058$ the solidity of the cone.

$$\frac{21\cdot2058}{\cdot01\times\cdot01\times\cdot7854}=270000 \text{ inch.}=4 \text{ miles, and } 460 \text{ yards, Ans.}$$

30. Suppose a pole to stand on a horizontal plane 75 feet in height above the surface: at what height from the ground must it be cut off, so as that the top of it may fall on a point 55 feet from the bottom of the pole, the end, where it was cut off, resting on the stump, or upright part?

As the whole length of the pole is equal to the sum of the hypotenuse and perpendicular of a triangle, (the 55 feet on the ground being the base) this, as well as the following question, may be solved by this

RULE. From the square of the length of the pole (that is, of the sum of the hypotenuse and perpendicular) take the square of the base; divide the remainder by twice the length of the pole, and the quotient will be the perpendicular, or height at which it must be cut off.

$$\frac{\overline{75\times75}-\overline{55\times55}}{75\times2}=17\tfrac{1}{3} \text{ feet, Ans.}$$

31. Suppose a ship sails from latitude 43°, north, between north and east, till her departure from the meridian be 45 leagues, and the sum of her distance and difference of latitude to be 135 leagues: I demand her distance sailed, and latitude come to?

$$\frac{\overline{135\times135}-\overline{45\times45}}{135\times2}=60 \text{ leagues, and } 60\times3=180 \text{ miles}=3 \text{ de-}$$

grees the difference of latitude, $135-60=75$ leagues the distance. Now as the vessel is sailing from the equator, and consequently the latitude is increasing: Therefore,

To the latitude sailed from	43°,00′ N.
Add the difference of latitude	3 ,00

And the sum is the latitude come to=46 ,00 N.

BOOK KEEPING.

BOOK KEEPING is a systematic record of mercantile transactions.

Every mercantile transaction consists in giving one thing for another. This change of property requires a distinct record in the books prepared for the purpose, so as to enable the man of business to know the true state of his affairs, and of his accounts with an individual.

The importance of a correct knowledge of Book keeping, to the man of business is obvious. His books should exhibit the result of each transaction, and the general result of the whole.

Book keeping may be performed either by Single or Double Entry.

The method of book keeping by single entry is the most simple, and is sufficient for the generality of Mechanics, Farmers, Retail Merchants, &c. The method by Double Entry is more systematic in its principles and more certain in its conclusions, and is much to be preferred for wholesale or any extensive business.

In Single Entry only, persons are entered as debtor and creditor.

BOOK KEEPING BY SINGLE ENTRY.

In the practice of single entry, two principal books, the Day Book or Waste Book, and the Leger, and one auxiliary book, the Cash Book, are necessary.

1. THE DAY BOOK OR WASTE BOOK.

The Day Book should begin with an account of all the property, debts, &c. of the person, and be followed by a distinct record of all the transactions of the trade in the order of time in which they occur.

Some accountants use also a Blotter, in which the changes of property are recorded, and the Day Book is only a copy of the Blotter, written in a more fair and plain manner.

Each page of the Day Book should be ruled with three columns on the right side for pounds, shillings, and pence, or with two columns for dollars and cents, as the accounts are to be kept in one or the other of these denominations of money.

The following is the order observed in making an account in the Day Book: First, the date; next, the name of the person with the abbreviation Dr. or Cr. at the right hand, as he is debtor or creditor by the transaction; and then, the article or articles with the price annexed, unless the article be money, and the value carried

out in the ruled columns, with the sum of the whole placed directly under, when there is more than one charged. Thus, for example,

	£	s.	d.
January 1st, 1820.			
David Bradley, Dr.			
To 2 yards of broadcloth at 42s. per yard,	4	4	0
To 10lbs. leaf sugar at 2s. 2d.	1	1	8
	5	5	8
January 2d.			
Simon Jones, Cr.			
By 3 bushels of wheat at 9s. 6d.	1	8	6

The following rule shows whether a person is to be entered as Dr. or Cr. on the Day Book. The person who receives any thing from me is Dr. to me, and the person from whom I receive is Cr.

Or, The person, who becomes indebted to me, whether by receiving goods or money or by my paying his debts, must be entered Dr.; and the person to whom I become indebted, whether by receiving from him goods or money, or by the payment of my debts, must be entered Cr.

The following general direction is to be observed in keeping the Day Book, viz.

Enter on the Day Book every case of debt or credit relating to the business in the order of time in which it takes place, and in language so explicit as not to be mistaken.

This rule is most important, because the Day Book is the decisive book of reference in case of any supposed mistake or error in the accounts in the Leger.

2. THE LEGER.

The various accounts of each person are collected from the Day Book, and placed or posted under his name, and on two opposite pages of the Leger, as they are Dr. or Cr. The name of the person is to be written in large and fair characters as a title, and the accounts in which he is Dr. are to be written on the left hand page, and those in which he is Cr. on the right hand page of the same folio. If the name be written only on the Dr. page, the title of the other page is to be, *Contra* or *Ca. Cr.* The Leger is ruled with a margin for the date of each transaction, or with a column for the page of the Day Book which contains the account, or with both. It must also be ruled with two or three columns on the right of each page for the denominations of money, as they may be Dolls. and Cents, or £ s. and d. If the Leger be a wide Folio, the accounts of Dr. and Cr. may be placed on the same page, as in the following example.

Dr.		James Fowler,						Cr.	
			$	c.				$	c.
Jan.1	3	Iron 3Cwt.	15	60	Jan.2	4	Wheat 12 bushels,	18	50
5	4	Rum 10 galls.	10	20	6	5	Corn 7 do.	3	71
11	8	Wine 3 galls.	6	36	10	7	Cash,	7	84
		Lot Ford, Dr.					Contra Cr.		
Jan.1	2	Broadcloth 2 yds.	9	50	Jan.3	4	Beef 110lbs.	4	40
6	5	Wine 1 gall.	2	12	7	6	Wood 3 cords,	6	00

In the preceding example, the two columns on the left both of the Dr. and Cr. accounts contain the date of the transaction and the page of the Day Book on which the original account is to be found. Next follows the article and its quantity, which *should* be written in few words, and then its amount in the money columns. Either the date or the page of the Day Book which contains the account, is amply sufficient in the Leger, and the latter is to be preferred.

The Leger exhibits at one view the accounts *with an individual*, as it contains on the Dr. side whatever he has received, and on the Cr. side whatever he has paid. The difference between the sums of Dr. and Cr. called the Balance, shows the state of the trade in this instance.

An *Index* must accompany the Leger, in which the names are arranged alphabetically, with the page of the Leger on which each account is to be found. See the Index to the Leger for Ex. 2.

The following general directions are to be observed in forming the Leger. Let each account be posted from the Day Book in its proper place in the Leger. If a mistake be made, let it be corrected by an account in the Day Book, clearly stating the correction, and then let this account be posted in its proper place in the Leger, that no blot or erasure may disfigure its pages.

THE CASH BOOK.

In this book are recorded the daily receipt and payment of money. For this purpose there are two columns, one for money received, and the other for money paid, in which should be recorded merely the date, to or by whom paid, and the sum. The Cash Book is convenient, but not absolutely necessary. By some accountants other auxiliary books are used, which are found to be useful or important in some particular business. These the accountant will readily form for himself, as circumstances may render necessary.

Note 1. As several of the preceding books may be necessary in the progress of business, they should be distinguished by lettering them in the following manner. Day Book A, Day Book B, &c. Leger A. Leger B. &c. And in posting accounts into the Leger, there must be a reference to Day Book A. or B. &c. as the account is found in the one or the other. See Example 2.

Note 2. In the following example the barter of any article, as well as the sale of an article for cash, is entered into the Day Book, although such accounts are not to be posted into the Leger. This is not generally practiced, but the accountant will often find a material benefit in recording even these changes of property.

EXAMPLE I. SINGLE ENTRY.

DAY BOOK.

	$	c.
January 1, 1820. My whole property is a debt of 400 dollars due me from Samuel Richards, the balance of my inheritance.		
Jan. 1, 1820. **Samuel Richards, Dr.** To balance from the estate,	400	
3rd. **Samuel Richards, Cr.** By broadcloth 105 yards at 3 dolls per yard,	315	
4th. **John Higgins, Dr.** To 55 yards of broadcloth, at $3 50c. per yard,	192	50
5th. Exchanged 40yds. of broadcloth, for 24Cwt. Iron at $5.		
6th. **John Higgins, Cr.** By cash, on account,	180	
7th. Sold S. M. 20Cwt. of Iron at $3¼ per Cwt. for cash,	105	

Note. The preceding example contains so few accounts, that the formation of the Cash Book is unnecessary. It is sufficient, however, to illustrate the principles of Single Entry; while it is so short that the whole may be easily comprehended by the pupil. The Leger, in which this Day Book is posted, is on the following page.

EXAMPLE I. SINGLE ENTRY.

LEGER.

Day Book	Dr.	$	c.	Day Book	Cr.	$	c.
	Samuel Richards				Contra Cr.		
1	Balance from estate,	400		1	Broadcloth 105yds.	315	
					Balance,	85	
						400	
	John Higgins, Dr.				Contra Cr.		
1	Broadcloth, 55yds.	192	50	1	Cash,	180	
					Balance,	12	50
						192	50

To balance the accounts, place the difference of the several accounts under the smaller side. Thus in the account with Samuel Richards, $85 is the Cr. to balance; and, in the account with John Higgins, $12 50c. is the balance on the same side. *It is obvious* that I have gained by the trade. Were not the gain evident, on inspection, it would be made so by the following inventory from the preceding Leger.

January 8, 1820.	$	c.
Due from Samuel Richards, - - - - -	85	00
——— John Higgins, - - - - -	12	50
On hand 10yds. broadcloth at $3 per yard, -	30	00
——— Cash from broadcloth and iron, -	285	00
——— 4Cwt. of iron at $5 - - - - -	20	00
Amount,	432	50
My property Jan. 1, - - - - -	400	00
Gain,	32	50

EXAMPLE II. SINGLE ENTRY.

DAY BOOK A.

(1

	£	s.	d.	£	s.	d.
January 1, 1821.						
Inventory of all my property and debts, taken this day by me, A. B.						
Ready money,	300	0	0			
House and Furniture,	500	0	0			
Williams Farm,	600	0	0			
Merchandise,	555	0	0			
Produce,	45	0	0			
	2000	0	0			
I owe, on accounts,						
To Henry Hardy, £15 10s.						
To Thomas Howe, 30 0	45	10	0			
My net estate.	1954	10	0			
January 1, 1821.						
Henry Hardy, Cr.						
By former account, balance				15	10	0
Thomas Howe, Cr.						
By former account, balance				30	0	0
Salmon Rogers, Dr.						
To 20 bushels of wheat, at 9s. per bushel,				9	0	0
To 6 yards of broadcloth, at 33s. per yd.				9	18	0
				18	18	0
John Wheat, Dr.						
To 20 gallons of Rum, at 6s. 9d. per gall.				6	15	0
John Taylor, Dr.						
To 6lbs. loaf sugar, at 2s. 7d. per lb					15	6
1gall. of Rum,					6	9
				1	2	3
2d.						
Simon Pond, Dr.						
To 5 bushels of wheat, at 9s. per bushel,				2	5	0
John Wheat, Cr.						
By cash, on account 60s.				3	0	0
3d.						
Henry Hardy, Dr.						
To 3 gallons of wine, at 12s. per gall.				1	16	0
30 bushels of wheat, at 9s. 6d. per bush.				14	5	0
				16	1	0

2)

January 3d, 1821.			
Titus Coale, Dr.			
To 20 gallons of Rum, at 7s. per gall.	7	0	0
¼cwt. of Havanna sugar, at 60s. per cwt.		15	0
	7	15	0
4th.			
John Wheat, Cr.			
By 50lbs. of nails, at 8d. per ℔	1	13	4
Simon Pond, Cr.			
By cash, on account 25s.	1	5	0
6th.			
Peter Owen, Dr.			
To goods delivered C. Paige, by your order,	3	16	8
Dixon Ferry, Dr.			
To 56 yards cotton cloth, at 1s. 4d. per yd.	3	14	8
8th.			
Henry Hardy, Dr.			
To 2 yards blue broadcloth, at 36s. per yd.	3	12	0
Salmon Rogers, Cr.			
By cash to balance,	18	18	0
Peter Pindar, Dr.			
To 30 galls. wine, at 13s. 6d. per gall.	20	5	0
9th.			
Titus Coale, Cr.			
By 25 bushels wheat, at 8s. 6d. per bush.	10	12	6
Hervey Brown, Dr.			
To 5yds. brown linen, at 1s. 9d. per yd.		8	9
10th.			
John Merrill, Dr.			
To 10℔ nails, at 10d. per ℔		8	4
16℔ brown sugar, at 11d.		14	8
	1	3	0
Thomas Howe, Dr.			
To cash, on account,	15	0	4
Titus Coale, Dr.			
To 16galls. Rum, at 7s. 3d.	5	16	0
12th.			
Peter Owen, Cr.			
By 20½ bushels wheat, at 9s.	9	4	6

January 13th, 1821.			(3
Simon Pond, Dr.			
To 3 gallons Rum, at 7s.	1	1	0
1½gall. wine, at 12s. 4d.		18	6
	1	19	6
14th.			
John Wheat, Cr.			
By cash in full, 41s. 8d.	2	1	8
17th.			
Thomas Howe, Dr.			
To 20galls. rum, at 7s. 3d. per gall.	7	5	0
5 do. wine, at 12s. 4d.	3	1	8
40lbs. nails, at 9d.	1	10	0
	11	16	8
18th.			
Charles Gray, Dr.			
To 3 gallons French Brandy, at 12s. per gall.	1	16	0
10¾lbs. loaf sugar, at 2s. 4d.	1	5	1
	3	1	1
Dixon Ferry, Cr.			
By 1 mahogany table, 72s.	3	12	0
1 wash stand, 17s.		17	0
	4	9	0
19th.			
Simon Pond, Cr.			
By cash, on account 43s. 6d.	2	3	6
20th.			
Peter Pindar, Cr.			
By 40 bushels of wheat at 9s.	18	0	0
cash, on account 45s.	2	5	0
	20	5	0
23d.			
John Taylor, Cr.			
By 20½lb butter, at 1s. 1d.	1	2	3
Henry Hardy, Dr.			
To 30 bushels of wheat, at 9s. 6d.	14	5	0
26th.			
Hervey Brown, Cr.			
By cash, on account 8s. 9d.		8	9
Titus Coale, Cr.			
By 6½ bushels wheat, at 8s. 6d.	2	15	3
cash, on account 3s. 3d.		3	3
	2	18	6

4)

		£	s.	d.
January 29th, 1821.				
Peter Owen,	Dr.			
To 1cwt. white Havanna sugar, at 63s.		3	3	0
10galls. Rum, at 8s.		4	0	0
	Cr.	7	3	0
By cash, on account 35s. 2d.		1	15	2
Dixon Ferry,	Dr.			
To 20lbs. brown sugar, at 11d.			18	4
John Merrill,	Cr.			
By cash in full 23s.		1	3	0
31st.				
Thomas Howe,	Dr.			
To 1cwt. Havanna sugar, at 63s.		3	3	0
Charles Gray,	Cr.			
By 8½ bushels wheat, at 8s. 6d.		3	12	3
Samuel Lyman,	Dr.			
To 100 bushels of wheat, at 9s.		45	0	0
	Cr.			
By cash, on account £40 10s.		40	10	0
Joshua Noble,	Dr.			
To 1cwt. Havanna sugar, at 63s.		3	3	0

LEGER INDEX,

FORMED FOR THE FOLLOWING LEGER, INTO WHICH THE PRECEDING DAY BOOK IS POSTED.

A.	B.	C.
	Brown, Hervey p. 2.	Coale, Titus p. 2.
D.	E.	F.
		Ferry, Dixon 2.
G.	H.	I.
Gray, Charles 3.	Hardy, Henry 1. Howe, Thomas 1.	
K.	L.	M.
	Lyman, Samuel 3.	Merrill, John 2,
N.	O.	P.
Noble, Joshua 3.	Owen, Peter 2.	Pindar, Peter 2. Pond, Simon 2.
Q.	R.	S.
	Rogers, Salmon 1.	
T.	U.	V.
Taylor, John 1.		
W.	X.	Y.
Wheat, John 1.		

Note. In the following Leger, both the date of the transaction and its page on the Day Book are given, merely to exercise the learner, as only one of them is essential.

EXAMPLE II. LEGER A.

(1)

1821.	P. D.B. A.	Henry Hardy, Dr.	£	s.	d.
Jan. 3	1	Wine 3 gallons at 12s.*	1	16	0
		Wheat 30 bushels at 9s. 6d.	14	5	0
8	2	Blue broadcloth 2yds. at 36s.	3	12	0
23	3	Wheat 30 bushels at 9s. 6d.	14	5	0
			33	18	0
		Thomas Howe, Dr.			
Jan. 10	2	Cash,	15	0	4
17	3	Rum 20 gallons at 7s. 3d.	7	5	0
		Wine 5 do. 12s. 4d.	3	1	8
		Nails 40lbs. 9d.	1	10	0
31	4	Havana Sugar 1Cwt.	3	3	0
			30	0	0
		Salmon Rogers, Dr.			
Jan. 1	1	Wheat 20 bushels,	9	0	0
		Broadcloth 6yds.	9	18	0
			18	18	0
		John Wheat, Dr.			
Jan. 1	1	Rum 20 gallons,	6	15	0
		John Taylor, Dr.			
Jan. 1	1	Loaf Sugar 6lbs.		15	6
		Rum 1 gallon,		6	9
			1	2	3

* In posting accounts into the Leger, some accountants write the article, quantity, and price, as is here done; others omit the price in the Leger, as the cor-

EXAMPLE II. LEGER A.

(1)

1821.	P. D.B. A.	Contra Cr.	£	s.	d.
Jan. 1	1	Balance of former account,	15	10	0
		Balance,	18	8	0
			33	18	0

		Contra Cr.			
Jan. 1	1	Balance of former account.	30	0	0

		Contra Cr.			
Jan. 8	2	Cash,	18	18	0

		Contra Cr.			
Jan. 2	1	Cash,	3	0	0
4	2	Nails 50lbs.	1	13	4
14	3	Cash,	2	1	8
			6	15	0

		Contra Cr.			
Jan. 23	3	Butter 20½lb,	1	2	3

rectness of each account is to be ascertained in the Day Book. The latter method is sufficient, and is generally followed in this Example.

Example II. LEGER A.

(2)

1821.	P. D.B. A.		£	s.	d.
		Simon Pond, Dr.			
Jan. 2	1	Wheat 5 bushels,	2	5	0
13	3	Rum 3 galls.=21s. and Wine 1½ gall.= 18s. 6d.	1	19	6
			4	4	6
		Titus Coale, Dr.			
Jan. 3	2	Rum 20 galls.=140s. and Sugar ¼Cwt.= 15s.	7	15	0
10	2	Rum 16 galls.	5	16	0
			13	11	0
		Peter Owen, Dr.			
Jan. 6	2	Goods per your order to C. Paige,	3	16	8
29	4	Havanna Sugar, 1cwt.=63s. and Rum 10 galls.=80s.	7	3	0
			10	19	8
		Dixon Ferry, Dr.			
Jan. 6	2	Cotton Cloth 56yds.	3	14	8
29	4	Brown Sugar 20lbs.		18	4
			4	13	0
		Peter Pindar, Dr.			
Jan. 8	2	Wine 30 gallons,	20	5	0
		Hervey Brown, Dr.			
Jan. 9	2	Brown Linen, 5yds.		8	9
		John Merrill, Dr.			
Jan. 10	2	Nails 10lb.		8	4
		Brown Sugar 16lb.		14	8
			1	3	0

EXAMPLE II. LEGER A.

(2)

1821.	P. D. B. A.	Contra Cr.	£	s.	d.
Jan. 4	2	Cash,	1	5	0
19	3	Cash,	2	3	6
		Balance,		16	0
			4	4	6
		Contra Cr.			
Jan. 9	2	Wheat 25 bushels,	10	12	6
26	3	Wheat 6½ do.	2	15	3
		Cash,		3	3
			13	11	0
		Contra Cr.			
Jan. 12	2	Wheat 20½ bushels,	9	4	6
29	4	Cash,	1	15	2
			10	19	8
		Contra Cr.			
Jan. 18	3	1 Mahogany Table 72s. and 1 Wash Stand 17s.	4	9	0
		Balance,		4	0
			4	13	0
		Contra Cr.			
Jan. 20	3	Wheat 40 bushels=£18 and cash £2 5s.	20	5	0
		Contra Cr.			
Jan. 26	3	Cash,		8	9
		Contra Cr.			
Jan. 29	4	Cash,	1	3	0

EXAMPLE II. LEGER A.

(3)

1821.	P. D. B. A.		£	s.	d.
		Charles Gray, Dr.			
Jan. 18	3	French Brandy, 3 gallons,	1	16	0
		Loaf Sugar, 10¾lbs.	1	5	1
		Balance,		11	2
			3	12	3
		Samuel Lyman, Dr.			
Jan. 31	4	Wheat 100 bushels,	45	0	0
		Joshua Noble, Dr.			
Jan. 31	4	Havanna Sugar, 1cwt.	3	3	0

EXAMPLE II. LEGER A.

(3)

1821.	P. D. B. A.	Contra Cr.	£	s.	d.
Jan. 31	4	Wheat 8½ bushels,	3	12	3
		Contra Cr.			
Jan. 31	4	Cash,	40	10	0
		Balance,	4	10	0
			45	0	0
		Contra Cr.			
		Balance,	3	3	0

CASH BOOK,

FORMED FROM THE DAY BOOK, EXAMPLE II.

Received.

		£	s.	d.
Jan. 2.	Of John Wheat,	3	0	0
4.	S. Pond,	1	5	0
8.	S. Rogers,	18	18	0
14.	J. Wheat,	2	1	8
19.	S. Pond,	2	3	6
20.	P. Pindar,	2	5	0
26.	H. Brown,		8	9
	T. Coale,		3	3
29.	P. Owen,	1	15	2
	J. Merrill,	1	3	0
31.	S. Lyman,	40	10	0
Amount received,		73	13	4
do. paid.		24	4	8
Excess,		49	8	8
Cash, Jan. 1.		300	0	0
Do. Feb. 1.		349	8	8

Paid.

	£	s.	d.
Jan. 10. Thomas Howe,	15	0	4
To this add my expences for the month, being	9	4	4
Amount paid,	24	4	8

In balancing the Leger, as in Example 2, draw two heavy black lines under the accounts in which the sums of Dr. and Cr. are equal, to show that the account is settled. Where the sums of Dr. and Cr. are unequal, place the sum to balance, under the smaller

account, writing against it the word, Balance. As there is to be no line drawn under these accounts, and as there is no reference in the marginal columns to the Day Book, it will be obvious on inspection, that such accounts are not settled.

TO FIND THE PROFIT OR LOSS.

An Inventory of the property and of the debts must be taken, as follows, from Example 2.

February 1, 1821.

Inventory of all my property, and of the sums due to me, or owed by me, taken this day by me A. B.

	£	s.	d.	£	s.	d.
Ready Money,				349	8	8
House and Furniture,				504	9	0
Williams Farm,				600	0	0
Merchandise,				480	0	0
Produce,				10	0	0
				1943	17	8
Due from Henry Hardy,	18	8	0			
S. Pond,		16	0			
D. Ferry,		4	0			
S. Lyman,	4	10	0			
J. Noble,	3	3	0			
	27	1	0			
I owe Charles Gray,		11	3			
Difference,	26	9	9	26	9	9
Net amount of property, Feb. 1.				1970	6	8
do, Jan. 1.				1954	10	0
Profit in the month,				15	16	8

GENERAL REMARK ON SINGLE ENTRY.

Book Keeping by Single Entry, shows clearly the state of accounts with individuals, but it does not exhibit the true state of his affairs to the book keeper himself. For this purpose, he must take an Inventory of all his property and debts, to ascertain the quantity of goods unsold, and the net amount of his property, and thence the profit or loss of trade, in the manner just taught. This is a work of much difficulty and trouble, if the business be extensive. It is for this reason, that book keeping by Double Entry is preferred in extensive trade.

SHORTER METHOD OF POSTING ACCOUNTS IN SINGLE ENTRY.

The method already given, of posting accounts from the Day Book into the Leger, is generally considered the most correct. The following shorter method is perhaps more commonly used.

The Leger is ruled as before, and merely the amount of an account is posted into the Leger, preceded by the page and letter of the Day Book on which the account is found. The first two accounts of the preceding Leger are here posted from the Day Book for an example of this method.

LEGER. SHORTER METHOD.

	£	s.	d.		£	s.	d.
Dr. Henry				**Hardy, Cr.**			
A 2. £16 1s. and £3 12	19	13	0	A 1. £15 10s.	15	10	0
A 4. £14 5s.	14	5	0	Balance,	18	8	0
	33	18	0		33	18	0
Dr. Thomas				**Howe, Cr.**			
A 3. £15 0s. 4d. and	15	0	4	A 1. £30.	30	0	0
£11 16s. 8d.	11	16	8				
A 4. £3 3s.	3	3	0				
	30	0	0				

In this Leger, A 1, A 2, &c. means that on page 1, 2, &c. of Day Book marked A, that particular account is to be found.

The manner of balancing the Books, and of ascertaining the Profit or Loss is the same as before taught. To make the subject familiar the learner should be directed to form a Day Book for himself, and to carry the accounts through the several books, according to the preceding principles.

SHORTEST METHOD OF KEEPING ACCOUNTS.

Only one Account Book is necessary in the practice of this method. It is formed precisely like the Leger in Single Entry, except that there is no column of reference to any other book. The transactions of trade are entered under the names against the date on which they take place. An alphabet for the arrangement of the names is found convenient for reference to the various accounts.

This Account Book is designed to answer the double purpose of Day Book and Leger. If the person be careful to enter every instance of debt and credit at the time it occurs, he will be able to ascertain at any time the state of his accounts in a particular case. This is the great object of this method, which is exhibited in the following example.

ACCOUNT BOOK.

Linden, Jan. 1, 1820.

1820.	John Wilson, Dr.	$	c.
Jan. 1	To 3 cords of wood, $1 50 per cord,	4	50
6	To 1½ days work, by hired man, at 66 cents,	1	00
9	To 5½ bushels rye, at 50 cents,	2	75
13	To 3 bushels wheat, at $1 75,	5	25
Feb. 1	To 5 cords wood, at $1 50,	7	50
March 9	To 7 bushels oats, at 31 cents,	2	17
15	To work with hired man and horses, one day,	1	66
		24	83
18	Cash to balance,		34
		25	17

1820.	Peter Paywell, Dr.		
July 1	To 1lb hyson tea,	1	56
9	To 10lbs. brown sugar, at 19 cents,	1	90
25	To 3 gallons Rum, at $1 17,	3	51
Sept. 9	To 9lbs. blister steel, at 10 cents,		90
11	To 6½yds. calico, at 54 cents,	3	51
Oct. 3	To 3yds. cotton cloth, at 18 cents,	0	54
13	To 2lbs. loaf sugar, at 31 cents,	0	62
1821.			
Jan. 1	To 1lb hyson tea,	1	46
3	To goods delivered by your order to E. T.	3	72
		17	72

SHORTEST METHOD.

Linden, Jan. 1, 1820.

1820.	John Wilson, Cr.	$	c.
Jan. 1	By 12lb shingle nails, at 10 cents,	1	20
5	By 25½lbs. cheese, at 7 cents,	1	78
13	By your order on John Gibbs, for goods,	8	75
Feb. 1	By 20½lbs. Butter, at 18 cents,	3	69
March 7	By 1½cwt. iron, at $6 50 per cwt.	9	75
		25	17

1820.	Contra. Cr.		
Aug. 4	By 12lbs. butter, at 12½ cents,	1	50
7	By work 2 days by your man,	1	33
Oct. 3	By 56lbs. cheese, at 7 cents,	3	92
10	By cash,	3	00
1821.			
Jan. 1	By 12 bushels rye, at 50 cents,	6	00
3	By cash to balance,	1	97
		17	72

BOOK KEEPING BY DOUBLE ENTRY.*

THIS method of Book Keeping differs from that by Single Entry in two important respects, viz, things, as well as persons, are entered as Dr. and Cr. and as Dr. and Cr. to each other, and each account is entered twice in the Leger. For the latter particular it is called Double Entry.

In the practice of Double Entry, three Principal Books, and four Auxiliary Books are necessary.

PRINCIPAL BOOKS.

These are the Day Book or Waste Book, the Journal, and the Leger.

I. THE DAY BOOK OR WASTE BOOK.

The Day Book begins, as in Single Entry, with an Inventory of all the property and debts of the merchant, and is followed by a regular account of the transactions in business, in the order of time in which they occur, stated in language so explicit and full that there can be no mistake. This book is in Double Entry, a mere record of the changes of property, and Dr. and Cr. are not introduced into it References are made in it to the auxiliary books, when it is necessary. It is ruled as in Single entry.

To exhibit the difference in the two methods of Book Keeping, the principles of Double Entry will be illustrated by Example I. of Single Entry.

(1) EXAMPLE I. DOUBLE ENTRY. DAY OR WASTE BOOK.

	$	c.
January 1, 1821 My whole property is a debt of $400 due me from Samuel Richards, the balance of my inheritance.		
Samuel Richards owes me the balance of my inheritance,	400	
3rd. Bought of Samuel Richards 105 yards of broadcloth at 3 dolls. per yard	315	
4th. Sold John Higgins 55yds. of broadcloth at $3 50c. per yard	192	50
5th. Bartered 40yds of broadcloth for 24cwt. of Iron, at 5 dolls. per Cwt.	120	
6th Received of John Higgins in part.	180	
7th. Sold 20cwt. of Iron to S. M. for Cash at 5¼ dolls per Cwt.	105	

* The general principles of this system of Book Keeping are taken from the System in Rees'Cyclopedia, which is generally adopted by the merchants of London.

Note. In recording transactions in the Day Book, the above order is generally to be preserved, viz. 1st the date; 2d the kind of transaction in the active voice, as owes, sold, bought, exchanged, &c.; 3d the name of the person; 4th the article and quantity; 5th the price; and 6th the amount in the columns of money.

II. THE JOURNAL.

The object of the Journal is to prepare the accounts for the Leger. To effect this, the Dr. and Cr. of every article contained in the Day or Waste Book, is ascertained and expressed in the Journal.

The Journal is ruled with two blank columns on the left, viz. one for the date and the other for the page in the Leger, and with the proper columns for money on the right, as in the following Journal of the preceding Day Book.

EXAMPLE I. DOUBLE ENTRY.

JOURNAL.

(1)

Jan.	P. L.		$	c.
1		Samuel Richards Dr. to Stock $400 00 For the balance of my inheritance,	400	
3		Broadcloth Dr. to Samuel Richards 315 00 For 105yds. broadcloth at 3 dolls. per yd.	315	
4		John Higgins Dr. to broadcloth 192 50 For 55yds. broadcloth at $3 50 per yd.	192	50
5		Iron Dr. to broadcloth 120 00 To 40yds broadcloth at 3 dolls. per yard for 24cwt. of iron,	120	
6		Cash Dr. to John Higgins 180 00 Received of him on account,	180	
7		Cash Dr. to Iron 105 00 Received for 20cwt. of iron at $5 25 per Cwt.	105	

To understand the method of forming the Journal, the following distinctions must be attended to. Accounts are distinguished into personal, real, and fictitious. Personal accounts are those in which a person is entered as Dr. or Cr. and are the same as in Single Entry. Thus, in the preceding Journal, Samuel Richards is Dr in one account.

Real accounts are those of property of any kind, as cash, houses, cloth, furniture, adventure, &c. In the preceding example Broadcloth is Dr. to Samuel Richards, and Iron Dr. to broadcloth, &c.

Fictitious accounts are those of stock, and profit and loss. Stock is used for the owner of the books. In the preceding Example,

Samuel Richards is Dr. to stock, i. e. to the owner of the books. Profit and loss is used for either gain or loss in the course of trade. This account does not appear in the Journal but in the Leger.

If two or more persons or things are included in the same account in the Journal, they are expressed by the term, Sundries.

Rules for distinguishing Dr. and Cr. in the Journal, are the following.

The person to whom or for whom I pay, or whom I enable to pay, is Debtor.

The person for whom or from whom I receive, or by whom I am enabled to pay, is Creditor.

Whatever comes into my possession or under my direction, is Dr.

Whatever passes out of my possession or from under my controul, is Cr.

The phrases, In debtor, and Out creditor, briefly express the points in these rules. Thus, in the preceding Journal, Broadcloth,

EXAMPLE I. DOUBLE ENTRY.

LEGER.

Date. 1820.	P. J.	Stock, Dr.	P. L.	$	c.
		Samuel Richards, Dr.			
Jan. 1	1	To Stock,	1	400	
		Broadcloth, Dr.			
3	1	To Samuel Richards, 105yds. at $3 per yard,	1	315	
		Profit and loss,		27	50
				342	50
		John Higgins, Dr.			
4	1	To broadcloth, 55yds. at $3 .50,	1	192	50
		Iron, Dr.			
5	1	To broadcloth, 24cwt. at $3,	1	120	
		Profit and loss,		5	
				125	
		Cash, Dr.			
7	1	To John Higgins,	1	180	
8		Iron, 20cwt. at $5¼,	1	105	
				285	
		Profit and Loss, Dr.			

which comes into my possession, is Dr. to Samuel Richards by whom it is paid in part for stock in his hands; John Higgins is Dr. to broadcloth; Iron is Dr. to broadcloth, and so on.

III. THE LEGER.

The object of the Leger is the same as in Single Entry. But as things, as well as persons, are introduced in the Journal, they must have separate accounts in the Leger also, where the respective Drs. and Crs. are to be arranged under their respective heads.

The Leger is ruled with columns for the denominations of money on the right side, immediately before which is a column for reference to the page of the Leger in which the corresponding account is found; and on the left side, is a column for dates, and another for the page of the Journal in which the account may be found. The following Leger for the preceding Example is formed on this plan. See foot of this and the preceding page.

EXAMPLE I. DOUBLE ENTRY.

LEGER.

Date. 1820.	P. J.	Contra Cr.	P. L.	$	c.
Jan. 1	1	By Samuel Richards,		400	
		Contra Cr.			
3	1	By Broadcloth,	1	315	
		Contra Cr.			
4	1	By John Higgins, 55yds. at $3 50,	1	192	50
5	1	Iron, 40yds. at $3		120	
		Contra Cr.			
6	1	By cash,	1	180	
		Contra Cr.			
5	1	By cash, for 20cwt. at $5 25,	1	105	
		Contra Cr.			
		Contra Cr.			
		By cloth,		27	50
		Iron,		5	
				32	50

In posting the Journal to form the preceding Leger, Samuel Richards is posted Dr. to Stock $400, and Stock is posted Cr. by Samuel Richards for the same sum; Broadcloth is then entered Dr. in the next account to Samuel Richards, and Samuel Richards Cr. by broadcloth; John Higgins is posted Dr. to broadcloth, and Broadcloth Cr. by J. Higgins, and so on. It is obvious that each Dr. must have a Cr. and each Cr. a Dr. and that every transaction relating to any one account, whatever may be its place in one account in the Leger, must be posted also on the proper side under the head to which it belongs. Thus, while Cash is Dr. to J. Higgins in the 6th account for $180, J. Higgins is Cr. by cash for the same sum in 4th account; and while Cash is Dr. also to iron, Iron is Cr. by cash to the same amount, in the 5th account.

On inspecting the preceding Leger it is evident, that in the personal accounts, as those of S. Richards and J. Higgins, all the articles for which they are indebted are posted on the Dr. side, and all the articles they pay are on the Cr. side of the account; in the real accounts, as those of broadcloth, and iron, the quantity bought is posted on the Dr. side, and the quantity sold on the Cr. side so that the quantity unsold and the profit or loss may be readily ascertained. In the fictitious account of Profit and Loss, the loss is to be posted on the Dr. side, and the profit on the Cr. side, so that the difference must show the net gain or loss, by which the stock has been increased or diminished in the course of trade.

Having ascertained that the accounts have been correctly posted, the next step is to balance the Leger. This is to be done in the following manner. To show this method more clearly, the preceding Leger is repeated, and the several steps in balancing the accounts are subjoined.

To the preceding Leger subjoin a new account, Balance Dr. and Cr. Begin with the next account to Stock, and place the balance of Dr. and Cr. under the smaller, to make them equal, viz. $85 on the Cr. side. Put this sum on the Dr. side of the account, Balance. For if S. Richards is Cr. by Balance 85 dolls. then Balance must be Dr. to S. Richards for the same sum. Next, to balance the Broadcloth account, the quantity on the two sides must first be made equal, and the value of the unsold cloth, at first cost, be placed under the amount sold, viz. 10 yards at $3, amounting to $30. The whole sum, viz. $342 50 must be equal to the amount bought and the profit on that sold. Then $30 must be placed on the Dr. side of Balance, for if Broadcloth be Cr. for the balance unsold, then Balance must be Dr. for the same sum. Proceed in this manner, through all the personal and real accounts.

Proof.*

Having balanced all the accounts except those of Stock, Profit and Loss, and Balance, in the first place, close the account of Profit and Loss, by making it Dr. to the stock gained, viz. $32 50, and then make Stock Cr. by the same sum. Next, let Stock be balanced by the necessary sum, viz. $432 50, and Balance be made Cr. by the same sum. If the sides of the account, Balance, are now equal, the work is right.

This proof is complete from this consideration. By this method, the cash in hand, the debts due, and the goods unsold, are contained on one side, and what is owed, is contained on the other side.

Another method of proof is to add the profit to, or subtract the loss from, the original stock, and the sum or difference placed to the Cr. of Balance, will be equal to the sum on the Dr. side of Balance.

General Remark.

The Journal should be kept up with the Day Book, and the accounts should be regularly posted into the Leger, that the books may be as nearly even as possible. And at the end of every month, the balance should be made, the Journal and Leger having been carefully examined to see that all the records of the Day Book have been carefully transferred into the Journal, and correctly posted from the Journal into the Leger.

The following Example is sufficient to exhibit the principles of Double Entry. It was designed to be so short that the student might have the whole before him at one view. It is too short, however, to render any of the auxiliary books necessary. In extensive business, however, these books are essential. In the practice too of Double Entry, the work will be shortened by forming the Journal, in the manner shewn in the next Example. An account of the auxiliary books will afterward be given, and a specimen of each one, as connected with the next Example of Double Entry.

* See pages 514 and 515.

EXAMPLE I. DOUBLE ENTRY.

LEGER.

Date. 1820.	P. J.		P. L.	$	c.
		Stock, Dr.			
		By Balance, my net estate,	1	432	50
Jan.		**Samuel Richards, Dr.**			
1	1	To Stock,	1	400	
		Broadcloth, Dr.			
3	1	To Samuel Richards, 105yds. at $3 per yard,	1	315	
		Profit and loss,		27	50
				342	50
		John Higgins, Dr.			
4	1	To broadcloth, 55yds. at $3 50,	1	192	50
		Iron, Dr.			
5	1	To broadcloth, 24cwt. at $5,	1	120	
		Profit and loss,		5	
				125	
		Cash, Dr.			
7	1	To John Higgins,	1	180	
		Iron, 20cwt. at $5¼,	1	105	
				285	
		Profit and Loss, Dr.			
		To stock gained,		32	50
		Balance, Dr.			
		By Samuel Richards,	1	85	
		Broadcloth, unsold,	1	30	
		John Higgins,	1	12	50
		Iron, unsold,	1	20	
		Cash,	1	285	
				432	50

EXAMPLE I. DOUBLE ENTRY.

LEGER.

Date. 1820.	P. J.	Contra Cr.		P. L.	$	c.
Jan. 1	1	By Samuel Richards,		1	400	
		Profit and loss,		1	32	50
					432	50
		Contra	**Cr.**			
3	1	By Broadcloth, 105 yards, at $3,		1	315	
		Balance,			85	
					400	
		Contra	**Cr.**			
4	1	By John Higgins, 55yds. at $3 50,		1	192	50
5	1	Iron, 40yds. at $3		1	120	
		Balance, 10yds. unsold, at $3			30	
		105yds.			342	50
		Contra	**Cr.**			
6	1	By cash,		1	180	
		Balance,			12	50
					192	50
		Contra	**Cr.**			
7	1	By cash, for 20cwt. at $5 25,		1	105	
		Balance, 4cwt. at $5,			20	
		24cwt.			125	
		Contra	**Cr.**			
		By balance,		1	285	
		Contra	**Cr.**			
		By cloth,		1	27	50
		Iron,			5	
					32	50
		Contra	**Cr.**			
		By stock, my net estate,		1	432	50

AUXILIARY BOOKS.

These are 1, the Cash Book; 2, the Bill Book; 3, the Invoice Book; and 4, the Sales Book.

These books are important to the accountant, as a record of particular transactions referred to in the Day Book, and as original and particular records of those transactions. They aid especially in posting accounts into the Leger. They may be considered as a kind of Day Book, in aid of the general Day Book, and it is obvious, that if all the particular accounts were arranged under general heads in separate books, the common Day or Waste Book would be unnecessary, except as exhibiting a general history of the changes of property.

I. THE CASH BOOK.

The Cash Book is a record of all money paid or received. It is referred to in the Day Book, by the initials C. B. It is formed like the Leger, with a Dr. and Cr. side, the Dr. side containing all money received, and the Cr. all money paid. The transactions are to be regularly entered into the Cash Book as they take place, with the dates, names, and all necessary particulars.

The man of business will find it convenient to have separate columns for some transactions, as of money accounts at a Bank, Brokers, &c. and for some small incidental expenses, as well as for money lent and repaid immediately.

The money accounts should be transferred to the Leger at least every month. When the cash account is entered into the Journal, it is written, Cash Dr. to Sundries, for money received, and Sundries Dr. to Cash, for money paid, mentioning all the necessary particulars.

The following Cash Book shows the manner in which this book is formed and kept. It belongs to Example 2, of Double Entry, and is the Cash Book referred to in the Day or Waste Book of that Example. The same remark applies to the auxiliary books which follow the Cash Book.

To post the Cash Book into the Leger,

Make Cash Dr. to Sundries for the amount received, and Cr. by Sundries for the amount paid. Then make each account Dr. to Cash, for the respective sums paid, and Cr. by Cash for the respective sums received. See Example 2, Leger.

CASH BOOK. JANUARY, 1820.

Jan.	Dr.	$	c.	Jan.	Cr.	$	c.
3	To interest for discounting Wm. Burr's Bill No. 13.	6	39	2	By charges on merchandise, per the Venus for Naples,	25	
14	Bills receivable, No. 11, Williams & Co.	1600		3	Bills payable, No. 13, Wm. Burr,	1440	
23	Ship Phebus, received for freight,	400	87	7	Charges on mer. per the Dolphin, for Bilboa,	32	50
25	Farm in Cambridge, received for produce,	160	35	12	Bills payable, No. 11, George Myers,	2222	
26	Bills receivable, No. 12, George Murray,	1300		21	Charges for sales, per the Betsy, pd. customs, &c.	489	24
31	Interest, half year's dividend at the bank,	300		23	Bills payable, No. 12, John Howe,	600	
31	Funded property, sold 9000 at $81\frac{1}{8}$,	7272		25	Ship Phebus, paid for repairs,	130	25
31	Debentures, received,	300		27	Charges on mer. per the Henry for Jamaica,	51	50
		11339	61	31	Expenses of House,	150	52
						5039	01

II. THE BILL BOOK.

The Bill Book is a record of all Bills of Exchange receivable or payable. The reference in the Day Book, is by the initials B. B. or, by B. R. for bills receivable, and B. P. for bills payable.

Bills Receivable are those paid or to be paid to the merchant.

Bills Payable are those drawn on the merchant or to be paid by him.

The particulars of each kind of bills are entered in the Bill Book under their separate heads of B. R. or B. P.

The records of the Bill Book are entered into the Journal, under the heads, Bills Receivable Dr. to Sundries, for all bills accepted, and Sundries Dr. to Bills Payable, for all bills to be paid; with all the necessary particulars of names, numbers, &c.

To post the Bill Book into the Leger, make bills receivable Dr. to sundries for their whole amount, and bills payable Cr. by sundries for their whole amount. Then make the persons for whom bills have been accepted, Dr. to bills payable for their respective amounts, and each person from whom bills received Cr. by bills receivable for their respective amounts.

The following is a copy of the Form of the Bill Book for Example 2, of Double Entry.

BILL BOOK. JANUARY 1820.

BILLS RECEIVABLE.

Page Jour.	No.	When received	From whom rec'd.	By whom drawn, and place.	On whom drawn, and place.	Date.	To whom payable.	Time.	Due.	Sum.
	1	Jan. 2.	Cyrus Coate.	Mills, Bordeaux.	Dillon, London.	Dec. 1.	George Cabot,	3 mo. sight.	March 5.	$411 00.
	2	Jan. 22.	Lemuel Rogers.	Pierre, Paris.	Child, Liverpool.	Dec. 15.	Wm. Boyle.	2 months.	Feb. 15.	2500 00.

BILLS PAYABLE.

P. J.	No.	By whom drawn, and place.	Date.	To whom payable.	Time,	Accepted.	Due.	Sum.	To whom paid, and time.
	1	Gilson & Co. London.	Jan. 2.	George Brown.	Feb. 1.	Jan. 15.	Feb. 1.	$800 00.	John Wild, Feb. 1.
	2	Joseph Lockwood, Hull.	Jan. 3.	Simon Pond.	31 days date.	Jan. 20.	Feb. 3.	3600 00.	Bank, Feb. 3.
	3	Smith & Son, Boston.	Dec. 1.	James Hull.	90 days date.	Jan. 24.	March 1.	2150 00.	S. Pond, March 1.

Copy of the Bill received from Cyrus Coate, and entered among bills receivable.

2466 Livres, at $16\frac{2}{3}$ cents. BOURDEAUX, DEC. 1, 1819.

Three months after sight, pay to the order of George Cabot, two thousand four hundred and sixty six livres, at sixteen two thirds cents exchange, for value received.

JAMES MILLS.

MR. GEORGE DILLON, London. Accepted, Jan. 1820. G. DILLON.

Copy of a Bill drawn on me by Gilson & Co. and entered bills payable.

$800. LONDON, JAN. 1, 1820.

Thirty days after date, pay George Brown, or order, eight hundred dollars, for value received,

GILSON & CO.

MR. M. N. Charleston. Accepted, M. N.

III. THE INVOICE BOOK.

An Invoice is an account of merchandise exported, with the charges on the shipment.

The Invoice Book contains every invoice of goods shipped abroad. It is referred to in the Day Book by the initials I. B.

As an invoice accompanies goods received also, this book is sometimes distinguished into two general heads, Invoice inward, and Invoice outward. If the former be preserved on file for reference, it will be sufficient to enter into the Invoice Book merely the invoices of goods exported.

The invoice contains the name of the ship, master, the place of destination, and of the person to whom the consignment is made, and then the quantity of goods and amount at prime cost, with the shipping charges. On this whole sum commission is charged. The commission, and insurance, if the merchandise be insured, is then added to the cost and charges, and the drawback, if any is allowed at the custom house, deducted.

The record of the Invoice Book is entered into the Journal in the following manner.

The person, for whom the the Invoice is sent, Dr. to Sundries, viz.

To merchandise, for goods shipped.
To charges, for shipping charges, &c.
To commission, for the commission.
To insurance, for the insurance.

The following Invoice, referred to on page 2 of Day Book, Example 2, of Double Entry, shows the method of forming the Invoice Book, and is a specimen of the other invoices referred to in the same Day Book, by the initials I. B.

To post the Invoice Book into the Leger, make the person to whom the invoice is sent Dr. to sundries, for the amount. Then make merchandize, charges, commission, &c. Cr. for the respective sums belonging to each.

The posting of the Invoice Book is rendered shorter and more simple by uniting several invoices when it can be done, as on page 3, of Journal, Example 2.

INVOICE OF SUGAR,

Shipped on board the Venus, W. Brown master, for Naples, by order of George Parish, merchant, on his account and risk, and consigned to him.

January 2, 1820.

G. P.			$	c.
	Cwt. qr. lb. Cwt. qr. lb.			
1 to 3	No. 1 Gross, 12 0 12 Tare 1 2 3			
	2 —— 12 2 16 —— 1 3 0			
	3 —— 11 3 24 —— 1 0 25			
	Gross 36 2 24 4 2 0			
	Tare 4 2 0			
	Neat 32 0 24 at 16 dolls. per Cwt.		515	43
	Charges.			
	Debenture Entry, - - -	16 00		
	Cost of hogsheads, - - -	4 75		
	Cartage, wharfage, bill of lading, -	2 25		
			23	
			538	43
	Commission on $538 43/100 at 2½ per cent. -		13	46
	Premium of Insurance, - - - -		8	40
			560	29
	Drawback allowed at the Custom House,		120	
	(Entered Journal page 2.)			
			440	29
	Deducting the drawback from the cost of the merchandise, the account in the Day Book would be as follows,			
	Merchandise, 395 43			
	Charges 23 00			
	Commission, 13 46			
	Insurance, 8 40			
	——440 29			
	See page 2, Day Book, Ex. 2, of Double Entry.			

IV. THE SALES BOOK.

The object of the Sales Book is to show the net proceeds of any goods received to be sold on commission. Its reference in the Day Book is S. B.

Each account of sales begins with the names of the goods, ship, and person by whom the consignment is made, and contains two general columns or pages. In the first page are recorded the various charges arising from duties, freight, landing, storeage, commission, &c. The second contains the quantity, price and amount

sold, with the names of the purchasers and the time of payment. The difference between the amount of the two columns or pages, is the net proceeds. The account is then to be transmitted to the owner of the goods, being copied from the Sales Book and signed by the agent to whom the goods were consigned.

If the goods are sold for ready money, the account must be entered into the Cash Book.

In entering an account of sales into the Journal, the person, or owner of the goods is made Dr. for the sales. Then, Sales per ship —— is Dr. to Sundries, viz.

To charges on merchandise, for the charges made.
Interest, if any money has been advanced.
Commission, for the commission.
To the consignee, —— for the net proceeds.

SALES BOOK.

ACCOUNT OF 8 PIPES OF PORT WINE, RECEIVED PER THE BETSEY, FROM OPORTO, AND SOLD ON ACCOUNT OF GEORGE GREAVES.

1820	Dr.	$	c.	1820	Cr.	$	c.
Jan. 21	To duty on 1118 galls. at 33⅓ cents per gall.	372	67	Jan. 26	By Smith & Son, sold them payable in 1mo.		
	Freight, primage, &c.	30	33		3 pipes.		
					No. 1, 139 galls.		
		403	00		2, 141		
	Cooperage at 50 cents per pipe,	4			3, 138		
	Cartage, wharfage, &c.	16			418 galls.		
	Storeage, and Insurance, and taking stock,	10			Ullage 1 do.		
	Brokerage, 78 cents per pipe,	6	24		417 do. at $154 per pipe of 139 gallons,	462	
	Amount of charges,	439	24	26	By Jos. Lockwood, sold him payable in 2mo.		
	Interest on $403 for 60 days at 6 per cent, advanced,	4	03		5 pipes.		
					No. 1, 140 galls.		
	Commission at 2½ per ct. on $1234 $\frac{14}{100}$	33	94		2, 140		
					3, 139		
					4, 140		
		477	21		5, 141		
26	Net proceeds, due this day to Geo. Greaves, Oporto,	756	93		700 galls.		
					Ullage 3 do.		
		1234	14		697 do. at $154 per pipe of 139 gallons,	772	14
						1234	14

To post the Sales Book into the Leger, make the persons to whom the consignment is made Dr. to Sales (per ship ——) for the amount. Then make the consigner, charges, commission, interest, &c. Cr. by sales for the sums belonging to each respectively.

Note. Besides the Auxiliary Books already mentioned, several others are occasionally employed, which the accountant can readily form for himself. The names and object of several follow.

1. The Book of Accounts current, is a record of accounts sent to your employers.

2. The Book of Commissions, is a record of Orders from correspondents.

3. Book of Charges contains accounts not charged to any thing else, as rent, wages, postage, incidental expenses.

4. Copy Book of Letters, sent to correspondents.

5. Book of Postage of letters, contains their date and cost.

6. Book of Ship Charges, contains the charges for each ship, which is to be carried to the proper account of the ship in the Leger.

7. Book of Receipts, for all receipts.

8. Memorandum Book, for particulars to be attended to afterwards.

EXAMPLE II. WASTE BOOK.

		$	c.
January 1, 1820.			
Inventory of all my property real and personal, with a list of the balances in my favour and against me, taken this day.			
Cash in hand,		13500	
Funded property $12000 in the 5 per cents, at 80½ per cent,		9660	
Farm in Cambridge,		4500	
House in the city,		2050	
Furniture,		1200	
Ship Phebus, my half,		9500	
Merchandise on hand,		6400	
Debentures for balance due at the Custom House,		1300	
Bills receivable, the following in hand.			
No. 11. On Williams & Co. due Jan. 14,	$1600 00		
12. On George Myers, 26,	1300		
		2900	
Balances in my favour, viz.			
James Greaves, Oporto,	$1700 00		
Cyrus Coate, Bordeaux,	1560 35		
Lemuel Rogers, Bilboa,	1175 00		
George Parish, Naples,	2200 00		
		6635	35
		57645	35
I owe as follows,			
Bills payable for bills accepted by me.			
No. 11. Drawn by George Myers, due Jan. 12,	$2222		
12. John Howe, 23,	600		
13. William Burr, 30,	1440		
		4262	

EXAMPLE II. WASTE BOOK.

Balances against me.			
To Smith & Son, London,	$2150 00		
To Gilson & Co. Do.	800 00		
To Spring & Jones, Jamaica,	1666 67		
To George Black, Do.	1175 00		
To James Broker, Do.	1450 58		
		7242	25
		11504	25
January 2d, 1820.			
Shipped on board the Venus, W. Brown master, for Naples, sugar on the account of George Parish, as per Invoice Book, viz.			
Merchandise,	$395 43		
Charges,	23		
Commission,	13 46		
Insurance,	8 40		
		440	29
2d.			
Received by post a bill from Cyrus Coate of 2466 livres, at 16⅔ cents, as per B. R.		411	
3d.			
Paid William Burr's bill, No. 13, as per C. B.		1440	
Received discount on the above for 27 days, at 6 per cent. as per C. B.		6	39
7th.			
Shipped on board the Dolphin, for Bilboa, goods on account of Lemuel Rogers, as per I. B.			
Merchandise,	$2000 00		
Charges,	32 50		
Commission,	60 75		
Insurance,	34 15		
		2127	40
12th.			
Paid George Myers bill, due this day, No. 11. as per C. B.		2222	
14th.			
Received the amount of Williams & Co's. bill, No. 11, as per C. B.		1600	
15th.			
Accepted a bill drawn by Gilson & Co. No. 1, B. P.		800	
16th.			
Bought of Joseph Lockwood, sundry goods amounting, as per bills of parcels, to		10000	87
20th.			
Samuel Lockwood has drawn on me, a bill, No. 2, as per B. P.		3600	

EXAMPLE II. WASTE BOOK.

January 21, 1820.		
Arrived the Betsey from Oporto, with 8 pipes of Port Wine, consigned by James Greaves, to me, to sell on his account, S. B.		
Paid sundry charges on landing.	439	24
22d.		
Received of Lemuel Rogers, a bill of Exchange, No. 2. as per B R.	2500	
23d.		
Paid John Howe's bill, No. 12, C. B.	600	
Received of G Seaman, my half share for freight on board the ship Phebus, C. B.	400	87
24th.		
Accepted a bill drawn by Smith & Son, of London. B. P	2150	
25th.		
Received of George Sabin, for produce of the farm in Cambridge, C. B.	160	35
25th		
Paid for repairs of the ship Phebus, C. B.	130	25
26th.		
Sold to Smith & Son, Port Wine, S. B.	462	
Sold to Joseph Lockwood, Port Wine. S. B.	772	14
Received cash of George Murray's bill. No. 12, C. B	1300	
27th.		
Shipped on board the Henry, Talbot, master, for Jamaica, sundry goods for sundry persons, as per I. B. viz.		
Spring & Jones,		
Merchandise, $1120 00		
Charges, 12 50		
Commission, 35 75		
Insurance, 37 25		
	1205	50
George Black,		
Merchandise, $1800 00		
Charges, 15 00		
Commission, 56 50		
Insurance, 62 50		
	1934	
James Broker and Shipper, each half a share,		
Merchandise, $2500 00		
Charges, 24 00		
Commission, 90 00		
Insurance, 105 48		
	2719	48

EXAMPLE II. WASTE BOOK.

January 31, 1820.		
Received a dividend at the Bank, half years interest on $12000 at 5 per cent. C. B.	300	
Sold $9000 of stock, at 81 per cent. commission ½ per cent. C. B.	7272	
Received debentures for goods shipped this month,	200	50
Received cash for debentures this month, C. B.	300	
Paid for house expenses this month, C. B.	150	52

EXAMPLE II. JOURNAL. JANUARY 1820.

(1)

D. Ja.	L. P.			$	c.
		Sundries Dr. to Stock,			
1	1	Cash in hand,		13500	
	1	Funded property $12000, at 80½ per cent,		9660	
	1	Farm in Cambridge,		4500	
	1	House in the city,		2050	
	1	Furniture,		1200	
	1	Ship Phebus, my half,		9500	
	1	Merchandise on hand,		6400	
	2	Debentures, balance due at the Custom House,		1300	
	2	Bills Receivable, bills due me, amount,		2900	
	2	James Greaves, Oporto,		1700	
	2	Cyrus Coate Bordeaux,		1560	35
	2	Lemuel Rogers, Bilboa,		1175	
	2	George Parish Naples,		2200	
				57645	35
		Stock Dr. to Sundries,			
1	2	To Bills Payable, bills accepted by me,		4262	
	2	Smith & Son, London,		2150	
	3	Gilson & Co, do.		800	
	2	Spring & Jones, Jamaica,		1666	67
	2	George Black, do.		1175	
	3	James Broker, do.		1450	58
				11504	25
		Cash Dr. to Sundries,			
		For sums received this month as per C. B.			
3	3	To Interest,	6 39		
31	3	do.	300 00		
				306	39
14	2	Bills Receivable, No. 11,	1600 00		
26	2	——— 12,	1300 00		
				2900	
23	1	Ship Phebus,		400	87
25	1	Farm in Cambridge,		160	35
31	2	Debentures,		300	
31	1	Funded property,		7272	
				11339	61

EXAMPLE II. JOURNAL. JANUARY 1820.

(2)

D. Ja.	P. L.			$	c.
		Sundries Dr. to Cash,			
		For sums paid this month as per C. B.			
2	1	Charges on merchandise, viz. per Venus,	23 00		
7		——— ——— Dolphin,	32 50		
21		——— ——— Betsey,	439 24		
27		——— ——— Henry,	51 50		
			———	546	24
3	2	Bills payable, No. 13,	1440 00		
12		——— 11,	2222 00		
23		——— 12,	600 00		
			———	4262	
25	1	Ship Phebus,		130	25
31	3	House expenses,		150	52
				5089	01
		Bills Receivable Dr. to Sundries,			
		For bills received this month, as per B. R.			
2	2	To Cyrus Coate, No. 1, due March 1,		411	
22	2	Lemuel Rogers, No. 2, Feb. 15,		2500	
				2911	
		Sundries Dr. to Bills Payable,			
		For bills accepted by me this month, as per B. P.			
15	3	To Gilson & Co. No. 1, due Feb. 1,		800	
20	3	Joseph Lockwood No. 2, —— 3,		3600	
24	2	Smith & Son, No. 3, March 1,		2150	
				6550	
		George Parish Dr. to Sundries,			
		For amount of Invoice of Sugar, per the Venus, for Naples, as per I. B. and W. B.			
2	1	To Merchandise,	395 43		
	1	Charges,	23 00		
	1	Commission,	13 46		
	2	Insurance,	8 40		
			———	440	29
		Samuel Rogers Dr. to Sundries,			
		Amount of invoice per Dolphin, for Bilboa, as per W. B. p. 2.			
	1	To Merchandise,	2000 00		
	1	Charges,	32 50		
	1	Commission,	60 75		
	2	Insurance,	34 15		
			———	2127	40

EXAMPLE II. JOURNAL. JANUARY 1820.

(3)

Ja.	L.		Merchan.	Charg.	Commis.	Insur.	$	c.
16	1	**Merchandise Dr. to Jos. Lockwood,**						
	3	To amount of goods bought of him, as per bills of parcels,					10000	87
	2	**Insurance Dr. to Globe Insur. Co.**						
		For amount of Insurance, as per I. B.						
2		Per Venus, for Naples, $8 40						
7		Per Dolphin, for Bilboa, 34 15						
27		Per Henry, for Jamaica, 205 23					247	78
	2	**Debentures Dr. to Merchandise,**						
31	1	For drawbacks received this month,					200	50
	3	**Sundries Dr. to Sales per the Betsey,**						
		For amount of 8 pipes of Port Wine, on account of James Greaves, as per S. B.						
26	2	To Smith & Son, 3 pipes at 1 month,					462	
26	3	Joseph Lockwood, 5 pipes at 2 months,					772	14
		Sundries Dr. to Sundries,						
		For amount of Invoices per the Henry, for Jamaica, as per W. B. p. 3.						
27			Merchan.	Charg.	Commis.	Insur.		
	2	Spring & Jones,	1120 00	12 50	35.75	37 25	1205	50
	2	George Black,	1800 00	15 00	56 50	62 50	1934	
	3	Adventure to Jamaica, in Co. with J. Broker, my half,	2500 00	24 00	90 00	105 48	1359	74
	3	James Broker, his half,					1359	74
		Sums,	5420 00	51 50	182 25	205 23	5858	98
	3	**Sales per the Betsey Dr. to Sundries,**						
26	1	To charges on merchandise, 439 24						
	1	Commission, 33 94						
	3	Interest, 4 03						
	2	James Greaves, for net proceeds of 8 pipes of Port Wine, as per S. B. 756 93					1234	14

ALPHABETICAL INDEX TO THE LEGER.

☞ Before attempting to balance the Leger, it must be ascertained whether the Journal has been correctly formed from the Day Book and Auxiliary Books, and whether the journal has been correctly posted into the Leger. In examining the books for this purpose, a point or some mark should be placed against the several accounts found to be correctly entered in the Journal and Leger, and this *pointing* or *marking* continued through all the accounts. It is then common to make a *trial* balance, on a separate piece of paper, before forming the account called Balance, in the Leger. If the books can be thus balanced, the several *balances* are then placed under Balance, and the work is finished.

Note. The examples here given are sufficient to illustrate the method of Double Entry. The teacher should not suffer the pupil to pass over any point, until it is well understood. If more examples should be desired, he can direct the learner to take Ex. 2. of Single Entry, and form from it the several books in Double Entry. After this has been done, the pupil should form, for himself a larger Day or Waste Book for Double Entry, and carry the account through all the forms, according to the principles taught in this system.

(1) EXAMPLE II. LEGER. JANUARY 1820.

D. Ja	J. P.		L.	$	c.	D. Ja	J. P.		L.	$	c.
		Dr. Stock,						**Contra . Cr.**			
1	1	To Sundries,		11504	25	1	1	By Sundries,		57645	35
		Balance,	3	47049	37			Profit and Loss,	3	908	27
				58553	62					58553	62
		Dr. Cash,						**Ca. Cr.**			
	1	To Stock,	1	13500			2	By Sundries,		5089	01
	1	Sundries,		11339	61			Balance,	3	19750	60
				24839	61					24839	61
		Dr. Funded						**Property, Cr.**			
1	1	To Stock 12000 a 80½	1	9660		31	1	By Cash 9000 at 80⅘	1	7272	
31		Profit and Loss,	3	27				Balance 3000 a 80½	3	2415	
				9687						9687	
		Dr. Farm in						**Cambridge, Cr.**			
1	1	To Stock,	1	4500		25	1	By Cash,	1	160	35
25		Profit and Loss,	3	160	35			Balance,	3	4500	
				4660	35					4660	35
		Dr House in the						**City, Cr.**			
1	1	To Stock,	1	2050				By Balance,	3	2050	
		Dr. Furniture,						**Ca. Cr.**			
1		To Stock,	1	1200				By Balance,	3	1200	
		Dr. Ship						**Phebus, Cr.**			
1	1	To Stock,	1	9500		23	1	By Cash,	1	400	87
25	2	Cash,	1	130	25			Balance,	3	9500	
		Profit and Loss,	3	270	62						
				9900	87					9900	87
		Dr. Merchandise,						**Ca. Cr.**			
1	1	To Stock,	1	6400		2	2	By Geo. Parish,	2	395	43
16	3	Jos. Lockwood,	3	10000	87		2	Lemuel Rogers,	2	2000	
							3	Sundries,		5420	
				16400	87		3	Debentures,	2	200	50
								Balance,	3	8384	94
										16400	87
		Dr. Charges on						**Merchandise, Cr.**			
	2	To Cash,	1	546	24	2	2	By George Parish	2	23	
						7	2	Lemuel Rogers,	2	32	50
						27	3	Sundries,		51	50
						21	3	Sales per Betsey,	3	539	24
										546	24
		Dr. Commission.						**Ca. Cr.**			
		To Profit and Loss,		290	40	2	2	By George Parish,	2	13	46
						7	2	Lemuel Rogers,	2	60	75
						27	3	Sundries,		182	25
						21	3	Sales per Betsey,	3	33	94
										290	40

EXAMPLE II. LEGER. JANUARY 1820. (2)

D. Ja	J. P.		L.	$	c.	D. Ja	J. P.		L.	$	c.
		Dr. Insurance,						Ca. Cr.			
	3	To Globe Insurance Company,	3	247	78	2	2	By George Parish,	2	8	40
						7	2	Lemuel Rogers,	2	34	15
						27	3	Sundries,		205	23
										247	78
		Dr. Bills						Receivable, Cr.			
	1	To Stock,	1	2900			1	By Cash,	1	2900	
	2	Sundries,		2911				Balance,	3	2911	
				5811						5811	
		Dr. Bills						Payable, Cr.			
1	1	To Cash,	1	4262		1	1	By Stock,	1	4262	
		Balance,	3	6550			2	Sundries,		6550	
				10812						10812	
		Dr. Debentures,						Ca. Cr.			
1	1	To Stock,	1	1300		31	1	By Cash,	1	300	
31	3	Merchandise,	1	200	50			Balance,	3	1200	50
				1500	50					1500	50
		Dr. James						Greaves, Cr.			
1	1	To Stock,	1	1700		26	3	To Sales per Betsey,	3	756	93
								Balance,	3	943	07
										1700	
		Dr. Cyrus						Coate, Cr.			
1	1	To Stock,	1	1560	35	2	2	By Bills Receivable,	2	411	
								Balance,	3	1149	35
										1560	35
		Dr. Lemuel						Rogers, Cr.			
1	1	To Stock,	1	1175		22	2	By Bills Receivable,	2	2500	
7	2	Sundries,		2127	40			Balance,	3	802	40
				3302	40					3302	40
		Dr. George						Parish, Cr.			
1	1	To Stock,	1	2200				By Balance,	3	2640	29
2	2	Sundries,		440	29						
				2640	29						
		Dr. Smith &						Son, Cr.			
24	2	To Bills Payable,	2	2150		1	1	By Stock,	1	2150	
26	3	Sales per Betsey,	3	462				Balance,	3	462	
				2612						2612	
		Dr. Spring &						Jones, Cr.			
27		To Sundries,		1205	50	1	1	By Stock,	1	1666	67
		Balance,	3	461	17						
				1666	67						
		Dr. George						Black, Cr.			
27	3	To Sundries,		1934		1	1	By Stock,	1	1175	
								Balance,	3	759	
										1934	

(3) EXAMPLE II. LEGER. JANUARY 1820.

D. Ja	J. P.	Dr.	L.	$	c.	D. Ja	J. P.	Cr.	L.	$	c.
		Dr. James						Broker, Cr.			
27	3	To Adventure to				1	1	By Stock,	1	1450	58
		Jamaica,	3	1359	74						
		Balance,	3	90	84						
				1450	58						
		Dr. Adventure						to Jamaica, Cr.			
27	3	To Sundries,		1359	74			By Balance,	3	1359	74
		Dr. Joseph				16	3	Lockwood, Cr.			
20	3	To Bills Payable,	2	3600				By Merchandise,	1	10000	87
26	3	Port Wine,		772	14						
		Balance,	3	5628	73						
				10000	87						
		Dr. Sales per the				26	3	Betsey, Cr.			
26	3	To Sundries,		1232	21			By Sundries,	1	1232	21
		Dr. Gilson &						Co. Cr.			
1	1	To Bills Payable,	2	800		15	2	By Stock,	1	800	
		Dr. Interest,						Ca. Cr.			
31		Profit and Loss,	3	310	42		1	By Cash,	1	306	39
								Sales per Betsey,	3	4	03
										310	42
		Dr. Globe Insur-						ance Co. Cr.			
		To Balance,	3	247	78			By Insurance,	2	247	78
		Dr. House						Expenses, Cr.			
31	1	To Cash,	1	150	52	31		By Profit & Loss,	3	150	52
		Dr. Profit and						Loss, Cr.			
31	2	To House Ex-						By Funded Prop.	1	27	
		penses,	3	150	52			Farm in Cam.	1	160	35
		To Stock,	1	908	27			Ship Phebus,	1	270	62
								Commission,	1	290	40
				1058	79			Interest,	3	310	42
										1058	79
		Dr. Balance,						Ca. Cr.			
		To Cash,	1	19750	60			By Bills Payable,	2	6550	
		FundedProperty,	1	2415				Spring & Jones,	2	461	17
		Farm in Cam.	1	4500				James Broker,	3	90	84
		House,	1	2050				Jos. Lockwood,	3	5628	73
		Furniture,	1	1200				Globe Insur. Co.	3	247	78
		Ship Phebus,	1	9500				Stock,	1	47049	37
		Merchandise,	1	8384	94						
		Bills Receivable.	2	2911						60027	89
		Debentures,	2	1200	50						
		James Greaves,	2	943	07						
		Cyrus Coate,	2	1149	35						
		Lemuel Rogers,	2	802	40						
		George Parish,	2	2640	29						
		Smith & Son,	2	462							
		George Black,	2	759							
		Adven. to Jamai.	3	1359	74						
				60027	89						

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